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THE MULATU NUMBERS, THE NEWLY CELEBRATED NUMBERS OF PURE MATHEMATICS

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ABSTRACT

The Mulatu numbers are sequences of numbers of the form 4, 1, 5, 6, 11, 17, 28, 45, …. The numbers have wonderful and amazing properties and patterns. In mathematical terms, it is defined by the following recurrence relation:

$$M_n = \begin{cases} 4 & \text{if } n = 0; \\ 1 & \text{if } n = 1; \\ M_{n-1} + M_{n-2} & \text{if } n > 1. \end{cases}$$

The first number of the sequence is 4, the second number is 1, and each subsequent number is equal to the sum of the previous two numbers of the sequence itself. That is, after two starting values, each number is the sum of the two preceding numbers. In this paper, we give summary of some important properties and patterns of Mulatu numbers. Its relations to the Fibonacci and Lucas numbers are also investigated.

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KEYWORDS

Mulatu numbers, Fibonacci Numbers and Lucas Numbers.

1. Introductions and Background

The Mulatu numbers are a sequence of numbers recently introduced by Mulatu Lemma, an Ethiopian Mathematician and Professor of Mathematics at Savannah State University, Savannah, Georgia, USA. The numbers are closely related to both Fibonacci and Lucas Numbers in its properties and patterns. Below we give the First 20 Mulatu, Fibonacci and Lucas numbers.

First 20 Mulatu, Fibonacci and Lucas Numbers (Tables 1 & 2).

Table 1

n :	0	1	2	3	4	5	6	7	8	9	10	11
M_n :	4	1	5	6	11	17	28	45	73	118	191	309
F_n :	0	1	1	2	3	5	8	13	21	34	55	89
L_n :	2	1	3	4	7	11	18	29	47	76	123	199

Table 2

n :	12	13	14	15	16	17	18	19	20
M_n :	500	809	1309	2118	3427	5545	8972	14517	23489
F_n :	144	233	377	610	987	1597	3584	4181	6765
L_n :	322	521	843	1364	2207	3571	5778	9349	15127

Background materials

In this paper M, L and F denote the Mulatu, Lucas and Fibonacci numbers respectively. We prove the theorems using induction only. The theorems are not new but the proofs are new.

The Fibonacci numbers are sequences of numbers of the form: 0, 1, 1, 2, 3, 5, 8, 13, …. In mathematical terms, it is defined by the following recurrence relation:

$F_n = F_{n-1} + F_{n-2}$ with $F_1 = F_2 = 1$ and $F_0 = 0$.

The first number of the sequence is 0, the second number is 1, and each subsequent number is equal to the sum of the previous two numbers of the sequence itself.

The Lucas numbers are sequences of numbers of the form: 2, 1, 3, 4, 7, 11, 28, 29, 47, 76, 123, ….

In mathematical terms, it is defined by the following mathematical recurrence relation

$$L_0 = 2,$$

$$L1 = 1,$$

$$L_{n+1} = L_n + L_{n-1} \text{ for } n > 1.$$

Each subsequent number is equal to the sum of the previous two numbers of the sequence itself. The following identities of *F* and *L* will be used in this paper.

(1) $L_n = F_{n-1} + F_{n+1}$

(2) $F_{n+1} = F_n + F_{n-1}$

(3) $F_{2n} = F_n L_n$

(4) $L_{2n} = F_n + 2F_{n-1}$

(5) $F_n = \dfrac{L_{n+1} + L_{n-1}}{5}$

(6) $L_{n+1} = L_n + L_{n-1}$

(7) $F_{n+k} = F_{n-1}F_k + F_nF_{k+1}$

(8) $5F^2{}_n - L^2{}_n = 4(-1)^{n+1}$

(9) $L_{n+m} = \dfrac{5F_nF_m + L_nL_m}{2}$

(10) $M_{n+k} = F_{n-1}M_k + M_nF_{k+1}$

2. The Main Results

Theorem 1(Partial sum of Mulatu numbers). If each $M_{i_i}(i \geq 0)$ are Mulatu numbers, then

$$\sum_{k=0}^{n} M_k = M_{n+2} - 1.$$

Proof: We use indication n

1. When n = 1, the formula is true as $M_0 + M_1 = 4 + 1 = 5 = M_3 - 1$.
2. Assume the formula is true for n=p
3. Verify the formula for n=p+1

$$\sum_{k=0}^{p+1} M_k = \sum_{k=0}^{p} M_k + M_{p+1}$$

$$= M_{p+2} - 1 + M_{p+1}$$

$$= M_{p+1} + M_{p+2} - 1$$

$$= M_{p+3} - 1$$

Hence the Theorem follows by induction

Theorem 2(Partial sum of the Mulatu numbers with odd indices). If each M_i $(i \geq 0)$ are Mulatu numbers, then

$$\sum_{\boldsymbol{k=0}}^{\boldsymbol{n}} \mathrm{M}_{2\mathrm{k}+1} = M_{2n+2} - 4$$

Proof: We use indication on n.

1. When n=1, the formula for n=1 is true $M_1 + M_3 = M_4 - 4$=7
2. Assume the formula is true for n=p.
3. Verify the formula for n=p+1

$$\sum_{k=0}^{p+1} M_{2k+1} = \sum_{k=0}^{p} M_{2k+1} + M_{2p+3}$$

$$= M_{2p+2} - 4 + M_{2p+3}$$

$$= M_{2p+4} - 4$$

Hence, the Theorem follows by induction.

Theorem 3(Partial sum of the Mulatu numbers with even indices). If each M_i $(i \geq 0)$ are Mulatu numbers, then

$$\sum_{\mathrm{k}=0}^{\mathrm{n}} M_{2k} = M_{2n+1} + 3.$$

Proof: We use indication on n.

1. When n=1, the formula for n=1 is true as $M_0 + M_2 = M_3 + 3 = 9$
2. Assume the formula is true for n=p

3. Verify the formula for n=p+1. Note that

$$\sum_{k=0}^{p+1} M_{2k} = \sum_{k=0}^{p} M_{2k} + M_{2p+2}$$

$$= M_{2p+1} + 3 + M_{2p+2}$$

$$= M_{2p+3} + 3$$

Hence by induction, the Theorem follows.

Theorem 4(Expressing M in terms of F). Let M_n and F_n be any Mulatu and Fibonacci numbers. Then we have

$$M_n = F_{n-3} + F_{n-1} + F_{n+2}.$$

Proof. We use induction on n.

(1) When n = 0, the formula is true as $M_0 = F_{-3} + F_{-1} + F_2$ and using $F_{-n} = (-1)^{n+1}F_n$, we have 4 = 2 + 1 +1 = 4.

(2) Assume the formula is true for n = 1, 2, 3, … , *k*-1, *k*.

(3) Verify the formula for n = *k* + 1.

Note that

$$M_{k+1} = M_k + M_{k-1} = F_{k-3} + F_{k-1} + F_{k+2} + F_{k-4} + F_{k-2} + F_{k+1}$$

$$= M_k + M_{k-1} = F_{k-4} + F_{k-3} + F_{k-2} + F_{k-1} + F_{k+2} + F_{k+1}$$

$$= F_{k-2} + F_k + F_{k+3}$$

Hence by Induction, the theorem follows.

Theorem 5. $L_n = \frac{M_n+F_n}{2}$

Proof: We use indication on n.

1. When n=1, the formula is true as $L_1 = \frac{M_1+F_1}{2} = \frac{1+1}{2} = 1$

2. Assume the formula is true for n=1,2,3… k-1, k
3. Verify the formula for n=k+1.

$$L_{k+1} = L_k + L_{k-1}$$

$$= \frac{M_k + F_k}{2} + \frac{M_{k-1} + F_{k-1}}{2}$$

$$= \frac{M_k + M_{k-1} + F_k + F_{k-1}}{2}$$

$$= \frac{M_{k+1} + F_{k+1}}{2}$$

Hence by induction, the Theorem follows.

Theorem 6. $M_n = 4F_{n+1} - 3F_n$

Proof: We use indication on n.

1. When n=1, the formula for n=1 is true as $M_1 = 4F_2 - 3F_1 = 1$
2. Assume the formula is true for n=k
3. Verify the formula for n=k+1. Note that

$$M_{k+1} = M_k + M_{k-1} = (4F_{k+1} - 3F_k) + (F_k - 3F_{k-1})$$

$$= 4(4F_{k+1} + F_k) - 3(F_k + F_{k-1})$$

$$= 4F_{k+2} - 3F_{k+1}$$

Hence by induction, the Theorem follows.

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