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MODELING AND ANALYSIS OF THE INTERACTION OF POLICE AND CRIMINAL POPULATIONS: A COMPETING SPECIES MODEL

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ABSTRACT

The rise of the criminal population in the United States has brought concern, debate, and contention to the modern world. The strategies of criminals are national and are no longer concentrated in a particular location. In this paper, we present a dynamical model of the interaction between criminal and police populations. The formulation is based on models of interactions between competing species type dynamics. An exploration of the long-term dynamics and stability of homogeneous equilibrium solutions and their stability is given. The paper is given in multiple parts. Part two presents the mathematical model and analyzes the situation for current population levels. Part three analyzes the situation when an additional number of members are introduced into the criminal population. Part four analyzes the scenario where the criminal population is reduced. Part five presents conclusions.

KEYWORDS

Criminal, Police, competing species model, equilibrium solutions, stability at equilibrium solutions.

Mathematica subject classification**:** 62J12, 62G99

Computing Classification: I.4

1- Introduction

Criminals are not a new phenomenon. However, there is a marked and exponential increase in the growth of criminals', which wreak havoc to native citizens. These criminals affect all areas of the economy, markets, and political and social policies. In addition, the strength and presence of criminal organization, activities create emigration issues. In particular, the rise of the criminals has caused the largest domestic crisis since the dawn of the Second World War. Consequently, countries are faced with extremely difficult, complex, and contentious political and social decisions on the addressing the criminal problem.

The acceptance of criminals provides a Trojan horse of issues, namely, violence against citizens, criminal conflict, extortion, protection, and their involvement in criminal activities. Despite these impending threats, there is not much literature that takes a dynamical systems approach to understanding the spread of criminals, at a population level. Our primary objective is to bridge the gap.

In our framework, we let C represent the criminal population. Of course, there will be some criminals that get clean and will be productive citizens. This populations called police and is denoted by P: P can be viewed as the total police population of an area. This paper is a first step in providing a mathematical modeling framework to study the evolution and interaction between this criminal and police population. The criminal population is modeled by standard population growth models

Also, we consider the addition to the criminal population of increased number of criminals this paper is organized as follows. In section 2, we develop and analyze the time-dependent autonomous criminal user ordinary differential equation (ODE) model. We examine the equilibrium solutions, the stability of the equilibrium solutions and investigate the dynamics numerically for the current population levels. In section 3, we consider the situation when more criminals are introduced into the system. We examine the equilibrium solutions, the stability of the equilibrium solutions and investigate the dynamics numerically for this situation also. In section 4, we analyze the scenario where the criminal population is decreased. In section 5 we present our conclusions.

2 Police-Criminal(P,C) ODE Model

Consider the mathematical model

$$C = a_1P(1+d_1P) – a_{NR}CP/(1+d_2P) – b_1C^2 = 0 = f_P(C, P) \quad (1)$$

$$P = a_2P/(1+d_3P) - a_{NR}CP/(1+d_2C) – b_2P^2 = 0 = f_C(C, P) \quad (2)$$

The populations C (t) and P (t) represent the populations of the criminal and police populations. New criminal are slowly coming into the criminal population. The parameters are all assumed to be positive and their descriptions are given in Table 1a.

Table 1a: List of parameters used in the differential equation model

Symbols	**Meaning**
a_1	Growth rate of the criminal population
a_2	Growth rate of the police population
b_1	Population loss in C due to intra-species competition and natural mortality
b_2	Population loss in P due to intra-species competition and natural mortality
a_{NR}	Maximum per capita loss in C due to recruitment by police
d_1	Measures the effectiveness of P in disrupting the growth rate of C
d_2	Measures the resilience of P to recruitment strategies by C
d_3	Measures the effectiveness of P in the decline of criminals

In the case of $d_i = b_i = 0$, the mathematical model becomes similar to the competing species model. The parameters d_i influence the carrying capacity of the individual populations. Or instance, if $d_1 >> 1$ then the growth rate of C is reduced. This is interpreted as: a highly effective police population can greatly hinder the growth rate of C. The growth rate of the criminal population depends on the successful recruitment from the neutral population. Notice, that if d_2>> 1 then the recruitment by P is small, Also, if d_3>> 1, new criminals members are introduced into the criminal population at a slower rate. The values chosen for the variables in this model are listed in Table 1b.

Table1b: Values of parameters

a_1	a_2	b_1	b_2	a_{NR}	d_1	d_2	d_3
2	2	0.5	0.5	2	2	2	3

2.1 Police Criminal (P, C) ODE Model

Consider the mathematical model

$$f_C(C, P) = (a_1/(1+d_3C) – a_{NR}P/(1+d_2P) – b_1P) P = 0 \quad (3)$$

$$f_P(C, P) = (a_2/(1+d_1P)) -a_{NR}P/(1+d_2P) –(b_2C)) C= 0 \quad (4)$$

Since this system is nonlinear, the first step is linearization using the Jacobian.

The Jacobian for this system is defined as

$$J = \begin{vmatrix} \partial f_p/\partial C & \partial f_p/\partial P \\ \partial f_C/\partial C & \partial f_C/\partial P \end{vmatrix}$$

The partial derivatives are:,

$$\partial P/\partial P = a_1/(1+d_1C) – a_{nr}C(1+d_2P) – a_{nr}d_2CP – 2b_1P$$

$$\partial P/\partial C = -a_1d_1P/(1+d_1C)^2 – a_{nr}P/(1+d_2P)$$

$$\partial C/\partial P = -a_2d_3C/(1+d_1C)^2 –a_{nr}P/(1+d_2P)$$

$$\partial C/\partial C = a_2/(1+d_3P) - a_{nr}C(1+d_2P) – an_rd_2CP – 2b_2C$$

Using the values in table for the parameters, the Jacobian becomes.

$$J = \begin{vmatrix} 2/(1+d_1C) – 2C(1+2P) – 4CP – P & P/(1+2C)^2 – 2P/(1+2P) \\ -4D/(1+2D)^2 –2P/(1+2P) & 2/(1+2P) - 2_rD(1+2P) – 4CP –C \end{vmatrix}$$

2.2 Equilibrium Points

Using the Maple CAS from Maplesoft, we obtained the following real valued equilibrium points

{C=0.,P=0.},
{C=4.,P=0.},
{C=0.,P=4.},
{C=0.6319394087,P=0.4891955799},
{C=-0.6082709305,P=-0.4325627635},
{C=0.1197573734,P=-0.4345884397},
{C=-2.874675564,P=-3.074988235}

2.1 Analyzing equilibrium points for stability

In this section we use the equilibrium points to generate the eigen values for the system and establish whether the equilibrium point is stable or unstable.

2.2 Summarization

Table 2 summarizes the results for the current population levels.

Table 2 – Results for Current Population Levels

Equilibrium Point	Eigen Values	Node Type	Stability
{C=0. P=0.},	2., 2	Repelling	Unstable
{C=4. P=0.},	-10., -7.77777777800000	Attracting	Unstable
{C=0. P=4.},	-4.91963492175763, 3.1418571439576	Saddle	Unstable
{C=0.6319394087, P=0.4891955799},	-.558388551229047, -2.41920366377095	Attracting	Unstable
{C=-0.6082709305, P=-0.4325627635},	-40.4097992538318, 62.9778396954318	Saddle	Unstable
{C=0.1197573734, P=-0.4345884397},	1.84772961863995, -.391190751339948	Attracting Spiral	Asymptotically Stable
{C=-2.874675564, P=-3.074988235}	-33.2330107120919, -34.5763031979081	Repelling	Unstable

3. Growth of the Criminal Population

In this section, we consider the situation where 25% more criminals are added to the criminal population. The mathematical model now becomes

$$F_P(P, C) = (a_1/(1+d_{1(}C(1.25)) - a_{NR(}C(1.25)P/(1+d_2P) - b_1P) P = 0 \quad (3)$$

$$F_C(P, C) = (a_2/(1+d_3P) - (a_{NR}P/(1+d_2P)) - b_2(C(1.25)) (C(1.25)) = 0 \quad (4)$$

Using Maple we obtain the following real valued equilibrium points:

{C = 0. P = 0.},
{C = 3.20, P = 0.},
{C = 0. P = 4.},
{C = .5055515270, P = .4891955799},
{C = -.4866167444, P = -.4325627635},
{C = 0.09580589875, P = -.4345884397},
{C = -2.299740451, P = -3.074988235},

3.1 Analyzing equilibrium points for stability

In this section we use the equilibrium points to generate the eigenvalues for the system and establish whether the equilibrium point is stable or unstable.

3.2 Summarization

Table 3 summarizes the results for an increased criminal population level.

Table 3 – Results for Increased Drug Levels

Equilibrium Point	Eigen values	Node Type	Stability
{C = 0. P = 0.},	2. 2.	Repelling	Unstable
{C = 3.200000000, P = 0.},	-7.60000000000000, -6.12972973000000	Attracting	Stable
{C = 0. P = 4.},	-4.91963492175763, 3.14185714395763	Saddle	Unstable
{C = .5055515270, P = .4891955799},	-1.98140290930199, -0.00859798059801042	Attracting	Stable
{C = -.4866167444, P = -.4325627635},	2619.70865593003, -2516.49288416003	Saddle	Unstable
{C = 0.09580589875, P = -.4345884397},	-2.28496565045305, 16.9937275912531	Saddle	Unstable
{C = -2.299740451, P = -3.074988235},	-26.5051470341675, -27.4237242158325	Attracting	Stable

4. Decline of the Criminal Population

In this section, we consider the situation where 25%are removed from the criminal population. The mathematical model now becomes

$$f_P(P, C) = (a_1/(1+d_1(C(0.75)) – a_{NR}(C(0.75))/(1+d_2P) – b_1P) P = 0 \quad (7)$$

$$f_C(P, C) = -2C/(1+3P)^2 - a_{NR}C/(1+d_2P) – b2(C(0.75))) (C(0.75)) = 0 \quad (8)$$

Using the Maple CAS on (7) and (8) we obtained the following equilibrium points:

{C=0.P=0.},
{C=5.333333333,P=0.},
{C=0.P=4.},
{C=0.8425858783,P=0.4891955799},
{C=-0.8110279073,P=-0.4325627635},
{C=0.1596764979,P=-0.4345884397},
{C=-3.832900753,P=-3.074988235},

4.1 Analyzing equilibrium points for stability

In this section we use the equilibrium points to generate the eigenvalues for the system and establish whether the equilibrium point is stable or unstable.

4.2 Summarization

Table 3 summarizes the results for an increased criminal population level.

Table 4 – Results for Decreased Drug Population Levels

Equilibrium Point	Eigen values	Node Type	Stability
{C=0. P=0.},	2., 2.	Repelling	Unstable
{C=5.333333333, P=0.},	-14., -10.49523810000	Attracting	Stable
{C=0. P=4.},	-4.91963492175763, 3.14185714395763	Saddle	Unstable
{C=0.8425858783, P=0.4891955799},	-1.42907252092108, -3.14804742107892	Attracting	Stable
{C=-0.8110279073, P=-0.4325627635},	1.36941934968578, 32.7338864003142	Repelling	Unstable
{C=0.1596764979, P=-0.4345884397},	-3.18376470408883, 15.9353268839888	Saddle	Unstable
{C=-3.832900753, P=-3.074988235},	-46.4672106267829, -44.5793122632171	Attracting	Stable

5. Conclusions

In this paper we modeled and analyzed the interaction of police and criminal populations. A comparison of the results in Table 2, and Table 3 and Table 4. Table 2 shows that any criminals make the system unstable. More criminals contribute to instability while a decreased criminal population reduces instabilities.

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