INTERNATIONAL JOURNAL OF MATHEMATICS
WWW.GPHJOURNAL.ORG
Research Article

Manuscript ID: #0377

The Mulatu Numbers in Actions

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ABSTRACT

The Mulatu numbers were introduced in [1]. The numbers are sequences of numbers of the form: **4, 1, 5,6,11,17,28,45…** The numbers have wonderful and amazing properties and patterns.

In mathematical terms, the sequence of the Mulatu numbers is defined by the following recurrence relation:

$$M_n := \begin{cases} 4 & if \ \ n = 0; \\ 1 & if \ \ n = 1; \\ M_{n-1} + M_{n-2} & if \ \ n > 1. \end{cases}$$

The double Angel Formulas for Fibonacci and Lucas numbers are given by the following formulas respectively**.**

$$(1)\, F_{2n} = F_n L_n \ \textbf{and} \ (2)\, L_{2n} = \frac{5F^2{}_n + L^2{}_n}{2}$$

Since both the Fibonacci and Lucas numbers have double angle Formulas, It is natural to ask if such formula exist for Mulatu Numbers. The answer is affirmative and produces the following paper.

2000 Mathematical Subject Classification: 11

KEYWORDS

Mulatu numbers, Mulatu sequences, Fibonacci numbers, Lucas numbers, Fibonacci sequences, and Lucas sequences.

1. Introduction and Background

In his As given in [1], the Mulatu numbers are a sequence of numbers recently introduced by Mulatu Lemma, Professor of Mathematics at Savannah State University, Savannah, Georgia, USA. The Mulatu sequence has wealthy mathematical properties and patterns like the two celebrity sequences of Fibonacci and Lucas.

In this paper, more interesting relationships of the Mulatu numbers to the Fibonacci and Lucas numbers will be presented.

Here are the First 21 Mulatu, Fibonacci, and Lucas numbers for quick reference.

Remark 1: Throughout this paper M, F, and L stand for Mulatu numbers, Fibonacci numbers, and Lucas number respectively.

The following well-known identities of Mulatu numbers, Fibonacci numbers, and Lucas numbers are required in this paper and hereby listed for quick reference.

(1) $L_n = F_{n-1} + F_{n+1}$

(2) $F_{n+1} = F_n + F_{n-1}$

(3) $F_{2n} = F_n L_n$

(4) $L_{2n} = F_n + 2F_{n-1}$

(5) $F_n = \dfrac{L_{n+1} + L_{n-1}}{5}$

(6) $L_{n+1} = L_n + L_{n-1}$

(7) $F_{n+k} = F_{n-1}F_k + F_n F_{k+1}$

(8) $5F^2{}_n - L^2{}_n = 4(-1)^{n+1}$

(9) $L_{n+m} = \dfrac{5F_n F_m + L_n L_m}{2}$

(10) $M_{n+k} = F_{n-1} M_k + M_{k+1} F_n$

The Main Results

We will state the following theorem proved in [1] as proposition 1 and use it.

Proposition 1. $M_n = F_{n-3} + F_{n-1} + F_{n+2}$

Theorem 1: The following are equivalent.

(1) M_n

(2) $F_{n-3} + F_{n-1} + F_{n+2}$

(3) $L_n + 2F_{n-1}$

(4) $F_n + 4F_{n-1}$

(5) $4F_{n+1} - 3F_n$

Proof: We will show that $(1) \Rightarrow (2) \Rightarrow (3) \Rightarrow (4) \Rightarrow (5) \Rightarrow (1)$

(i) $(1) \Rightarrow (2)$ follows by Proposition 1.

(ii) $(2)) \Rightarrow (3)$ follows as shown:

$$F_{n-3} + F_{n-1} + F_{n+2} = F_{n-3} + F_{n-1} + F_{n+1} + F_n$$

$$= F_{n-3} + F_{n-1} + F_{n+1} + F_{n-1} + F_{n-2}$$

$$= F_{n-1} - F_{n-2} + F_{n-1} + F_{n+1} + F_{n-1} + F_{n-2}$$

$$= 2F_{n-1} + L_n$$

(iii)$(3) \Rightarrow (4)$ follows as shown:

$$L_n + 2F_{n-1} = F_{n+1} + F_{n-1} + 2F_{n-1}$$

$$= F_n + F_{n-1} + F_{n-1} + 2F_{n-1}$$

$$= F_n + 4F_{n-1}$$

(iv) $(4) \Rightarrow (5)$ follows as shown:

$$F_n + 4F_{n-1} = F_n + 4(F_{n+1} - F_n)$$

$$= 4F_{n+1} - 3F_n$$

(v) (5) $\Rightarrow$(1) follows as shown:

$$4F_{n+1}-3F_n = 4F_{n+1}-3(F_{n+1}-F_{n-1}) = F_{n+1}+3F_{n-1} = F_{n+1}+F_{n-1}+F_{n-1}+F_{n-1}$$

$$= F_{n+1}+(F_n - F_{n-2})+F_{n-1}+F_{n-3}+F_{n-2} = F_{n+1}+F_n - F_{n-2}+F_{n-1}+F_{n-3}+F_{n-2}$$

$= F_{n+2}+F_{n-1}+F_{n-3} = M_n$ by Proposition 1 and hence) (5) $\Rightarrow$(1).Thus the theorem is proved.

Theorem 2.

$$L_{2n}+2F_{2n-1} = M_nL_n + 5F^2{}_n - L^2{}_n$$

Note that Using (9) above, we have $L_{2n}+2F_{2n-1} = \dfrac{5F^2{}_n + L^2{}_n}{2} + 2F^2{}_n + 2F^2{}_{n-1} =$ $\dfrac{9F^2{}_n + L^2{}_n + 4F^2{}_{n-1}}{2}$.

Now Observe that

$$\frac{9F^2{}_n + L^2{}_n + 4F^2{}_{n-1}}{2} = \frac{5F^2{}_n + L^2{}_n}{2} + 2F^2{}_n + 2F^2{}_{n-1}$$

$$= L_{2n} + 2F_{2n-1} = F_{2n} + 2F_{2n-1} + 2F_{2n-1} = F_nL_n + 4F_{2n-1}$$

$$= F_n(F_n + 2F_{n-1}) + 4F^2{}_{n-1} + 4F^2{}_n = 5F^2{}_n + 4F^2{}_{n-1} + 2F_{n-1}F_n =$$

$$=(F^2{}_n + 8F^2{}_{n-1} + 6F_{n-1}F_n) + 5F^2{}_n - (F^2{}_n + 4F_{n-1}F_n + 4F^2{}_{n-1})$$

$$= (F_n + 4F_{n-1})(F_n + 2F_{n-1}) + 5F^2{}_n - (F_n + 2F_{n-1})^2$$

$$= M_nL_n + 5F^2{}_n - L^2{}_n$$

Theorem3. $M_{2n} = M_nL_n + 5F^2{}_n - L^2{}_n$

Note that

$$M_nL_n + 5F^2{}_n - L^2{}_n = M_nL_n - L^2{}_n + 5F^2{}_n$$

$$= M_nL_n - (F_n + 2F_{n-1})^2 + 5F^2{}_n$$

$$= M_nL_n - (F^2{}_n + 4F_{n-1}F_n + 4F^2{}_{n-1}) + 5F^2{}_n$$

$$= M_nL_n - (F_n + F_{n-1})(F_n + 4F_{n-1}) + F_nF_{n-1} + 5F^2{}_n$$

$$= M_nL_n - F_{n+1}M_n + F_nF_{n-1} + 5F^2{}_n$$

$$= M_n(L_n - F_{n+1}) + F_n(F_{n-1} + 5F_n)$$

$$= M_nF_{n-1} + F_nM_{n+1}$$

$= M_{n+n} = M_{2n}$, by (10) above

Theorem 4.

$M_{2n} = M_{2n} = M_nL_n + 5F^2{}_n - L^2{}_n = M_nL_n + + 4(-1)^{n+1}$, using (8) above.Hence, the theorem follows.

Theorem5.

(a) If M_n is divisible by 2, then $M^2{}_{n+1} - M^2{}_{n-1}$ is divisible by 4

(b) If M_n is divisible by 3, then $M^3{}_{n+1} - M^3{}_{n-1}$ is divisible by 9.

Proof: Note that: Using $M_{n+1} = (M_n + M_{n-1})$, we have:

(a) $M^2{}_{n+1} - M^2{}_{n-1}$
$= (M_{n+1} - M_{n-1})(M_{n+1} + M_{n-1}) = M_n(M_n + M_{n-1} + M_{n-1}) = M^2{}_n + 2M_nM_{n-1}$.

Now it is easy to see that if M_n is divisible by 2, then $M^2{}_{n+1} - M^2{}_{n-1}$ is divisible by 4

(b) $M^3{}_{n+1} - M^3{}_{n-1} = (M_{n+1} - M_{n-1})(M^2{}_{n+1} + M_nM_{n-1} + M^2{}_{n-1})$

$$= M_n(M^2{}_{n+1} + M_{n+1}M_{n-1} + M^2{}_{n-1})$$

$$= M_n((M_n + M_{n-1})^2 + M_{n-1}(M_n + M_{n-1}) + M^2{}_{n-1})$$

$$= M_n(M^2{}_n + 3M_nM_{n-1} + 3M^2{}_{n-1})$$

$$= M^3{}_n + 3M^2{}_n M_{n-1} + 3M_n M^3{}_{n-1}$$

Hence M_n is divisible by 3 $\Rightarrow M^3{}_{n+1}$ - $M^3{}_{n-1}$ is divisible by 9.

Notable Honor and Dedication

This interesting paper is in an honor of the2021 Black History Month and is also notable dedicated Professor Darrell Deloach who recently passed away. Professor Deloach was an outstanding professor of the SSU Math Department and he is always remembered. Let him rest in peace.

Acknowledgements

Special recognition to**:**

1. Agegnehu Atena
2. Samuel Dolo
3. Jonathan Lambright
4. Shalonda Millidge
5. Tamara Gray
6. Abyssinia Mulatu
7. Alex Lambright
8. Jordan Brown
9. Jonathan Millidge
10. Michael Tessema
11. Gelila Aseffa

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