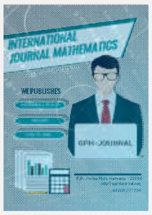

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Research Paper

Modeling and Analysis of the Interaction of Neutral and Gang Populations: A Competing Species Model

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Abstract:
The rise of the gang population in the United States has brought concern, debate, and contention to the modern world. The strategies of gangs are local and are concentrated in major metropolitan areas. In this paper, we present a dynamical model of the interaction between gang and native populations. The formulation is based on models of interactions between competing species[3] type dynamics. An exploration of the long-term dynamics and stability of homogeneous equilibrium solutions and their stability is given. The paper is given in multiple parts. Part two presents the mathematical model. Part three analyzes the current populations. Part four analyzes the situation when an additional number of gang members are introduced into the gang population. Part five analyzes the scenario where there is a decline in the gang population. Part six presents conclusions

Keywords: Gangs, competing species model, equilibrium solutions, stability at equilibrium solutions.

Mathematics subject classification: 62J12, 62G99

Computing Classification: I.4

1. Introduction:

Gangs are not a new phenomenon. However, there is a marked and exponential increase in the growth of gangs. Gangs wreak havoc to native citizens. These gangs affect many areas of the economy, markets, and political and social policies. Consequently, major metropolitan areas are faced with extremely difficult, complex, and contentious political and social decisions in addressing the gang problem.

The acceptance of gangs provides a Trojan horse of issues, namely, violence against citizens, inter-gang conflict, extortion, protection, and their involvement in drug activities. Despite these impending threats, there is not much literature that takes a dynamical systems approach to understanding the spread of undocumented aliens, at a population level. Our primary objective is to bridge the gap.

In our framework, we let G represent the gang population. Of course, there will be some gangs that are peaceful and will be productive citizens. This native population called neutral and is denoted by N: N can be viewed as the total native population of a metropolitan area. This paper is a first step in providing a mathematical modeling framework to study the evolution and interaction between this gangs and native population. The gang population is modeled by standard population growth models

Also, we consider the addition to the gang population of increased admission of gang members. The paper is organized as follows. In section 2, we develop and analyze the time-dependent autonomous gang ordinary differential equation (ODE) model. In section 3, we consider the situation when more gang members are introduced into the system. We examine the equilibrium solutions, the stability of the equilibrium solutions and investigate the dynamics numerically for this situation. .In section 4 we analyze the scenario where the gang population declines. In section 5 we present our conclusions based on the analysis in sections 2, 3, 4, and 5.

2. Neutral Gang (N,G) ODE Model

Consider the mathematical model

$$N = a_1N/(1+d_1G) – a_{NR}GN/(1+d_2N) – b_1N^2 = 0 = f_N(G, N)G \qquad (1)$$

$$G = a_2G/(1+d_3N) -a_{NR}GN/(1+d_2N) \quad – b_2G^2 = 0 = g_R(G, N)N \qquad (2)$$

The populations N (t) and UG (t) represent the populations of the neutral and Gang populations. New gang members are slowly coming into the gang population. The parameters are all assumed to be positive and their descriptions are given in Table 1a.

Table 1a: List of parameters used in the differential equation model

Symbols	Meaning
a_1	Growth rate of the gang population
a_2	Growth rate of the police population
b_1	Population loss in N due to intra-species competition and natural mortality
b_2	Population loss in R due to intra-species competition and natural mortality
a_{NR}	Maximum per capita loss in N due to recruitment by gangs
d_1	Measures the effectiveness of N in disrupting the growth rate of G
d_2	Measures the resilience of N to recruitment strategies by G
d_3	Measures the effectiveness of G in the growth of gangs

In the case of $d_i = b_i = 0$, the mathematical model becomes similar to the competing species model. The parameters d_i influence the carrying capacity of the individual populations. Or instance, if $d_1 >> 1$ then the growth rate of G is reduced. This is interpreted as: a highly effective radicalized population can greatly hinder the growth rate of G. The growth rate of the gang population depends on the successful recruitment from the neutral population. Notice, that if $d_2 >> 1$ then the recruitment by G is small, Also, if $d_3 >> 1$, new gang members are introduced into the gang population at a slower rate. The values chosen for the variables in this model are listed in Table 1b.

Table1b: Values of parameters

a_1	a_2	b_1	b_2	a_{NR}	d_1	d_2	d_3
2	2	0.5	0.5	2	2	2	3

3.0 Neutral, Gang Model

Consider the mathematical model

$$f_N(N, G) = (a_1/(1+d_1G*1.25) – a_{NR}N/(1+d_2N) – b_1N) N = 0 \qquad (3)$$

$$f_R(N, G) = (a_3/(1+d_3N)) -a_{NR}N/(1+d_2N) –(b_2G*1.25)) G*1.25= 0 \qquad (4)$$

Since this system is nonlinear, the first step is linearization using the Jacobian.

The Jacobian for this system is defined as

$$J = \begin{vmatrix} \partial f_N/\partial N & \partial f_N/\partial G \\ & \\ \partial f_R/\partial N & \partial f_R/\partial G \end{vmatrix}$$

The partial derivatives are:

$$\partial N/\partial N = a_1/(1+d_1G) – a_{nr}G*(1+d_2N) – a_{nr}d_2G5N – 2b_1N$$

$$\partial N/\partial G = -a_1d_1N/(1+d_1G)\text{^}2 – a_{nr}N/(1+d_2N)$$

$$\partial G/\partial N = -a_2d_3G/(1+d_1G)\text{^}2 – a_{nr}N/(1+d_2N)$$

$$\partial G/\partial G = a_2/(1+d_3N) - a_{nr}G(1+d_2N) – an_rd_2GN – 2b_2G$$

Using the values in table 1b for the parameters, the Jacobian becomes

$$J = \begin{vmatrix} 2/(1+2G) – 2G(1+2N) – 4GN – N\text{-}4N/(1+2G)\text{^}2 – 2N/(1+2N)) & \\ -4G/(1+2G)\text{^}2 – 2N/(1+2N) & - 2/(1+2N) - 2G(1+2N) – 4GN – G \end{vmatrix}$$

3.1 Equilibrium Points

Using the Maple CAS on (3) and (4) we obtained the following real valued equilibrium points:

{G = 0., N = 0.},
{G = 4., N = 0.},
{G = 0., N = 4.},
{G = .6319394087, N = .4891955799},
{G = -.6082709305, N = -.4325627635},
{G = .1197573734, N = -.4345884397},
{G = -2.874675564, N = -3.074988235},

Thus there are seven real equilibrium points for this system.

3.2 Analyzing equilibrium points for stability

In this section we use the equilibrium points to generate the eigenvalues for the system and establish whether the equilibrium point is stable or unstable. Substituting equilibrium points into the Jacobian and solving for eigenvalues, we get the results in Table 2.

Table 2 summarizes the results for the current population levels.

Table 2 – Results for Current Population Levels

Equilibrium Point	Eigen values	Node Type	Stability
(G = 0., N = 0	2, 2.	Repelling	Unstable
(G = 4., N = 0.)	-10. -7.77777777800000	Attracting	Stable
(G = 0., N = 4.)	-4.91963492175763, 3.14185714395763	Saddle	Unstable
(G = .6319394087, N = .4891955799)	-.558388551229047, -2.41920366377095	Attracting	Stable
(G = -.6082709305, N = -.4325627635)	-40.4097992538318, 62.9778396954318	Saddle	Unstable
(G = .1197573734, N = -.4345884397),	-2.62016378611131, 16.5910505440113	Saddle	Unstable
(G = -2.874675564, N = -3.074988235)	-33.2330107120919, -34.5763031979081	Attracting	Stable

4.0 Growth of the Gang Population

In this section, we consider the situation where 25% increases in gang members are added to the gang population. The mathematical model now becomes

$$f_N(N, G) = \left(a_1/(1+d_1(G*1.25)) - a_{NR}(N)/(1+d_2N) - b_1N \right)(N) = 0 \qquad (7)$$

$$f_R(N, G) = a_2/(1+d_3N) \; -(a_{NR}N(G*1.25)/(1+d_2(G*1.25)) - b_2(1.25*G)(1.25)G) = 0 \qquad (8)$$

Using the Maple CAS on (7) and (8) we obtained the following real valued equilibrium points:

{G=0.N=0.},
{G=4.N=0.},
{G=0.N=4.},
{G=0.6319394087, N=0.4891955799},
{G=-0.6082709305,N=-0.4325627635},
{G=0.1197573734,N=-0.4345884397},
{G=-2.874675564,N=-3.074988235},

4.1 Analyzing equilibrium points for stability

In this section we use the equilibrium points to generate the eigenvalues for the system and establish whether the equilibrium point is stable or unstable.

Table 3 summarizes the results for an increased gang population level.

Table 3 – Results for Increased Gang Population

Equilibrium Point	Eigen Values	Type of Node	Stability
{G=0., N=0.},	-2. 2.500	Saddle	Unstable
{G=3.20, N=0.},	-11.7831183980041, -9.49465938199590	Attracting	Stable
{G=0., N=4.},	-7.60, -6.129729730	Saddle	Unstable
{G=0.5055515270, N=0.4891955799},	-5.15842697804865, -.791233670951349	Attracting	Unstable
{G=-0.4866167444, N=-0.4325627635},	11.8478492025000+75.3302521839959*I, 11.8478492025000-75.3302521839959*I	Repelling	Unstable
{G=0.09580589875, N=-0.4345884397},	-5.20171705170115, 19.1560650757012	Saddle	Unstable
{G=-2.299740451, N=-3.074988235}	-.426456457639759, -34.1355867353602	Attracting	Stable

5.0 Decline of the Gang Population

In this section, we consider the situation where 25%decreased gang members are deleted from the gang population. The mathematical model now becomes

$$f_N(N, G) = \left(a_1/(1+d_1(G*0.75)) - a_{NR}(G(0.75))/(1+d_2N) - b_1N \right)N = 0 \qquad (9)$$

$$f_R(N, G) = \; a_3/(1+d_3N) - a_{NR}(N)/(1+d_2(N)) - b2(G*0.75)) \; (G*0.75) = 0 \qquad (10)$$

Using Maple on (7) and (8) we obtained the following real valued equilibrium points
{G=0.,N=0.},
{G=5.333333333,N=0.},
{G=0.,N=4.},
{G=0.8425858783,N=0.4891955799},
{G=-0.8110279073,N=-0.4325627635},
{G=0.1596764979,N=-0.4345884397},
{G=-3.832900753,N=-3.074988235}

5.1 Analyzing equilibrium points for stability

In this section we use the equilibrium points to generate the eigenvalues for the system and establish whether the equilibrium point is stable or unstable.

Table 3 summarizes the results for decreased gang population level.

Table 4 – Results for Decreased Gang Population Levels

Equilibrium Point	Eigen Values	Type of Node	Stability
{G=0. N=0.},	1.50, 2.	Repelling	Unstable
{G=4., N=0.},	-7.500, -5.7142857140	Attracting	Stable
{G=0. N=4.},	-4.01796567807023, 2.18463234477023	Saddle	Unstable
{G=0.6319394087, N=0.4891955799},	-.173690628143177, -1.81764011345682	Attracting	Stable
{G=-0.6082709305, N=-0.4325627635},	228.921617296507, -182.127661886507	Saddle	Unstable
{G=0.1197573734, N=-0.4345884397},	-1.48833880891855, 12.5604382597185	Saddle	Unstable
{G=-2.874675564, N=-3.074988235}	-24.8752324056236, -25.5010356543764	Attracting	Stable

6. Conclusions

In this paper we modeled and analyzed the interaction of neutral and radical populations. A comparison of the results in Table 2 indicates that the system is already unstable, and Table 3 indicates that with a increase in gang population the system becomes more unstable Table 4 indicates that with a decline in gang population the system becomes more stable,

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