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THE GEOMETRICAL CONSTRUCTION OF THE REGULAR POLYGONS USING < A RULER AND A COMPASS > & THE HOW GRAVITATIONAL - FORCE KEEPS THEM STILL.

By
Markos Georgallides

[1]Larnaca–Cyprus (Expelled from Varosha-Famagusta town occupied By the Barbaric Turks , in Aug -1974) Cyprus , Civil - Structural Engineer (NATUA) , Athens .
Email address : georgallides.marcos@cytanet.com.cy

Abstract : The Method

a). Document Article 132-REGPOIY, is The NEW Method of Constructing All Polygons and Based on Three Triangles &Three Circles{1=Apothems , 2=Constant–Differences-Squares 3=Constant-Sum-Squares}where this Mechanism of these Triangles-Circles is Drawn Step by Step with < A Ruler and a Compass only >.In Mechanism and onto Any Even-Sequence -(n , n+2)-- of Polygons , The Inbetween Odd -(n+1)- is Detected on the Two Circles and Defined by the Third one on the main circle. The Resolution of Constructibility with Ruler and a Compass for All the Regular Polygons was achieved with , the Clear Step by Step Geometrical Analysis and Proofs , while is Highlighted The Erroneous Solution at the time.

b)..In the **System** of a circle [**O , OK**] **,** of Diameters **KK`⊥ aa` = 2R** , **KK`⊥ K**$_{(E)}$, and a Chord **KA** = **KE**$_{A}$ on K$_{(E)}$ , **Represents The Fixed Twin - Couple of the Triangles** KAK`, KAE$_{A}$ **. Simultaneously** ,The Midpoint **O**$_{A}$ of side KE$_{A}$, The Isosceles triangle KAE$_{A}$, **The** Base Point D$_{A}$ of Perpendicular AD$_{A}$, and **The** Perpendicular triangles KAD$_{A}$, KAK` **Are Collectively and Constantly Related to each other**;....**It is Proved**

b1). On Triangle **KAK`** any Chord KA is related to its Angle φ_A as **KA = 2R. sin** $\boldsymbol{\varphi_A}$, and Squared |**KA**|² =**2R. h**$_{A}$, While on the Triangles KAD$_{A}$,KAE$_{A}$, sin φ_A= AD$_{A}$ / KA , and Differences |**KA**|²-|**KE**$_{A}$|² = |KA|.|**O**$_{EA}$D$_{A}$|= **k²** , where Point **O**$_{EA}$ is Midpoint of KE$_{A}$= KA.

b2). On Any 2-Even Polygon Set-[n,n+2]--of Sides KA,KB is **Defined** the Constant Apothems on circle &Archimedes 2n-Sided-Vertices Polygon from n+1**.** Midpoints $\mathbf{M_{EAB}}$,$\mathbf{M_{BB}}$ from Distance |E_A-E_B| **And** |BB`| **, and** The Golden-Ratio-Point $\mathbf{A_{2.C}}$ on Apothems ($\mathbf{r_A - r_B}$) Sector |$A_{1.A}$ , $A_{1.B}$| which Denotes Apothem $\mathbf{A_C}$ , and $\mathbf{KA_C}$ Cuts [O,OK] Circle at Point **C** such that **Side KC** IS The **ODD-Polygon`s** , &The Gravitational-Force as Stress **σ** =2Gπrf/Φ ≡ The Holding **Energy**[Fig-7-3] .

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The Prior :

a... The Unsolvability of the Three famous Ancient Greek Problems—Trisecting the Angle Doubling the Cube , Squaring the Circle and The Construction of the Regular Polygons **With a Ruler and a Compass , Stands on the Base justification** of the **Algebraic Field Theory** (**specifically Galois Theory**) and the **Theory of Constructible Numbers** , which were developed in the 19 th century .

b... The Greeks often used other Techniques (**like Conic sections** or **Mechanical tools**) to Solve these Problems , But **their self-imposed** "Euclidean" constraint with **a Ruler and a Compass** ,(straightedge and compass) made these Specific Problems impossible to solve.

c... In the Published Articles [123],[124],[126] , it is **clearly evident** that the Solution of the Ancient , Unsolved Greek Problems, using **a Ruler and a Compass [**as the constraint has been set by Euclid] , **Has Become Possible**, and is in the Critique of Both , **Human Logic Thinking and , The Artificial Intelligence** when it uses **The Path of Knowledge** to the Truths of Nature , **and which is the Dialectic Logic** of the Euclidean Geometry **.**

In article [**123**] , are given on the **2-Vectors , 3-Poles** Rotation Squares **Mechanism ,** where $\bar{\bar{E}} = \bar{\bar{M}}$ **,** such the Geometrical as the Mechanical Proof , By using the **Conjugate circles** of

Polhode and **Herpolhode** which consist the Spin of Photons and which is their motion .The frequency needed for the velocity vector $\bar{\mathbf{c}}$ **to Rotate** , is used from Kepler`s **Unit meter of Time** $\mathbf{k = f^2_e\, a^3}$, where , a = λ / 2 ,and which is **the clock** measuring the changes of motions, **In article** [**124**] , It was Noted that **When** in Photons , $\bar{\bar{\mathbf{E}}} = \bar{\bar{\mathbf{M}}}$ **, these acquire the common Plane-meter** the Square CMNH = $[CM]^2 = \pi.[EC]^2$, where EC is the circle`s Radius **, Using the Bellow-Motion Method** by creating the Square CMNH = $[CM]^2$ from E , M , and from $[\mathbf{E}/\sqrt{2}]^{\mathbf{2}} = \pi.[M/\sqrt{2}\,]^2$, $\boldsymbol{\pi \equiv \frac{[\mathbf{E}/\sqrt{2}]^{\mathbf{2}}}{[\mathbf{M}/\sqrt{2}]^{\mathbf{2}}}}$ **and Thus Photon is Squared to an UNID SQUARE . In article** [**126**] , It is important to Note that **When** in Photons , $\mathbf{E \neq M}$ **, these acquire the common Space-meter** $\sqrt[3]{2}$ **, Using Quantum-Cloning Method for** creating a Perfect-Copy of E , M , as are when E = 2.M and , $[\,\mathbf{E \neq M}\,]^{\mathbf{3}} = 2.[\,\bar{\bar{\mathbf{E}}} = \mathbf{2}.\bar{\bar{\mathbf{M}}}\,]^{\mathbf{3}}$ **. With this Way Photon is Dublicated to a New-One .** The simplified But a Rigorous Proof of the Problem , ***The Squaring of the Circle using a Ruler and a Compass*** , as it was first Posed by the Ancient Greeks , is followed by The Photons which Square their , Energy circle ≡ The **Herpolhode ≡ SPIN** , to equal an **Energy Unit Square** , which **UNIT-Square** they Promote , **common Plane - meter** , either as the ***Speed of the Photon which is their Electric Field*** , or they store it Perpendicularly to the motion in an equal area , ***The Anti-Square which is their Magnetic Field*** **.** Promotion is done at the **Birefringence angle** of 45^o , and thus The **Bellow-motion** is their Torsional motion . At Phase Angle where $\bar{\bar{E}} = 2.\bar{\bar{M}}$,Energy Cube $[M / 2]^3$ is Dublicated and enters INTO . $[E]^3 = [2.M]^3 = 8.M^3 \equiv$**Tetrahedron-Cube-Sphere-Mechanism. [**102**]**

d...From [40]--The Special Problems of Euclidean Geometry [47] consist the , ***Moulds of Quantization*** **, of E - Geometry in it , to become → Monad , through mould of Space Anti-Space in itself ,** ***which is the material Dipole in inner monad Structure and which is identical with the Electromagnetic cycloidal field*** **→ Linearly through the mould of the Parallel Theorem [44- 45] ,** ***which are the equal distances between Points of Parallel and line*** **→ In Plane , through mould of Squaring the circle [46] ,** ***where the Two Equal and Perpendicular Monad-Vectors*** **, E = M ,** ***consist a Plane acquiring The common Plane -meter,*** π **, and in Space (volume) through mould of Duplication of the Cube [46] ,** ***where Any Two Unequal Perpendicular monads*** **E≠M ,** ***acquire the common Space-meter*** $\sqrt[3]{2}$**,** ***Using Quantum-Cloning Method to be Twice each other*** as analytically Proved-explained**.** The Unification ***of → Space and Energy ← Becomes through*** **[STPL]** ***Geometrical Mould Mechanism of Elements , the minimum Energy - Quanta ,*** **In monads →** ***Particles , Anti – Particles , Bosons , Gravity – Force , Gravity - Field , Photons , Dark Matter , and Dark – Energy , consisting the Material Dipoles in inner monad Structures ,*** **i.e.**

→ the innate Electromagnetic Cycloidal Field of monads ← [39-41]

Euclid's Elements consist of assuming a small Set of intuitively appealing Axioms , Proving many other Propositions. Because No One until [9] succeeded to Prove the Parallel Postulate By means of **Pure Geometric Logic** , many self consistent Non -Euclidean Geometries have been discovered , Based on Definitions , Axioms or Postulates , in order that Non of them contradicts any of the other Postulates . It was Proved [39] , that the only Space - Energy Geometry is Euclidean , agreeing with the **Physical Reality** on Unit AB ≡ Segment ≡ Vector which is The Electromagnetic field of the Quantized on $\overline{AB}$ Energy Space Vector of Angular Momentum= Spin, on the contrary to the General Relativity of Space-time which is based on the Rays of the Non-Euclidean Geometries to the limited velocity of light in Planck`s cavity . Euclidean Geometry elucidated the Definitions of its geometry - content , **i.e**. { [for Point ,

Segment , Straight Line , Plane , Volume , Space [S] , Anti-space [AS] , Sub - space [SS] , Cave , The Space-Anti-Space Mechanism of the Six-Triple-Points-Line , that Produces and transfers Points of Spaces , Anti-Spaces and Sub-Spaces in a Common Inertial Sub-Space , and a cylinder , in Gravity field [MFMF] Particles} and describes the Space-Energy vacuum beyond Plank´s length level [Gravity`s Length 3,969.10¯62 m] , reaching the absolute Point

$L_v = e^{i.\left(\frac{N\pi}{2}\right)b=10^{-}N=-\infty}$ m = 0 m , which is Nothing , and the Absolute

Primary Neutral Space [PNS] = cave [r = $10^{-35} \sim\sim 10^{-62}$ m [43-46] .

In Physics , there is No single Process called "Duplication of Energy" for Photons Because Energy must always be conserved , or when Two Photons are "merged" into a single Photon.

In Mechanics , The Gravity-cave *Energy Volume quantity* $|\bar{c}| \equiv$ [wr] is Doubled , and is Quantized in Planck`s - cave Space quantity $(h/2\pi)$ = The Spin = 2.[wr]³ → **i.e.** Energy Space quantity [wr] is Quantized , *doubled* , and becomes the Space quantity **h/π** following Euclidean Space-moulds of *Duplication of the cube*, in Sphere volume V = $(4\pi/3).[wr]^3$ and follows the *Squaring of the circle* $\boldsymbol{\pi}$, and in Sub-Space-Sphere volume $\sqrt[3]{2}$, as *Trisection* **.**

Keywords **: The Quadrature of The Electromagnetic Fields in Dual - Photons** ,
→ **The Squaring of the circle , Using a Ruler and a Compass** [124]
The Dublication of Photon , Using The Quantum - Cloning Method ,
→ **The Dublication of the Cube , Using A Ruler and a Compass** [126]
The Trisection of the Energy Octave , Using an Directional -Triangle
→ **The Trisection of any Angle , Using a Ruler and a Compass** [128]
The Construction of The Regular-Polygons Using 2-Directional-Triangles
→ **The Construction of Regular-Polygons Using a Ruler & a Compass** [130]
THE NEEDED Directional -Triangles FOR ,
→ (1) **The Squaring of the circle , Using a Ruler and a Compass** [124]
→ (2) **The Dublication of the Cube , Using a Ruler and a Compass** [126]
→ (3) **The Trisection of any Angle , Using a Ruler and a Compass** [128]
→ (4) **The Construction of Regular-Polygons Using Ruler & Compass** [130]
→ (5) **The Construction of Reg - Polygons By The Gravitational Force** [132]

1… The **Isosceles Right Angle Directional -Triangle** which **Hypotenuse** lies on the Diameter of the circle with relation of Base-Angles 1 : 2 = 45^e / 90^e , & it is the **Material-Mould for the Squaring The circle** & **Energy**-circle [124].

2.. The **Right Angle Directional - Triangle** which **Hypotenuse** lies on the Diameter of the circle with relation of Base - Angles 1 : 2 = 30^e / 60^e , and it is [126].
→ **The Material - Mould for The Dublication of the Energy Cube** ←

3.. The **Right Angle Directional - Triangle** which **Hypotenuse** lies on the Diameter of the circle with relation of Base - Angles 1 : 2 = 30^e / 60^e , and it is the **Material-Mould forThe Trisection of any Angle** ←
& → **The Trisection of Energy - Octava** ← [128] .

4… The **Isosceles Directional - Triangle** which **Base is a Chord** of the circle with relation of Base - Angles $\varphi_A + 2.\varphi_{EA} = 180^e$, and it is the , **Material-Mould for Constructing the Regular Polygons of the circle .**
& The Polygons of the **Energy**-circle **on STPL Forces .**

The **Two Right Angle** at Points **B , B` Directional - Triangles** which **Foot** form Their **Apothems** on Archimedes 2-n.Sided Vertices Circle to **B , B`** arc . [130].

A... THE ALGEBRAIC SOLUTION : [63]

It has been Proved by De Moivre's , that the n-th roots on the unit circle AB are represented by the vertices of the Regular n-sided Polygon inscribed in the circle .
It has been Proved that the Resemblance Ratio of Areas , of the circumscribed to the inscribed squares (Regular quadrilateral) which is equal to 2 , leads to the squaring of the circle. [124]
It has been also proved that , Projecting the vertices of the Regular n-Polygon on any tangent of the circle , then the Sum of the Heights y_n is equal to **n ∗ R** . [63M]
This is a linear relation between Heights , **h** , and the radius **R** , of the circle , the monad .
This property on the circle yields to the Geometrical construction (As Resemblance Ratio of Areas is now controlled) , and the Algebraic measuring of the Regular Polygons as follows :

For : R = The Radius of the circle , with a random Diameter AB .
a = The Side of the Regular n -Polygon inscribed in the circle
n = Number of sides , a , of the n -Polygon , then exists :

$$n \cdot R = 2 \cdot R + 2 \cdot y_1\ 2 \cdot y_2 + 2 \cdot y_3 + \ldots\ldots 2 \cdot y_n \ldots\ldots\ldots\ldots (n)$$

The Heights y_n are as follows :

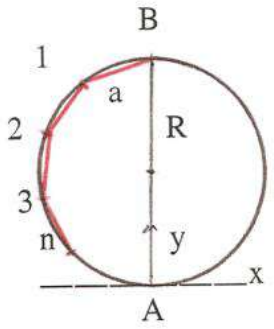


Figure-1-

$y_B = [\ 2 \cdot R\ ]$

$y_1 = [\ 4.R^2 - a^2\ ]\ /\ (\ 2 \cdot R\ )$

$y_2 = [\ 4.R^4 - 4.R^2.a^2 + a^4\ ]\ /\ (\ 2.R^3\ )$

/

$$y_3 = \frac{[\ 8.R^6 - 10.R^4.a^2 + 6.R^2.a^4 - a^6\ ] - a^2 \cdot \sqrt{64.R^8 - 96.R^6.a^2 + 52.R^4.a^4 - 12.R^2.a^6 + a^8}}{2.R^5}$$

$y_n = [\ \ldots\ldots\ldots\ldots\ldots\ ]\ /\ 2.R^n$

THE ALGEBRAIC EQUATIONS OF THE REGULAR n - POLYGONS

(**a**) REGULAR TRIANGLE ۞ :

The Equation of the vertices of the Regular Triangle is :

$$3.R = 2.R + \left[\ \frac{4.R^2 - a^2}{R}\ \right] \quad \text{or} \quad >>> \quad R^2 = 4 \cdot R^2 - a^2 \quad >>> \quad a^2 = 3.R^2$$

The side $\mathbf{a_3} = R \cdot \sqrt{3}$ …….(1).

(**b**) REGULAR QUADRILATERAL ۞ (SQUARE) :

The Equation of the vertices of the Regular Square gives :

$$4.R = 2.R + \left[\frac{4.R^2 - a^2}{R} \right] \quad \text{or} \quad >>> \quad a^2 = 2 \cdot R^2$$

The side $\mathbf{a_4} = R \cdot \sqrt{2}$ …….(2)

(**c**) REGULAR PENTAGON ۞ :

The Equation of the vertices of the Regular Pentagon is :

$$5.R = 2.R + \left[\frac{4.R^2 - a^2}{R} \right] + [\, 4.R^4 - 4.R^2.a^2 + a^4. \,] \quad >>> \quad a^4 - 5.R^2.a^2 + 5.R^4 = 0$$

$$\overline{\qquad\qquad} R^3 \overline{\qquad\qquad}$$

Solving the equation gives :

Solving the equation gives :

$$R^2 = \frac{5.R^2 - \sqrt{25.R^4 - 20.R^4}}{2} = \frac{5.R^2 - R^2.\sqrt{5}}{2} = [\, \{ 5.R^2 - R^2.\sqrt{5} \} / 2] = \frac{R^2}{2}(5 - \sqrt{5}) \quad ..(3)$$

$a^2 = \{ R^2 \} . [\, 10 - 2\sqrt{5} \,]$ >>> The side $\mathbf{a_5} = \left| \frac{R}{2} \right| \sqrt{10 - 2.\sqrt{5}}$

(**d**)

REGULAR HEXAGON ۞ :

The Equation of the vertices of the Regular Hexagon is :

$$6.R = 2.R + [\, 4.R^2 - a^2 \,] + [\, 4.R^4 - 4.R^2.a^2 + a^4 \,] \quad >> \quad a^4 - 5.R^2.a^2 + 4.R^4 = 0$$

$$\overline{\quad} R \overline{\quad} \qquad \overline{\qquad\qquad} R^3 \overline{\qquad\qquad}$$

Solving the equation gives :

$$a^2 = \frac{5.R^2 - \sqrt{25.R^4 - 16.R^4}}{2} = [\, 5 - 3 \,] . R^2 = R^2$$ The side $a_6 = R$

…..(4)

(**e**) REGULAR HEPTAGON ۞ :

The Equation of the vertices of the Regular Heptagon is :

$$7.\ R = 2.R + \left[\frac{4.R^2 - a^2}{R}\right] + \left[\frac{4.R^4 - 4.R^2.a^2 + a^4}{R^3}\right] + \left[\frac{8.R^6 - 10.R^4.a^2 + 6.R^2.a^4 a - a^6}{2.R^5}\right] -$$

$$- \left[\frac{a^2}{2.R^5}\right].\sqrt{64.R^8 - 96.R^6.a^2 + 52.R^4.a^4 - 12.R^2.a^6 + a^8}$$

Rearranging the terms and solving the equation in the quantity **a** , obtaining :

$$R^2.a^{10} - 13.R^4.a^8 + 63.R^6.a^6 - 140.R^8.R^4 + 140, R^{10}.a^2 - 49.R^{12} = 0 \ , \text{for } a^2 = x$$

$$x^5 - 13.R^2.x^4 + 63.R^4.x^3 - 140.R^6.x^2 + 140.R^8.x^1 - 49.R^{10} = 0 \quad .....(7)$$

Solving the 5 nth degree equation the Real roots are the following two :

$$x1 = R^2.[3 - \sqrt{2}\ ] \ , \ x2 = R^2.[3 + \sqrt{2}\ ]$$ which satisfy equation (7)

Having the two roots , the Sum of roots be equal to 13 , their combination taken 2 ,3, 4 at time be equal to 63 , - 140 , 140 , the product of roots be equal to – 49 , then equation (7) is reduced to the third degree equation as :

$$z^3 - 7.z^2 + 14.z - 7 = 0 \quad .....(7a)$$

by setting $\psi = z - (-7/3)$ into (7a) , then gives $\psi^3 + \rho.\psi + q = 0$ .... (7b) where ,

$$\rho = 14 - (-7)^2 / 3 = 14 - 49/3 = -7/3 \ > \ \rho^2 = 49/9 \ > \ \rho^3 = -343/27$$
$$q = 2.(-7)^3/27 + 14.(-7)/3 - 7 = 7/27 \ > \ q^2 = 49/729$$

Substituting ρ , q then $\psi^3 - (7/3).\psi + (7/27) = 0$ … (7b)
The solution of this third degree equation (7b) is as follows : $\rho = -7/3$
$q = 7/27$

Discriminant $D = q^2/4 + \rho^3/27 = (49/729.4) - (343/27.27) = -[49/108] < 0$

$$D = -49/108 = i^2(3.21^2/4.27^2) = i^2(21.\sqrt{3}/2.27)^2 = i^2(21.\sqrt{3}/54)^2$$

$$D = [7.\sqrt{3}/18]^2.i^2 \quad \text{and also} \quad \sqrt[2]{D} = \left|\frac{7.\sqrt{3}}{18}\right|.i$$

Therefore the equation has three Real Roots :

Substituting $\psi = w - \rho/3.w = w + 7/9.w \ > \ \psi^2 = w^2 + 49/81.w^2 + 14/9$

$$> \quad \psi^3 = w^3 + 343/729w^3 + 49/27w + 7w/3$$

to (7b) then becomes $w^3 + 343/729\, w^3 + 7/27 = 0$

and for $z = w^3$ $\quad z + 343/729\, z + 7/27 = 0$

$$z^2 + 7.z/27 + 343/729 = 0 \qquad \ldots(7c)$$

The Determinant $D < 0$ therefore the two quadratic complex roots are as follows :

$$Z1 = [-7/27 - \sqrt{49/27.27 - 4.343/729}\,]/2 = [-7/27 - \sqrt{49/27.27.4 - 49.7.4/27.27.4}\,]/2$$

$$= [-7/27 - \sqrt{(49 - 49.28)/27.27.4}\,]/2 = [-7 - 7.\sqrt{-27}\,]/27.2$$

$$= [-7 - 21.\sqrt{-3}\,]/3^3.2 = [-7/2].(1 - 3.i.\sqrt{3})/27 = (-7/54).[1 - 3.i.\sqrt{3}\,]$$

$$Z2 = [-7/2.(1 - 3.i\sqrt{3}\,]/27 = (-7/54).[1 + 3.i.\sqrt{3}\,]$$

The Process is beginning from the last denoting quantities to the first ones :

$$\text{Root } W1,2 = \sqrt[3]{3} = \frac{1}{3}\sqrt[3]{\frac{-7 \pm 21.i.\sqrt{3}}{2}} = \frac{1}{3}\sqrt[3]{-(7).[1 \pm 3.i.\sqrt{3}\,]} \quad \ldots(1)$$

$$\text{Root } \psi = W + 7/9.W = \frac{1}{3.\sqrt[3]{\frac{-7 \pm 21.i.\sqrt{3}}{2}}} + \sqrt[3]{\frac{-7 \pm 21\,i.\sqrt{3}}{3.2}} \qquad \ldots.(2)$$

$$\text{Root } X = \psi - \rho/3 = \psi + 7/3 = \frac{7}{3} + \frac{1}{3}.\sqrt[3]{\frac{-7 \pm 21.i.\sqrt{3}}{2}} + \sqrt[3]{\frac{-7 \pm 21.i.\sqrt{3}}{2}}$$

$$X = \frac{1}{3}\left| \sqrt{7.\left(\frac{-7 \pm 21.i.\sqrt{3}}{2}\right)} + \sqrt{\frac{-7 \pm 21.i.\sqrt{3}}{2}} + 7 \right| \ldots\ldots (3)$$

$$\left| \frac{}{\sqrt[3]{-7 \pm 21.i.\sqrt{3}}} \right| . R^2$$

The root a 7 of equation (7) equal to the side of the Regular Heptagon is a $7 = \sqrt{X}$

$$a_7 = \sqrt{\frac{1}{3}}\left[\sqrt[3]{\frac{-7 \pm 21.i.\sqrt{3}}{2}} + \sqrt[3]{\frac{-7 \pm 21.i.\sqrt{3}}{2}} + 7\right].R \quad \ldots. (4)$$

$$\sqrt{\frac{1}{3}}\left[\sqrt[3]{\frac{-7 \pm 21.i.\sqrt{3}}{2}} + \sqrt[3]{\frac{-7 \pm 21.i.\sqrt{3}}{2}} + 7\right]$$

Instead of substituting $\psi = w - \rho / 3.w$ into (7.b), is substituted $\psi = u + v$ and then gives the equation of second degree as $z^2 + 7.z / 27 + 343 / 729 = 0$ which has the two complex roots as follows :

$$z_{1,2} = \frac{7}{54}.[-1 \pm 3.i.\sqrt{3}] = \frac{1}{27}.[(-7 \pm 21.i.\sqrt{3}) / 2]$$ and the side a_7 is as ,

$$a_7 = \sqrt{\sqrt[3]{\frac{z_1}{}} + \sqrt{\sqrt[3]{\frac{z_2}{}}} + \frac{7}{3}}$$ and by substituting Z1 , Z2 into (7b) becomes the same formula as in (4) .

It is easy to see that $\sqrt[3]{-(7/2).[1 - 3.i.\sqrt{3}]} * \sqrt[3]{-(7/2).[1 + 3.i.\sqrt{3}]} = 7$

Analytically is :

$x = R^2 . [\ 0{,}753\ 020\ 375\ 967\ 025\ 701\ 777\ ] \quad >> \quad x^2 = 0{,}56704$

$a_7 = \sqrt{x} = R . [\ 0{,}867\ 767\ 453\ 193\ 664\ 601\ \ldots\ ]$

By using the formula of the Real Root of equation (7a) then :

$a.x^3 + b.x^2 + c.x + d = 0$ >> for $a = 1$, $b = -7$, $c = 14$, $d = -7$ then $x^3 - 7.x^2 + 14.x - 7 = 0$

$$x = -\frac{b}{3} - \frac{2^{1/3}.(-b^2 + 3.c)}{3\left[-2b^3+9bc-27d+\sqrt{4(-b^2+3c)^3+(-2b^3+9bc-27d)^2}\right]^{1/3}} + \frac{\left[-2b^3+9bc-27d+\sqrt{4(-b^2+3c)^3+(-2b^3+9bc-27d)^2}\right]^{1/3}}{3^2}$$.

Substituting the coefficients to the upper equation becomes :

$-b^2 + 3.c = -(-7)^2 + 3.14 = -49 + 42 = -7$ $-2.b^3 + 9.b.c - 27.d = -2.(-7)^3 + 9.(-7).14 - 27.(-7) = 686 - 882 + 189 = -7$

$4.(-b^2+3.c)^3 = 4(-7)^3 = -1372$ $(-2.b^3 + 9.b.c - 27.d)^2 = (-7)^2 = 4932\frac{1}{3} = \sqrt[3]{8.4} = 2.\sqrt[3]{4}$

$$X = \frac{7}{3} - \frac{\sqrt[3]{2}.(-7)}{\sqrt[3]{3.\sqrt{-7 + 21.i.\sqrt{3}}}} + \frac{\sqrt[3]{-7 + 21.i.\sqrt{3}}}{2.\sqrt[3]{4}}$$ and

$$a_7 = \sqrt{X} = \sqrt{\frac{7}{3} + \frac{7.\sqrt[3]{2}}{\sqrt[3]{3.\sqrt{-7 + 21.i.\sqrt{3}}}} + \frac{\sqrt[3]{-7 + 21.i.\sqrt{3}}}{2.\sqrt[3]{4}}}$$

The Side of the **Regular Heptagon**

(4.a)

Further Analysis to the Reader

(f) REGULAR OCTAGON ⯃ :

The equation of vertices of the Regular Octagon is

$$8.R = 2.R + (a^2) + (4.R^2.a^2 - a^4) + 10.R^4.a^2 - 6.R^2.a^4 + a^6 + a^2.\sqrt{64.R^8 - 96.R^6a^2 + 52.R^4.a^4 - 12.R^2.a^6 + a^8}$$

Rearranging the terms and solving the equation in the quantity **a ,** is a 10th degree equation , and by reduction ($x = a^2$) is find the 5^{th} degree equation as follows :

The equation of vertices of the Regular Octagon is

$$a^{10} - 13.R^2.a^8 + 62.R^4.a^6 - 132.R^6.a^4 + 120.R^8.a^2 - 36.R^{10} = 0$$

$$x^5 - 13.R^2.x^4 + 62.R^4.x^3 - 132.R^6.x^2 + 120.R^8.x^1 - 36.R^{10} = 0 \ldots. (a)$$

Solving the 5th degree equation is find the known algebraic root of Octagon of side a as

The roots are >>>>>>>>>> $x1 = R^2.[2 - \sqrt{2}]$, $x2 = R^2.[3 - \sqrt{3}]$

$$a8 = \sqrt{x} = R.\sqrt{2 - \sqrt{2}} \quad \ldots\ldots\ldots (b)$$

Verification :

$x = a^2 = R^2 (2 - \sqrt{2})$ $x^2 = R^4 . (6 - 4\sqrt{2})$ $x^3 = R^6 . (20 - 14.\sqrt{2})$

$x^4 = R^8 . (68 - 48\sqrt{2})$ $x^5 = R^{10} . (232 - 164\sqrt{2})$ ……(c)

by substitution (c) in (a) becomes :

$R^{10}. [232 - 164 . \sqrt{2}] = R^{10}. [232 - 164 .\sqrt{2}\ 2]$

$- R^{10} . [884 - 624 . \sqrt{2}] = R^{10}. [-884 + 624 . \sqrt{2}]$

$R^{10} . [1240 - 868 . \sqrt{2}] = R^{10}. [1240 - 868 . \sqrt{2}]$

$- R^{10} . [792 - 528 . \sqrt{2}] = R^{10}. [-792 + 528 . \sqrt{2}]$

$R^{10}. [240 - 120 . \sqrt{2}] = R^{10} . [240 - 120 . \sqrt{2}]$

$- R^{10} . [36] = R^{10} . [- 36]$

$R^{10} . [1712 - 1712 + (1152 - 1152) . \sqrt{2}] = 0$

$R^{10} . [0+0] = 0$ therefore Side $a_8 = R . \sqrt{2 - \sqrt{2}}$ …… (b)

CONCLUTIONS :

By summation the heights y on any tangent in a circle ,which hold for every Regular n-sided Polygon inscribed in the circle as the next is :

$n . R = 2.R + 2. y_1 + 2. y_2 + 2. y_3 + ... 2 . y_n$ ……(n)

the sides a_n of all these Regular n-sided Polygons are Algebraically expressed .
The Geometrical Construction of all Regular Polygons has been proved to be based on the solution of the moving Segment ZD of the figure of page 8 and it is the Master Key of Geometry , because so , the nth degree equations are the vertices of the n-polygon .
In this way , all Regular p - gon are constructible and measureable .
The mathematical reasoning is based on Geometrical logic exclusively alone .
As the Resemblance Ratio of Areas on the 4 - gone is equal to 2 , the Problem of squaring

the circle has been approached and solved [124] by extending Euclid logic of Units (under the restrictions imposed to seek the solution , with a ruler and a compass ,) on the unit circle AB , to unknown and now the Geometrical elements . (the settled age-old question for all these problems is not valid) .

The Regular Heptagon :

According to Heron , the Regular Heptagon is equal to six times the equilateral triangle with the same side and is the approximate value of $\sqrt{3} \cdot R / 2$

According to Archimedes , given a straight line AB we mark upon it two points C , D such that $AD.CD = DB^2$ and $CB.DB = AC^2$, without giving the way of marking the two Points .

According to the Contemporary Method , the side of the Regular Heptagon is the Root of a third degree equation with three real roots , one of which is that of the Regular Heptagon as analytically Presented.

B.1... THE GEOMETRICAL SOLUTION OF THE POLYGONS :

b-1.. The Even and Odd n-Polygons Properties : [63]

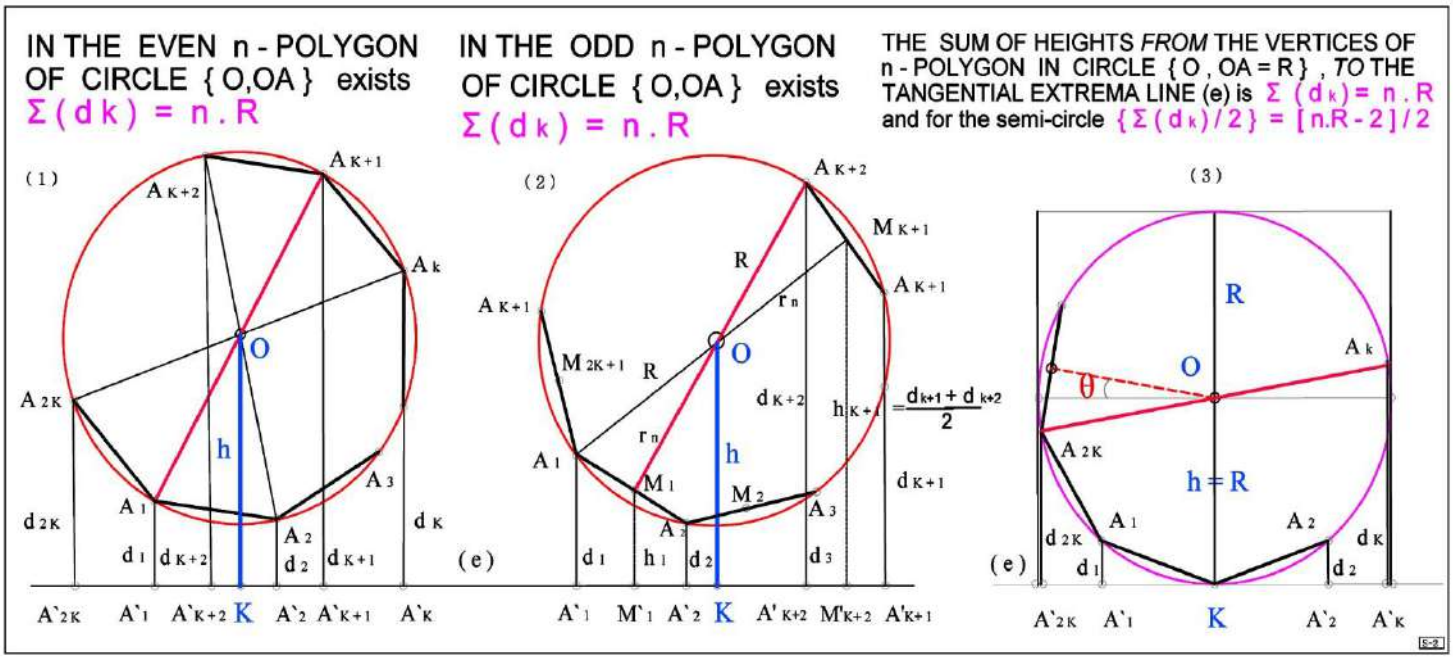


Figure-2- An Even and an Odd n-Polygon in circle [O ,OA] with diameters , $A_k\ A_{2k}$, Passing from A_{2k} , as Vertex (apex) of the Polygone , and the Diameters , $A_{k+2}\ M_1$, which are Perpendicular to Side A_1A_2 .

Let Be the n-Polygon A_1 , A_2 , A_3 , A_k , A_{k+1} , A_{k+2} , A_{2k} , in circle (O , OA_1) ,
(e) a straight line not intersecting the circle ,

d_1 , d_2 , d_{2k} , The Heights of the Vertices to (e) line ,
h_1 , h_2 , h_{2k+1} , The Heights of the midpoints $M_k\ M_{k+1}$ of the Sides to (e) line and
OK = h , The Height from the center O to (e) tangential line .

$n_1, n_3, n_5, \ldots\ldots n_{n+1}$, The Number of Sides in an Odd Regular – Polygon.
$n_2, n_4, n_6, \ldots\ldots n_n$, The Number of Sides in an Even Regular – Polygon.

To Proof :
In any n - Polygon , The Sum , $\Sigma = \Sigma(h)$, of the Heights , d_1 , d_2 , d_{2k} , of the Vertices A_1 , A_2 , A_3 , A_k , A_{k+1} , A_{k+2} , A_{2k} , where $n = 2k$, from any straight line (e) is equal to

$$\mathbf{\Sigma = \Sigma(h) = n \cdot OK = n \cdot h}$$

The Proof Fig-1- :
From any Vertex A_k , of the n-Polygon Draw the Diameter ($A_k O A_{2k}$)
a.. When $n = 2.k$ → then Vertex A_{2k} belongs to the Polygon
b.. When $n = 2.k + 1$ → then line $A_k O$, is mid-perpendicular to one of the sides .

Case a.. $n = 2.k$ = The Even-Polygons Fig-2- (1)
Exists $\frac{n}{2} = \frac{2k}{2} = k$, and are the Pairs of vertices in opposite diameters as A_1 , A_{k+1} , and the k , Trapezium which has bases the Heights of the vertices in opposite diameters from (e) line , and which have Height OK = h , as Common Height from their Diameter , i.e.

From Trapezium A_1 , $A`_1$ , $A_{k+1}$ , $A`_{k+1}$ exists $d_1 + d_{k+1} = 2.h$ and analogically is ,
$d_2 + d_{k+2} = 2.h$
$d_3 + d_{k+3} = 2.h$ …. $d_k + d_{k+1} = 2.h$
And by Summation ,

$$d_1 + d_2 + \ldots. d_k + d_{2k} = 2.h \quad \text{or} \quad \mathbf{\Sigma = (2k) \cdot h = n \cdot h = n \cdot OK} \quad \ldots\ldots(\mathbf{1})$$

Case b.. $n = 2.k + 1$ = The Odd - Polygons Fig-2-–(2)
$A_1 A_2$, $A_2 A_3$, ……. $A_{2k+1} A_1$, the Sides of the Polygon .
M_1 , M_2 , M_{2k+1} , are the Midpoints of sides from line (e)
h_1 , h_2 , …. h_{2k+1} The corresponding Heights of midpoints from (e) .
The Diameter from vertex A_1 is Perpendicular to side $A_{k+1} A_{k+2}$ which has the midpoint M_{k+1} , while the other $A_1 M_{k+1} = A_1 O + O M_{k+1} = \mathbf{R + r_n}$, The Radius plus the The Apothem In Trapezium $A_1 A`_1 M`_{k+1} M_{k+1}$ with Bases $A_1 A`_1$ and $M`_{k+1} M_{k+1}$, both Perpendicular to (e) line is Parallel to Height OK = h and bisects $A_1 O = R$ and $O M_{k+1} = r_n$ and from figure , exists

$$OK = h = \frac{R h_{k+1} + r_n \cdot d_1}{R + r_n} \qquad \ldots\ldots\ldots(\mathbf{2}) \qquad \textbf{i.e.}$$

Height OK is common to all 2k+1 Trapezium which are formed as $A_1 A`_1 M`_{k+1} M_{k+1}$ and

OK Height divides also the corresponding to $A_1 M_{k+1}$ side with the same analogy as $\frac{R}{r_n}$.

By Summation of 2k+1 Parts of (2) which are all equal to OK = h , then from the 2k+1 different Between them Trapezium referred exists ,

The Summation of Heights of the Regular Polygons ,

$$\mathbf{(2k+1).\ h} = \frac{\mathbf{R}\left\{ \mathbf{h_{k+1}+h_{k+2}+h_{k+1+h_1}+\cdots h_k} \right\} + \mathbf{r_n}.\{\mathbf{d_{1+}\ d_2+\cdots d_{k+1}+\ d_{2k+1}}\}}{\mathbf{R+r_n}} = \mathbf{n\ .\ h} = \frac{\mathbf{R.S+\ r_n\ \Sigma}}{\mathbf{R+\ r_n}} \ldots.(\mathbf{3})$$

Where $S = h_1 + h_2 + \ldots. h_k + d_{2k+1}$. Since h_1 , h_2 , ... h_k , d_{2k+1} are the diameters of Trapezium with bases d_1 , d_2 to h_1 , d_2 , d_3 to h_2 and so on and also d_{2k+1} , d_1 to h_{2k+1} ***then***

$$\mathbf{S} = \frac{\mathbf{d_1 + d_2}}{\mathbf{2}} + \frac{\mathbf{d_2 + d_3}}{\mathbf{2}} + \ldots.. \frac{\mathbf{d_{2k} + d_1}}{\mathbf{2}} = \frac{\mathbf{2\{\ d_1+\ d_2+\cdots\ d_{2k+1}\}}}{\mathbf{2}} = \mathbf{d_1 + d_2 + \ldots.\ d_k + d_{2k} = \Sigma}$$ and

(**3**) becomes → (**4**)

$$\mathbf{n\ .\ h} = \frac{\mathbf{R.S+\ r_n\ \Sigma}}{\mathbf{R+\ r_n}} = \left[\ \frac{\mathbf{R+\ r_n}}{\mathbf{R+\ r_n}}\ \right]\mathbf{.\ \Sigma = \Sigma}$$ **i.e. Σ = n . h for all Even and Odd n-Polygons** .

And is a Relation between Heights and the Number of the Regular Polygons .

Case c.. Line (e) is **Extrema as Tangential to circle** Fig-2-–(**3**)
In this case height , h , is equal to radius R and OK = h = R .
Since the Sum of Heights of the vertices in any n-Polygon is $\Sigma = n\ .\ h = n\ |OK|$ then $\Sigma = n\ .\ R$
This remark helps to construct Geometrically i.e. with a Ruler and a Compass , all the Regular n-Polygons Because gives the relation of the Mid-Perpendicular to the radius r_n of the inscribed circle which is related to the Interior angle $w = \{\frac{n-2}{n}\}.180^o$. $w = \{\frac{360}{n}\}$.
i.e. Angles , **w** , in a circle of radius , R , define the n-Sides , $A_1\ A_2$, of the Regular Polygon which in turn define the Sum , Σ , of their Heights equal to $\Sigma = n\ .\ R$
Since also the relation of radius R , between the Circle and r , of the Inscribed circle is extended to Heights , this helps Extrema - Method to be applicable on the solution which follows .

The Theory of Means :
It was known from Pappus the how to exhibit in a semicircle all three means , namely ,
The Arithmetic , The Geometric , and The Harmonic mean .

In Fig.-3--(1a) → Is given Segment AC and is defined Harmonic mean **DE** for any BA , BC
Draw at any Point B , **BD** at right angles to AC meeting the circle (O , OA = OC)
Join DO and from Point B Draw the Perpendicular **BE** on it .
Show that **DE** is the Harmonic - Mean between **BA , BC** .

The Proof :
Since ABC is a Right – angled triangle , and DE = AB then ,
$AB^2 = AD\ .\ AE = AD\ .\ (AD + DE) = AD\ .\ (AD + AB) = AD^2 + AD\ .\ AB$ therefore ,
$AD^2 = AB^2 - AD\ .\ AB = AB\ .\ (AB - AD)$ or $AD^2 = AB\ .\ MB$

That is , AM is the Harmonic Mean in AB Segment , or between AB and MB .

In Fig.-3--(1b) → On the Diameter AB of circle (O,OA= OB) , is the Point M .
Draw BC at Right angles to AB meeting circle (B,BO at C .
Join AC meeting circle (C,CB) at Points D,E , and Draw the
circle (A,AD) .meeting Diameter AB at Point M .
Show that **AM** is the Harmonic - Mean between **AB , BM**

Proof :
Since ADC is a right – angled triangle , and BD is Perpendicular to AB then issues ,
$BD^2 = BA\ .\ BC$ and since BEO is a right-angled triangle , and BD is Perpendicular to OB

then issues $BD^2 = DE \cdot DO$. By rearranging $\mathbf{DE} = BD^2 / DO = BD^2 / OA$,because DO=OA .
So issues $\mathbf{DE} = BD^2 / OA = \mathbf{[BA \cdot BC] / OA}$ that is ,

→ **DE is the Harmonic Mean Between BA and BC** ← .

b-2.. The Theory , On Segments and Angles Relation :

The Three Circles [O,OK] – [K,KA] – [Δh =Diameter] **– Method** :

In Fig.-3--(2) → Two Even n and n+2 Regular Polygons on the same circle (O , OA) where,

n , n +2 are the Number of sides differing by an Even number

λ_a = The length of a Side of a – [n - Polygon].

λ_b = The length of a Side of b – [n+2 Polygon].

r_a = Apothem = Απόστημα = Radius of inscribed circle of **a**–Polygon

r_b = Apothem = Απόστημα = Radius of inscribed circle of **b**–Polygon).

h_A = The Height of $K A_1$ Side of **a** – Polygon .

h_B = The Height of $K B_1$ Side of **b** – Polygon .

$\Delta h = h_A - h_B$, the Difference of Heights .

$\Delta r = r_a - r_b$, the Difference of Apothems .

S = The Sum of interior angles equal to (n-2) .180° =

$(n-2).\pi$ $\frac{h_A}{\lambda_a} = \sin\varphi_a$, $\frac{h_B}{\lambda_b} = \sin\varphi_b$, $\frac{h}{\lambda} = \varphi$,

$w_a = [\frac{2}{n}].180 = [\frac{2}{n}]\,\pi$, The Interior angle of the [n - Polygon].

$w_b = [\frac{2}{n+2}].180 = [\frac{2}{n+2}].\pi$, The Interior angle of the [n+2 Polygon].

w_o = An Extrema-angle between w_a , w_b which is related to Heights.

$\varphi_a = [\frac{n-2}{2.n}]\,\pi$, The angle of side λ_a to (e) line for Even , n-Polygon.

$\varphi_b = [\frac{n}{2(n+2)}]\,\pi$, The angle of side λ_b to (e) line for Even , n+2 Polygon.

$\varphi_o = [\frac{n-1}{2(n+1)}]\,\pi$, The angle of side λ_o to (e) line for Odd – Polygon .

Show that , The Extrema - angle w_o and the complementary angle φ_o , Define for the In -between Odd-Regular Pentagon in the same circle (O , OA) , and by Scanning the Δh , difference of Heights on the *Circles - Heights - System* , then Is succeeded on Δh , the *Harmonic Ratio of Mean on Sides* .

The Proof : Figure.-3--(2 - 3)

a.. Draw on OK circle the Tangent (e) at Point K , and from K any two Chords KA and KB . Draw the Perpendiculars AA`, BB` from Points A , B and Parallels AA_1,$BB_1$to Tangent (e).

b.. Draw the circle of Heights (A_1 , A_1B_1) In right angles triangles KAA` , KBB` , the ratios $\frac{AA`}{KA} = \frac{h_A}{\lambda_a} = \sin\varphi_a$ and $\frac{BB`}{KB} = \frac{h_B}{\lambda_b} = \sin\varphi_b$, where $h_A = \lambda_a \cdot \sin\varphi_a$ and $h_B = \lambda_b \cdot \sin\varphi_b$ and the difference $\Delta h = h_A - h_B$, or

$$\mathbf{\Delta h = h_A - h_B = \lambda_a \cdot \sin\varphi_a - \lambda_b \cdot \sin\varphi_b} \quad \ldots\ldots\ldots (1)$$

Since Between the two Sequent , n , n + 2 , Even – Regular – Polygons exists the Geometric logic of AB Monads , **i.e.** In a Segment the whole is equal to the Parts , and to the two halves , and for angle φ_a to become φ_b is needed to Pass through another one **Angle φ_o** , which is between the Two , ***Therefore*** ,

a.. Between the Two Sequence Even-Regular-Polygons exists another One Regular-Polygon

b.. According to Pappus theory of Proportion and Means , between the three terms h , λ , φ exists one of the three means .

c.. Since the Sum { **it is algebraically n + (n+2) = 2n + 2 = 2.(n+1)** } must be an Integer and which can be divided by 2 as exists . ***Therefore*** ,

d.. Between Any Two Numerical Even -Regular-Polygons n , n +2 exists The Only One (n+1) Odd – Regular - Polygon .

For the commonly Divergence angle , φ , equation (1) becomes $\mathbf{h_{\varphi}}$ between $h_A - h_B$,

From $\Delta h = h_A - h_B = \lambda_a \sin\varphi_a - \lambda_b . \sin\varphi_b$, and for the In-between is

$$\mathbf{\Delta h = h_A - h_B = (\lambda_a - \lambda_b) . \sin\varphi = \lambda_{\varphi} . \sin\varphi} \quad \ldots\ldots (2)$$

or , $h_A - h_B = (r_a . \sin\varphi - r_b . \sin\varphi) . \sin\varphi = (r_a - r_b) . \sin^2\varphi$ **i.e.**

$\mathbf{h_A - h_B} = (r_a - r_b) . \sin^2\varphi = \Delta h = \lambda_{\varphi} . \sin\varphi$ or , $\frac{h_A - h_B}{\sin\varphi} = \frac{\sin\varphi}{(r_a - r_b)} = \lambda_{\varphi}$ ...(**3**)

That is , $\mathbf{\sin\varphi} = (\frac{h_{\varphi}}{\lambda_{\varphi}})$, is the **Harmonic - Mean** between [$\mathbf{h_A - h_B}$] , [$\mathbf{r_a - r_b}$]

And since $[\mathbf{r_a}] = \frac{1}{2}\sqrt{4R^2 - \lambda^2_a}$, $[\mathbf{r_b}] = \frac{1}{2}\sqrt{4R^2 - \lambda^2_b}$, **Harmonic – Mean** is to ($\lambda_a - \lambda_b$).

Show that , the Extrema-angle , w_o , formulates the Complementary angle , φ , defining the In-between Odd - Regular n-Polygons on the same circle (O , OK) , using the Extreme cases of this System { $\Delta h = h_A - h_B = A_1B_1$ } , on The Circles of difference of Height .

Analysis **:**

1.. From above relation of Heights and circle radius for two Sequent–Even - Polygons then

$\Sigma\ h_n = n . R = n . OK$ (**a**) and $\Sigma\ h_{n+2} = (n+2) . R = (n+2) . OK$ (**b**)

By Subtraction (a) , (b)

$$\mathbf{\Sigma\ h_{n+2} - \Sigma\ h_n = (n+2)\ R - n\ R = 2.R \quad \rightarrow \text{ constant}}$$

By Summation (a) , (b)

$$\mathbf{\Sigma\ h_{n+2} + \Sigma\ h_n = (n+2)\ R + n\ R = (n+1)\ .2.R \quad \rightarrow \text{ constant}}$$

i.e. In the System of Regular-Polygons the Even Vertices are Divided on Diameters , while the Odd Vertices on the Foot of Height (Apothem) &extended on the Diameter.

The Interior angles (**w**) and Gradients φ , Heights (**h**) and their differences , **Δh** , – Summation and Subtraction of Heights are Interconnected and Intertwined on their **Odd Vertices on the Foot of Heights** and on Common Circle [A , ($\mathbf{\Delta h = h_A - h_B}$)]

Where thus **is Produced The Common (n +1) , Odd – Regular - Polygon** .

2.. From Fig.-3--(2-3) → $O_k A$, $O_k B$ are the Perpendicular chords on the circle which form the same angles at Point K and are $\lambda_a = 2R.\sin\varphi_a$, $\lambda_b = 2R.\sin\varphi_b$, and their Power - Product [POP] = ($\lambda_a . \lambda_a$) = $4R^2.[\sin\varphi_a . \sin\varphi_b]$ …… (**5**)

The Sum of Heights for the n and n+2 Even Regular Polygon is $\mathbf{\Sigma h_A = n.R}$ and $\mathbf{\Sigma\ h_B = (n + 2).R}$ and the In-between Odd Regular Polygon $\mathbf{\Sigma h_o = (n + 1) .\ R}$.

The corresponding Interior angles are ,

$$w_a = [\frac{2}{n}]\pi \quad \text{and} \quad \varphi_a = [\frac{n-2}{2.n}]\pi \quad \text{or} \quad \varphi_a = [\frac{\pi}{n}]$$

$$w_b = [\frac{2}{n+2}]\pi \quad \text{and} \quad \varphi_b = [\frac{n}{2.(n+2)}]\pi \quad \text{or} \quad \varphi_a = [\frac{\pi}{n+2}]$$

$$w_o = [\frac{2}{n+1}]\pi \quad \text{and} \quad \varphi_o = [\frac{n-1}{2.(n+1)}]\pi \quad \text{or} \quad \varphi_o = [\frac{\pi}{n+1}]$$

The Power of Point O to the circle of diameter Δh is for $\lambda_o = 2R.\sin\varphi_o$, $\lambda`_o = 2R.\sin\varphi_o$ & , $[POP] = [\lambda_o . \lambda`_o] = 4R^2 . \sin^2\varphi_o$..(6) which is Equal to the Power from (5) and this Because the Perpendiculars from the two Centers , O . O_k , form the same Angles φ_a , φ_b

Therefore , $\sin\varphi_a \cdot \sin\varphi_b = \sin^2\varphi_o$ or $\frac{\sin\varphi_a}{\sin\varphi_o} = \frac{\sin\varphi_o}{\sin\varphi_b}$ .....(7)

i.e. Angle φ_o follows ***The Golden Ratio Harmonic Mean of Sides*** Between angles φ_a , φ_b on the Δh = ***The Golden Ratio Difference of Heights .***

3.. Since the Product of magnitudes $\lambda_a \cdot \lambda_b$ = **constant** and also $(\lambda_a - \lambda_b) \cdot (\lambda_a + \lambda_b)$ constant therefore , The Power of Any Point IN and OUT of the circle of Heights is also Constant , meaning that exists another one Regular – Polygon , Between the two Even - Sequence **i.e.**

The Outer are The Two Even - Regular n and n+2 Polygons ,

and The Inner is → The n + 1 ODD – Regular Polygon ←

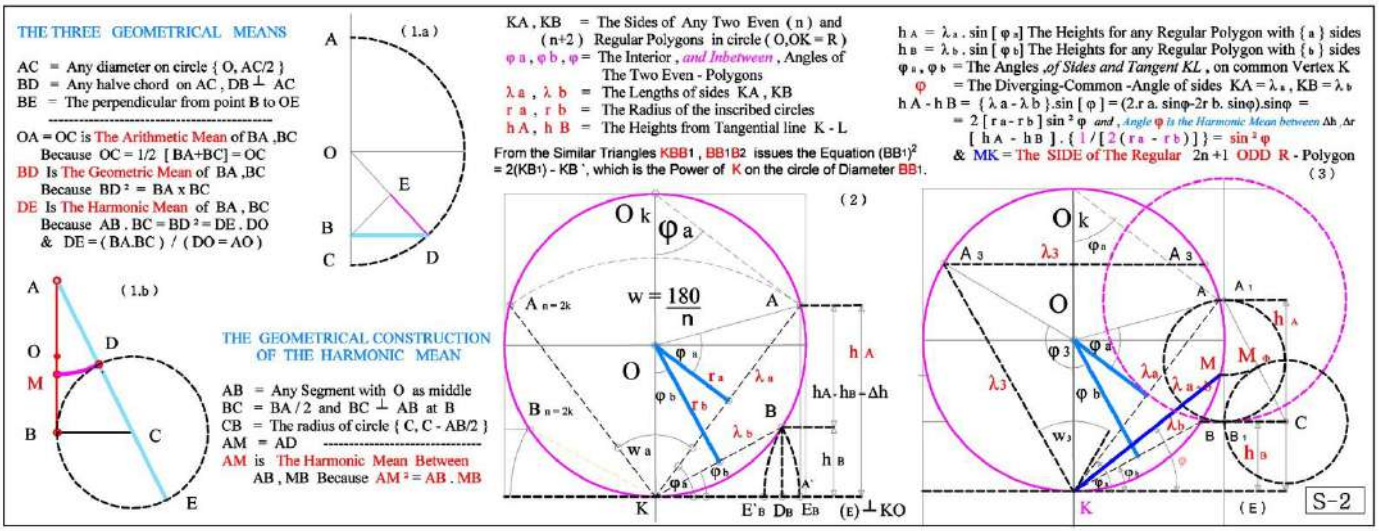


Figure-3-

In (1a) are shown the two ways for constructing the three Means on One or Two Segments .

In (1b) is The Geometrical Construction of the Golden Ratio AM² = AB . MB

In (2) **AB** is an Arbitrary Chord on circle (O , OK = O_k) and circle (K , KB) intersects the (e) line at Point E_B . The line-Segment $\mathbf{K\,B_1 = KB}$ and the Chord $\mathbf{B\,B_1 = B\,B_2}$.

The triangles K B E_B , B E_BE`$_B$ occupy the 2-common angles $K\widehat{E`_B}B = B\widehat{E`_B}E_B = K\widehat{B}E_B$

Therefore are Similar triangles and is $\frac{KB}{B\,E_B} = \frac{B\,E_B}{KB - E_B E`_B}$ or $[KB]^2 - KB.[E_B E`_B] = [BE_B]^2$

If we choose the Pentagon then the central angle $\mathbf{w_5}$ = 360° / 5 = 72° and φ_5 = 36° .

As in the Dublication of the Cube [126] and Trisection of angle [128] , the **Right Angle Directional -Triangle** was of the relation 1 : 2 = 30° / 60° , **in The case of the Isosceles Directional -Triangle** B E_BE`$_B$ , is needed the relation 1 : 2 = 36° / 72° , where then issues $E_B E`_B = B\,E_B$ and Becomes $\frac{KB}{B\,E_B} = \frac{B\,E_B}{KB - B\,E_B}$ or $[KB]^2 - KB.[B\,E_B] = [BE_B]^2$.

This second degree equation is rearranged as $\mathbf{[BE_B]^2} + KB.\mathbf{[BE_B]} - [KB]^2 = 0$, with Two Solutions the Real and the Negative Root .

The Solution of the Real Root is $\mathbf{[B\,E_{B1}]} = [\frac{KB}{2}].(\sqrt{5} - 1) = \mathbf{|KB|.[}\frac{\sqrt{5}-1}{2}\mathbf{]} = \mathbf{|KB|\,.\Phi}$ ..(f)

Equation (f) is the Divergency of Sides to Heights of Two n , and (n+2) Even Polygons .

This issues in Heights because , $\sin\varphi_a = \frac{h_a}{R}$, $\sin\varphi_b = \frac{h_b}{R}$, and or Proportional to Heights .
In (3) is shown the locus of the Two - Circles on Heights (A_1, A_1B_1) and the Parallels to (e) .
to be Extrema case for the Two Segments **KA , KB** . On the circle of Diameter $\mathbf{A_1, B_1}$, is Applied the (1a) Method for the Harmonic Mean and Point **M** , defines on circle the **KM** Chord and , the **n+1 , Odd Regular Polygon .**

Analysis :

1… The Geometrical Construction of the Regular Tetragon and Hexagon is in Fig.5--1,2-

2… The Geometrical Construction of the Regular Pentagon is as the Step-4 & Fig.5--3--

3… To Prove that Triangle $\mathbf{KBE_B}$ is the **Directional-Triangle of The Regular-Polygons .**

For the Point **B** of circle (K,KB =KO) issues $BK^2 - BE_B^2 = KD_B^2 + BD_B^2 - BD_B^2 - D_BE_B^2$ and $\mathbf{[BK^2 - BE_B^2] = KD_B^2 - D_BE_B^2}$ …..(a) . From Pythagorean 2nd Theorem of Diamesus on the Isosceles Triangle KBE_B issues $\mathbf{[BK^2 - BE_B^2] = 2.\ KE_B\ .[O_BD_B]}$ ……....(b)

where $\mathbf{O_B}$ is the midpoint of $KE_B = KB$ **.** Rearranging the equation (b) then ,

$\mathbf{[O_BD_B] = [BK^2 - BE_B^2]\ /\ 2.[\ KE_B = KB\ ] == [\ KD_B^2 - D_BE_B^2\ ]\ /\ 2.[KB]}$ .....(c)

i.e. The Distance $[O_BD_B] = k^2 =$ A Constant BD_B at Foot-Height Point D_B.

Since the Points **K , B , $\mathbf{E_B}$, $\mathbf{D_B}$, $\mathbf{O_B}$, are Fix & Constants** for Triangle $\mathbf{KBE_B}$, then →

The Geometrical locus of Points $\mathbf{E_B}$, which difference of squares $\mathbf{BK^2 - BE_B^2 = k^2}$ from the Two constant Points K , $\mathbf{E_B}$, is the Constant Height $\mathbf{h_B}$ equal to $\mathbf{|BD_B|}$ at the Constant Point $\mathbf{D_B}$ which Distances from the midpoint $\mathbf{O_B}$ of $\mathbf{KE_B}$ as

→ $\mathbf{[O_BD_B] = k^2 / 2.\ KE_B = constant}$ ← In the same way for an Point **A** issues ,

The Geometrical locus of Points $\mathbf{E_A}$, which difference of squares $\mathbf{AK^2 - AE_A^2 = k^2}$ from the Two constant Points K , $\mathbf{E_A}$, is the Constant Height $\mathbf{h_A}$ equal to $\mathbf{|AD_A|}$ at the Constant Point $\mathbf{D_A}$ which Distances from the midpoint $\mathbf{O_A}$ of $\mathbf{KE_A}$ as

→ $\mathbf{[O_AD_A] = k^2 / 2.\ KE_A = constant}$ ← (d) ← The Proof from triangle [KBK1]

On Right angle Triangle KAK` issues $|KA|^2 = |KD_{A1}|.\ KK` = h_A.2R$ and $|KB|^2 = h_B.2R$,

By Subtraction $|KA|^2 - |KB|^2 = 2R.[\ h_A - h_B\ ]$ or $\mathbf{\Delta h = h_A - h_B = [KA^2 - KB^2] / 2R}$ .....(e)

And The Sum of Heights $[\ h_A + h_B\ ] = [\ |KA|^2 + |KB|^2\ [\ / 2R$

i.e. **The Geometrical locus of any Two-Even-Sequence-Sets of Points A , B , is On their constant Difference of Heights** $\Delta h = h_A - h_B$ **.** (Conclusion-1) **And from** ..(d)

3a... When the constant difference $[O_AD_A] = k^2 / 2.\ KE_A = AK^2 - AE_A^2 > 0$ **, Then** $[O_AD_A]$ Belongs to the Half-Distance O_AE_A , which is the corresponding **Index** of Chord KA .

3b... When the constant difference $[O_AD_A] = k^2 / 2.\ KE_A = AK^2 - AE_A^2 < 0$ **, Then** $[O_AD_A]$ Belongs to the Half-Distance O_AK , which is the corresponding **Index** of Chord KA .

3c... When the constant difference $[O_AD_A] = k^2 / 2.\ KE_A = AK^2 - AE_A^2 \equiv 0$ **, Then** $[O_AD_A]$ Belongs to Point D_A which is an **Indeterminacy** of Point A and Chord KA us at D_A .

3d... For a Chord **KX** , **Between Any Even - Sequence-Set** , **KA – KB** , and when issues $\mathbf{KX = KE_X \equiv KD_A}$, then **Indeterminacy** is transferred to Pole $\mathbf{D_A}$ where $|XK^2 - XD_A^2| = 2.|KD_A.O_XD_X$ & → $\mathbf{[O_X\ D_X] = |XK^2 - XD_A^2|\ /\ 2.KD_A}$ = Constant ← ..(Conclusion-2)

3e... For The Chord **KX** , **Between The Even - Sequence-Set** [4-6] , which is the Regular

Tetragon with $|KA| \equiv |Ka| = R\sqrt{2}$ and Regular Hexagon with $|KB| \equiv KD_A = |R|$ and BD_A Rotates to BE_b to Construct KBE_b from where issues $KE_b = KD_A = R$, and from (b) $[O_B D_B] = [BK^2 - BE_B^2] / 2.KE_B \rightarrow [BK^2 - BD_A^2] / 2.BK$, Or $[BK^2 - BE_B^2] = 2.KE_B .[O_B D_B] \rightarrow [BK^2 - BD_A^2] = 2|BK|.[O_B D_B]$ ….(g) **i.e.** Since the Points K , B , E_B , D_B , O_B , **ARE Fix & Constant** for Triangle $KBE_B \equiv KBD_A$, and **The Geometrical locus of Points E_B Exist also in the Prior KBD_A Therefore** In Even - Sequence-Set [4 - 6] , Triangle KBD_A is **Fix & Constant** .

In the same Locus AD_A and from the Side of the Regular n = 4-Polygon Exists and is Calculated the Side of the Regular 2n = 8-Polygon , and in the Case of Inbetween , n+1 = 5 Regular Pentagon Exists and is Calculated the Side of the Regular 2.| n+1 | = 10 Regular Decagon . At the same time ,

The Geometrical locus of Points E_A , which difference of squares $AK^2 - AE_A{}^2 = k^2$ from the Two constant Points K , E_A , is the Constant Height h_A equal to $|AD_A|$ at the Constant Point D_A , which Distances from the midpoint O_A of KE_A are as

$$\rightarrow \mathbf{[O_B D_B] = k^2 / 2.KD_A = [BK^2 - BD_A^2] / 2.KD_A = constant} \leftarrow$$

3f…For Point **C** of circle (O , OK) where Angle φ_C exists on Chord **KC** , issues

$\sin\varphi_C = \frac{KC}{2.R}$, & $|\frac{KB}{\sin\varphi_B}| = 2.R$, $|\frac{KC}{\sin\varphi_C}| = 2.R$, and $|\frac{KA}{\sin\varphi_A}| = 2.R = Constant$,

From (c) $h_A - h_B = \frac{|KA|^2 - |KB|^2}{2.R}$,, $h_A - h_C = \frac{|KA|^2 - |KC|^2}{2.R}$,, $h_C - h_B = \frac{|KC|^2 - |KB|^2}{2.R}$,

By division $\mathbf{[h_A - h_C]^2} = \frac{|KA|^2 - |KC|^2}{|KA|^2 - |KB|^2 . |KC|^2 - |KB|^2} [h_A - h_B].[h_C - h_B] = \frac{|KA|^2 - |KC|^2}{R.|KC|^2 - |KB|^2} \mathbf{[h_C - h_B]}$

And this Because $h_A - h_B = R - R/2 = R/2$ and $|KA|^2 - |KB|^2 = |R\sqrt{2}|^2 - R^2 = R^2$ ..... ∴

The above Relation Denotes the Correlation of **Heights and Sides** . From above issues , $|KA|^2 = 2R.h_A$ and $|KB|^2 = 2R.h_B$ .The Difference of Squares $[|KA|^2 - |KB|^2] = 2R.[h_A - h_B]$ and the Sum of Squares $[|KA|^2 + |KB|^2] = 2R.[h_A + h_B]$. Applying on Proofs

$\Sigma h_{n+2} - \Sigma h_n = (n+2)R - nR =$ **2R** $\rightarrow$ **constant**

$\Sigma h_{n+2} + \Sigma h_n = (n+2)R + nR =$ **(n+1) .2R** $\rightarrow$ **constant** Then **,**

$2R.[h_A - h_B] = [|AK|^2 - |BK|^2] = \{|AE_A|^2 - |AE_B|^2\} + 2.[AK,|O_A D_A| - BK,|O_B D_B|\}$, and

$2R.[h_A + h_B] = [|AK|^2 + |BK|^2] = \{|AE_A|^2 + |AE_B|^2\} + 2.[AK,|O_A D_A| + BK,|O_B D_B|\}$, **i.e….**

On Any Two Sequence Even-Regular-Polygons Set –-(n , n+2)-- of Sides KA , KB , **The**

1..**Difference of The Squares** $[|AK|^2 - |BK|^2] = 2R.[h_A - h_B] = k^2 =$ Constant in Circle [O,OK],

2..$[|AK|^2 - |BK|^2] = \{|AE_A|^2 - |BE_B|^2\} + 2.[AK,|O_A D_A| - BK,|O_B D_B|\}$, in Circles [K,KA],[K,KB],

3..**Sum of The Squares** $[|AK|^2 + |BK|^2] = 2R.[h_A + h_B] = k^2 =$ Constant in Circle [O,OK] ,

4..$[|AK|^2 + |BK|^2] = \{|AE_A|^2 + |BE_B|^2\} + 2.[AK,|O_A D_A| + BK,|O_B D_B|\}$,in Circles [K,KA],[K,KB],

5..$[|BK|^2 + |BE_B|^2] = |O_{BE}B|^2$. in Circle $[O_{BE}, O_{BE}A]$ **&** $[|AK|^2 + |AE_A|^2] = |O_{AE}A|^2$.in $[O_{AE}, O_{AE}A]$,

6..$[|BE_A|^2 + |BE_B|^2] = |M_{EAB}B|^2$. in Circle $[M_{EAB}, M_{EAB}B]$ where $|M_{EAB}E_A| = |M_{EAB}E_B|$, **And Difference & Sum of The Squares Must BOTH Be on those three Circles , Which are Required in Order to Remain Both Constant , from The Two Fixed Points** E_A , E_A .

4…The Apothem Relations . Fig.6--4--

Apothem is the Distance of Center O to the Midpoint $\mathbf{A_B}$ of Sector KB , and to any other Sector KA . When Apothem is extended to The Circle [O, OK] , Intersects this circle at Point $\mathbf{A_{1B}}$, **i.e.** Point A_B is on The Locus of Apothems → { ***The Circle of Diameter KO*** } & according to Archimedes , **Point $\mathbf{A_{1.B}}$ is The Vertex of the Regular 2n-Regular Polygon on The Circle [O , OK]** where issues , **n.2R = constant .** (Conclusion-3)

5…On The Isosceles Twin-Couple Triangle KBE_B issues , $\mathbf{|BK|^2 - |BE_B|^2 = 2.|BK|.|O_BD_B|}$

Which is a Constant , and the same for Triangle KAE_A , $\mathbf{|AK|^2 - |AE_A|^2 = 2.|AK|.|O_AD_A|}$ For Any **Two-Even-Sequence-Sets** , **n - n+2** , of Points A , B , of circle (O, OK) and Sides KA , KB with Difference of Heights $\mathbf{\Delta h = h_A - h_B}$, or Projections , And Angle $A\overset{\frown}{K}B$ is the Difference of Angles $\boldsymbol{\varphi_A - \varphi_B = \varphi_{AKB}} = A\overset{\frown}{K}B$. The Corresponding Foots - Difference of the Heights $[D_AD_B]$ **is Equal** to that of Apothems $K.A_A - K.A_B$, and this Because correspond to the same Angle φ_{AKB} .. (Conclusion-4) . **Since for the n and n+2 Even Regular Polygon issues $\Sigma h_{n+2} + \Sigma h_n = (n+2)R + nR = (n+1).2R$ → constant , Relation $\Sigma h_o = (n+1).2R$ Corresponds to The In-between Odd Regular Polygon Therefore → → →**

By Extending OA_A , OA_B [where $|KA| = 2.|KA_A|$, $|KB| = 2.|KA_B|$] on [O, OK] circle at Points $\mathbf{A_{1A}}$, $\mathbf{A_{1B}}$, which are the Vertices of their **2n-Polygons** , ***Then*** according to the above $\mathbf{A_{1C}}$ **is Their Inbetween (n+1).2R Odd Regular Polygon** . (Conclusion-5)

6…On Circle [K,KB], All Points occupy the Property of **Constant Sum Squares** from the two Constant Points E_A , E_B , as , $|AK|^2 + |AE_A|^2$ **= Constant ,** $|BK|^2 + |BE_A|^2$ **= Constant** .For Middle Point M_{AB} of Arc $\overset{\frown}{AB}$ issues also $|M_{AB}K|^2 + |M_{AB}E_A|^2$ **= Constant** , (Conclusion-6).

b-3.. The 3-Geometrical Methods for Constructing ALL the Regular Polygons :
Using the Coupler Curves [63] 16 / 06 / 2017 .

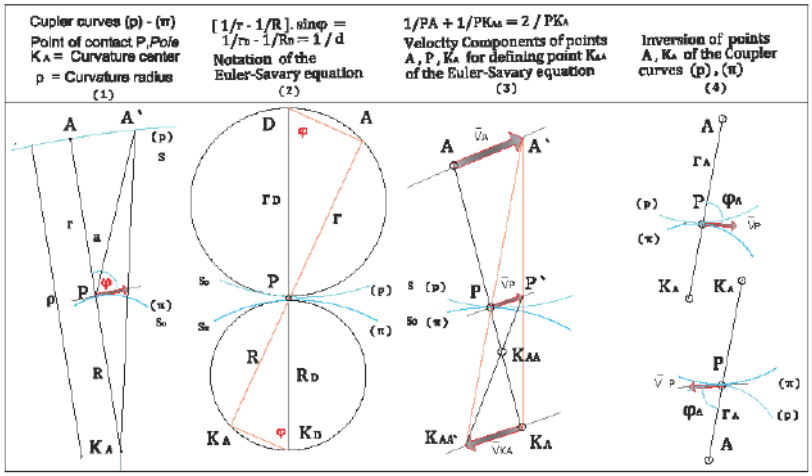


Figure-4-

a... The [Antiphon – Archimedes] Ancient Greek – Polygons method .

b.. The [Euler - Savary] Coupler-Curves curvature - centers method .

c.. The [Markos Geometrical , Three – Circles - Method , in Polygons .

In (1) A Point A on Coupler-curves (p) , (π) define the Point of curvature KA , The Instaneous Point P , the Pole on (π) .

In (2) is the case of Point A lying on radius of curvature of Polar Path (Point D) where then the Paths of Points A in S , System have the Instaneous center of curvature KA on the fixed System So .

In (3) The Velocity Instaneous center for curvature Point K_A , in S_o System is Point K_{AA}

In (4) The two Points A , K_A , of Coupler-curves (p) , (π) , follow the inversed motion where the two Poles of rotation , A and K_A , are inverted .

C.. THE NEEDED PROOFS for The Geometrical Construction of Regular Polygons :

The Prior Proofs : The Analysis of the Geometrical Construction . Fig.--6--

1-c.. The Sum **Σ** of Heights **h** from any line (e) , and the Number **n** of the Regular Polygons are Related as **Σ = n . h , for all Even and Odd n-Polygons** ., and for any tangent at K(e) of circle [O,OK = R] is → **Σ = n . h = n . R** ……(1)

1-c1.. On Right angle Triangles KAK`, KBK`, of the circle [O,OK= R] with Perpendicular Diameters KK`, a-a` , for any Chords |KA| , |KB| issues $|KA|^2 = |KD_{A1}| \cdot KK` = h_A \cdot 2R$ and $|KB|^2 = h_B \cdot 2R$ where $D_{A1}$ , $D_{B1}$ , are the Foots of the Vertices A , B on KK` axis .

By **Subtraction** $|KA|^2 - |KB|^2 = 2R \cdot [h_A - h_B]$, or $\mathbf{\Delta h = h_A - h_B = [KA^2 - KB^2] / 2R}$ ..(2)

And By **Summation** of Heights $\mathbf{[h_A + h_B] = [KA|^2 + |KB|^2] / 2R}$ .....(2c)

1-c2.. On the Isosceles Triangles KAE_A , KBE_B issues $[AK^2 - AE_A^2] = 2. KE_A .[O_AD_A]$, $[BK^2 - BE_B^2] = 2. KE_B.[O_BD_B]$ **&** for The Foots of the Vertices A , B on K(e) axis issues $\mathbf{[O_AD_A] = [AK^2 - AE_A^2] / 2.(KE_A = KA)}$ **„** $\mathbf{[O_BD_B] = [BK^2 - BE_B^2] / 2.(KE_B = KB)}$ ....(3)

i.e. In an [**n , n + 2**], Even Sequence Set The Differences of Each-Chords is Constant from midpoints $\mathbf{O_A , O_B}$, and the constant Foots Points $\mathbf{D_A , D_B}$, on tangent K(e) .

1-c3.. From (2-2c) the **Sum of Squares** is $[|KA|^2 + |KB|^2] = 2R.[h_A+h_B]$ **, and** when Applied on an Even Sequence Set of Chords [n , n + 2] then issues ,

$\Sigma\ h_{n+2} - \Sigma\ h_n$ = (n+2) R – n R = **2R** → **constant** ....(4-4c)

$\Sigma\ h_{n+2} + \Sigma\ h_n$ = (n+2) R + n R = **(n+1) .2R** → **constant** meaning that **,**

Between Any Two Numerical Even-Regular-Polygons [n , n +2] exists The Only

One (n+1) Odd – Regular - Polygon .

c1.. THE NEEDED PROOFS :

c(1**a)-** Since the **Algebrical Sum** { **n + (n+2) = 2n + 2 = 2.(n+1)** } must be an Integer and which also can be divided by 2 as this exists . ***Therefore*** ,
Between Any Two Numerical Even - Regular-Polygons n , n +2 exists The Only One (n+1) Odd – Regular - Polygon .

c(1**b)-** From (4-4c) the Difference and Sum of Squares in an **[n & n+2] , Even Sequence Set Reveals** The Inbetween Odd Polygon therefore , **are Both together needed for the Simultaneousely Issuing above Properties** .

The construction of all the Even - Regular - Polygons is Possible by dividing the circle (O ,OK) in 2 , 4 , 6 , 8 , 10 , 12 , 14 ..2n Parts as $w_a = [\frac{2}{n}]\pi$ and $\varphi_a = [\frac{\pi}{n}]$, for n = 1 , 2 , 3 , n ,
The construction of all the Odd - Regular -Polygons is Possible by Applying the Three Circles System method , on the Heights of the Chords of the Even-Sequence-Sets of these known Polygons as [2 , 4] , [4 , 6] , [6 , 8] , [8 ,10] …[(n)--(2n)] as formulas $w_o = [\frac{2}{n+1}]\pi$ $\varphi_o = [\frac{\pi}{(n+1)}]$, and is applicable from Point K **.** On it is formed the Harmonic-Mean of the Difference from the 2- Heights .

c(1**c**)- **THE PROOF FOR THE SUM OF HEIGHTS** :

Figure.-6--3–

-(**1**)-- The Line , $A_1\ A_{k+1}$, is Parallel to K-(e) line and ⊥ KO therefore $KA_1 = KA_{k+1}$.
The mid-Perpendicular OM_{k+1} Passes from A_1 Vertex , therefor the Straight line-Segment $A_1\ A_{k+1} = A_1\ A_{k+2}$. The Segment $A_1\ M_{k+1} = A_1O + OM_{k+1} = R + r_{k+1}$
In Trapezium $A_1\ M_{k+1}\ M`_{k+1}\ A`_1$, which is on the Vertice A_1 and with the Bases $A_1A`_1$ and $M_{k+1}\ M`_{k+1} = h_{k+1}$, the Distance KO = h is Parallel to Bases and divides the $A_1\ M_{k+1}$ Segment into the Parts R , r_{k+1} . From Trapezium- Lemma →

$$\text{Distance } \mathbf{KO = h} = [\frac{R.h_{k+1} + r_{2k+1}.\ d_1}{R + r_{2k+1}}] \ \ldots\ldots. (\mathbf{1})$$

Distance KO is Common to all 2k+1 Trapeziums as that on Vertex A_1 , and keeps the Ratio $[R / r_{2k+1}]$ = Constant ……. (**2**) . By adding all the homonymous Distances of (1) which all give KO = h from the Between them Different Trapeziums then issues ,

$$[2k+1].h = [\frac{R.[h_{k+1}+h_{k+2}+h_{2k+1}+h_1+h_k] + r_{2k+1}.[d_1+d_2+d_{k+1}+d_{2k+1}]}{R + r_{2k+1}}] = [\frac{R.Sh_i + r_{2k+1}.\Sigma d_\iota}{R + r_{2k+1}}]$$

$$= [2k+1].h = (n+1).h = [\frac{R.Sh_i + r_{2k+1}.\Sigma d_\iota}{R + r_{2k+1}}] = \frac{R.S + r_n \Sigma.}{R + r_n} \quad \ldots/\ldots. (\mathbf{3})$$

Where $S = h_1 + h_2 + \ldots h_{2k+1}$, and since $h_1 = (d_1 + d_2)/2$, then S becomes ,

$$S = h_1 + h_2 + .h_{2k+1} = \frac{1}{2}[(d_1+d_2)+(d_2+d_3)+..(d_{2k}+d_1) = \frac{1}{2}[2(d_1+d_2+..d_{2k+1})] = \Sigma$$

And the above equation becomes (n+1).h $= \frac{R.S + r_n \Sigma.}{R + r_n} = \frac{[R + r_n].\Sigma.}{R + r_n} = \mathbf{\Sigma = (n+1).h}$ …(**4**)

i.e. a).. For any n-Even Regular Polygon and (n+1) Odd Regular Polygon ,The Summation Σ The Heights is equal to → **n .KO = n .R** ← for Number **n** of the Polygon.

b).. For an Even **n**- Polygon exist **n** Apothems $\mathbf{r_n}$ with **2.n** Trapeziums ,
For an Even n+2 - Polygon exist **n+2** Apothems $\mathbf{r_{n+2}}$ with **2.n+2** Trapeziums ,
From above **n-gone-difference** -n+2– n = **2** - Apothems $\mathbf{r_{n+2} - r_n = 2}$, Trapeziums 2n+2 -2n= **2**

Or , **The In-between Odd Polygons are Independent of the Even - Sequence – Sets .**

And Their Side λ_{n+1} also , are in their In-Between Even Sequence Sets .

-(**2**)- In Figure.-6--3 , **The Twin-Couple of Triangles** $K\ A_{k+1}\ A_{k+2}$ & $A_{k+2}\ A_{k+1}E$, where DE = DA_{k+1} and the Side , $K`A_{k+1}$, of the 2-n Polygon Parallel to $A_{k+2}D$.
The General Pythagorean theorem for Side $A_{k+1}\ A_{k+2} = A_{k+1}\ E$ gives ,

$[A_{k+1}\ A_{k+2}]^2 = [E\ A_{k+2}]^2 = [K\ A_{k+2}]^2 + KE^2 - 2.[K\ E]$, or
$[A_{k+1}\ A_{k+2}]^2 = [K\ A_{k+2}]^2 - [K\ E] * \{2.[K\ D] - K\ E\}$, or
$[A_{k+1}\ A_{k+2}]^2 = [K\ A_{k+2}]^2 - [K\ E] * \{K\ D] + D\ E\}$, or
$[A_{k+1}\ A_{k+2}]^2 = [K\ A_{k+2}]^2 - [K\ E] * \{K\ D] + D\ A_{k+1}\}$, or
$\mathbf{[A_{k+1}\ A_{k+2}]^2 = [K\ A_{k+2}]^2 - \{K\ A_{k+1} * K\ E\}}$.…..(**5**) , **i.e.**

The Tangent Sector [K E ≡ K (e)] , controls The Construction of The Odd Polygons .

On the Isosceles Triangles KAE_A , KBE_B , exist $\mathbf{KA = K\ E_A = \lambda_A}$ & $\mathbf{KB = K\ E_B = \lambda_B}$,
The $\sin\varphi_A = \frac{h_A}{\lambda_A} = \frac{\lambda_A}{2R_A}$ because K`A ⊥ KA and , $\sin\varphi_B = \frac{B}{\lambda_B} = \frac{\lambda_B}{2R_B}$ because K`B ⊥ KB .
From above $[\lambda_A]^2 = 2R_A.\ h_A$ and , $[\lambda_B]^2 = 2R_B.\ h_B$ …..(1)
Applying above relation to the **Even Sequence Sets** then for **their Odd KC** , issues ,

$$\sin\varphi_A = \frac{KA}{2.R_A.} = \frac{\lambda_A}{2R_A.} = \frac{h_A}{\lambda_A} \text{ and then } [\lambda_A]^2 = 2R_A\ .\ h_A \quad \ldots.(2a)$$

$\sin\varphi_B = \frac{KB}{2.R_B.} = \frac{\lambda_B}{2R_B.} = \frac{h_B}{\lambda_B}$ and then $[\lambda_B]^2 = 2R_B \,.\, h_B$ ….(2b)

$\sin\varphi_C = \frac{KC}{2.R_C} = \frac{\lambda_C}{2.R_C} = \frac{h_C}{\lambda_C}$ and then $[\lambda_C]^2 = 2R_C \,.\, h_C$ ….(2c)

From $\Delta h = h_A - h_B = KA.\sin\varphi_A - KB.\sin\varphi_B$, and for their In-between Index is $\mathbf{\Delta h = h_A - h_B = (KA - KB).\sin\varphi = AB.\sin\varphi}$, or

$h_A - h_B = (r_A.\sin\varphi - r_B.\sin\varphi).\sin\varphi = (r_A - r_B).\sin^2\varphi$ and

Placing the difference of Heights $\Delta h = h_A - h_B$, then

$\mathbf{h_A - h_B} = (r_A - r_B).\sin^2\varphi = \Delta h = KC.\sin\varphi$ or , $\frac{h_A - h_B}{\sin\varphi} = \frac{\sin\varphi}{(r_A - r_B)}$ .. ....(φ)

i.e. → $\mathbf{\sin\varphi} = (\frac{C\,D_C}{KC})$ ← is the **Harmonic - Mean** Between $[\Delta h]$, $[\mathbf{r_A - r_B}]$

Placing the Difference of Heights on **KO Axis** then , $\mathbf{\Delta h} = h_A - h_B = \mathbf{D_{A1}\,D_{B1}}$...(φ1)

-**(3)-** In Fig.-6-4 , For Regular Hexagon issues $D_{A1}\,D_{B1} = R - R/2 = R/2 = D_{B1}A_A = \mathbf{\Delta h}$

And the Projection of Point A on $D_{B1}A_A$ axis is $A_AA_1 = AA_1 = A_ADB_1 = R/2$, or

$h_A - h_B = R - R/2 = R/2$, and $r_B - r_A = \sqrt{3}.|R/2| - |R/2| = |R/2|.|\sqrt{3} - 1|$

Since The Harmonic-Mean which issues on Radius $AO = AD_A$, issues also on Halve Radus AA_1 Analogically on BD_{B1} Sector and at D_{B1} Foot of Apothem for Decagon .

These Points are the Constant Point-Positions on the two Axis , ***Therefore*** **,**

The Projection D_C of the ODD In-Between Polygon follows (φ) , where Angle φ Is the Mean of difference of Heights $h_A - h_B$, to CD_C Pole line .

c(1**d)- THE PROOF FOR The Directional–Triangles $KA.E_A$, $KB.E_B$,**

Figure.--6—4-–

-**(d1)-** From the Pythagoras theorem On Right-angle-Triangle [K,AK`] issues

$(AK)^2 = (KK`)^2 - (AK`)^2 = (2R)^2 - (AK`)^2 = 4R^2 - (2R\cos\varphi_A)^2 =$

$4R^2 - 4R^2.\cos^2\varphi_A = 4R^2[1 - \cos^2\varphi_A] = \mathbf{4R^2.\sin^2\varphi_A = (KA)^2}$ …..(1) or

$\mathbf{\sin\varphi_A = \frac{KA}{2.R.}}$,, $\mathbf{\sin^2\varphi_A = |\frac{KA}{2.R.}|^2}$,, $\mathbf{|\frac{KA}{\sin\varphi}| = 2.R}$, $\mathbf{|\frac{KA}{\sin\varphi}|^2 = 4.R^2 = Constant}$,

-**(d2)-**Applying (1) on the Isosceles Triangle KAE_A , then $\mathbf{\sin^2\varphi_A} = |\frac{KA}{2.R.}|^2 = |\frac{(AD_A)^2}{(KA)^2}| = |\frac{AD^2_A}{2R.h_A.}|$

or , $\sin^2\varphi_A = |\frac{AD^2_A}{2R.h_A.}| = [KA^2/4R^2]$ and rearranging $|\frac{KA^2}{AD^2_A.}| = |\frac{2R}{h_A}| = |\frac{2R}{R}| = 2$ ...(2) Besides

$\varphi_A = 180^o - 2.\varphi_a$ and then $2.\varphi_a = 180^o - \varphi_A$, **And** $\sin 2.\varphi_a = 2.\sin\varphi_a.\cos\varphi_a$ **. i.e.**

Issues on the Isosceles Triangle → $|\frac{AD_A}{AE_A.}| = 2.|\frac{AD_A}{KA.}|.|\frac{KD_A}{KA.}|$ ← OR $\mathbf{|KA|^2 = 2.|KD_A|.|AE_A|}$

The same for KB , KC → $\mathbf{|KB|^2 = 2.|KD_B|.|BE_B}$ **&** $\mathbf{|KC|^2 = 2.|KD_C|.|CE_C|}$ ← ....(3)

-**(d3)-** Applying (5) On Isosceles Triangle $[AKE_A]$ then $|AK|^2 - AE_A|^2 = 2.|KD_A - D_AE_A|.|KE_A|$

and if O_A is the middle of KE_A then issues $|KA|^2 - |AE_A|^2 = 2|KA|.|O_AD_A|$ or ,

$2.|KA|.|O_AD_A|$, and issues → $\mathbf{|AK|^2 - |AE_A|^2 = 2.|AK|.|O_AD_A|}$ ← ……...(4)

i.e. The **Difference** of the Squares $|AK|^2 - |AE_A|^2$ on the Isosceles Triangle KAE_A, $KA = KE_A$ from Point **A ,** to the Two Constant Points K , E_A , is Constant and Equal to the Product of the Double Chord KA to the Distance of Midpoint to the Foot of A . The same for **B** The Difference of the Squares $|BK|^2 - |BE_B|^2$ on the Isosceles Triangle KBE_B, $KB = KE_B$ from Point **B ,** to the Two Constant Points K , E_B , is Constant and Equal to the Product of Twice the Chord KA multiplied by the Length between the Midpoint O_A and the

Foot D_A of Point A . ***Therefore*** ,

The Difference of Squares is Constant for all Even - Sequence– Sets of Chords . And are Required The Three Circles in order to Remain Constant On E_A , E_B .

2-**c- THE PROOF FOR CONSTANT Sum Squares in The Directional-Triangles .**

By applying the Median Theorem for Triangles to the Directional Triangles then is Revealed the Locus of the Sum Squares .

On Triangle KBE_B , issues $|BK|^2 + |BE_B|^2 = 2.|O_{BE}B|^2 + \frac{|KE_B|^2}{2.}$ = **Constant-s^2** , and since $KE_B = |KB|$ and $|O_{BE}B| = |KB| / 2$, then Becomes $\rightarrow$ $|O_{BE}B|^2 = s^2 / 2 - \frac{|KE_B|^2}{4.}$ $\leftarrow$ or

Radius $|O_{BE}B| = \sqrt{s^2/2 - \frac{|KE_B|^2}{4.}} = \frac{1}{2}\sqrt{2.s^2 - |KB|^2}$ = **Constant** . .(e-1) **i.e.**

Point B on Circle $|O_{BE}, O_{BE}B|$ formulates Triangle KBE_B , such that The Squares from B To the Two Constant Points K , E_B , is Constant as (e-1)

And by Changing the Two Constant Points to **E_B , E_A ,** their New Sum is , $[BE_A|^2 + BE_B|^2] = s^2$, and their Center O_{BE} is their New Midpoint M_{EAB} ,

Therefore **, All Points of this Circle Occupy**

The Constant **Radius** $\rightarrow$ $|M_{EAB}B| = \frac{1}{2}\sqrt{2.s^2 - |KB|^2}$ = **Constant** .... .(e-2) **OR** ,

The Locus of Any Point **X** of the Plane **For Constant Sum-Squares** from Constant Edge Points **E_B , E_A ,** of Line - Sector , **Is The Circle** with **Center** the Mid-Point $|M_{EAB}|$ of The Sector , **And Radius** The Diamesus $|XM_{EAB}|$ of Triangle $|XE_B E_A|$.

$\Rightarrow$ **In** Fig.-5-1-- from Point K is Drowned the Diameter KK` , and from the center O the Perpendicular Diameter a a`**.** By Connecting Points K , a , K` , a`, which are the **Vertices** of Reg–Tetragon , line Segments K a , a K`, K`a` , a`K , are the **Sides** of **Regular -Tetragon** .

$\Rightarrow$ **In** Fig.-5-2-- The Circle (K , KO) intersects circle (O , OK) at Point B_1 , **and** from B_1 Circle (K ,KO) , is Drowned to Point B_2 ,a*nd from each intersection Point* the same Circle (K ,KO) , defines the last Point B_6 at Point K which belongs to the Regular Hexagon **.** By connecting the Points K , B_2 , B_4 , K , which are the **Vertices** of **Regular - Triangle ,** the Between line Segments , KB_2 , B_2B_4 , B_4K , are the **Sides** of the Regular -Triangle .

$\Rightarrow$ **In** Fig.-5-2=3- The Circle (K , KO) intersects The circle (O , OK) at Point **B** and the Tangent K(e) at Point **E_B .** From Point **B** are Drowned $\rightarrow$ **a**).The circle (B , BE_B =$BE`_B$) intersecting line K(e) at Point $E`_B$ and **b**).The Perpendicular **BD_B** at Point **D_B** of K(e) Tan .

It was Proved , c(1d)-(d3) , **that** the Geometrical locus of all **Points B** on circle **(O,OK)** on which the difference of squares **[**BK^2- $BE_B{}^2$] from the Two constant Points **K , E_B ,** and where $KE_B = KB$, **IS constant AND is the Height BD_B** at Point **D_B** which distances from the midpoint **O_B** of **KE_B** to the Point **D_B** ,**AND** is Equal to $\rightarrow$ **$O_B D_B = k^2 / 2.KD_B$** which belongs to **Semi-Straight line Sector $O_B E_B$** where when $\rightarrow$ **$k^2 > 0$, And** to **Semi Straight line Sector $O_B K$** when $\rightarrow$ **$k^2 < 0$.** For Any other **Point A** also **issue the above and for Points , A , B on the $\Delta h = h_A - h_B$.**

It was Proved , c(1c)-(c2) , **that Difference** Δh **, Defines on , KA , KB Segments The** Chord **KC** Between the Even **, KA , KB ,** which is The Side of **Regular–Odd-Pentagon**.

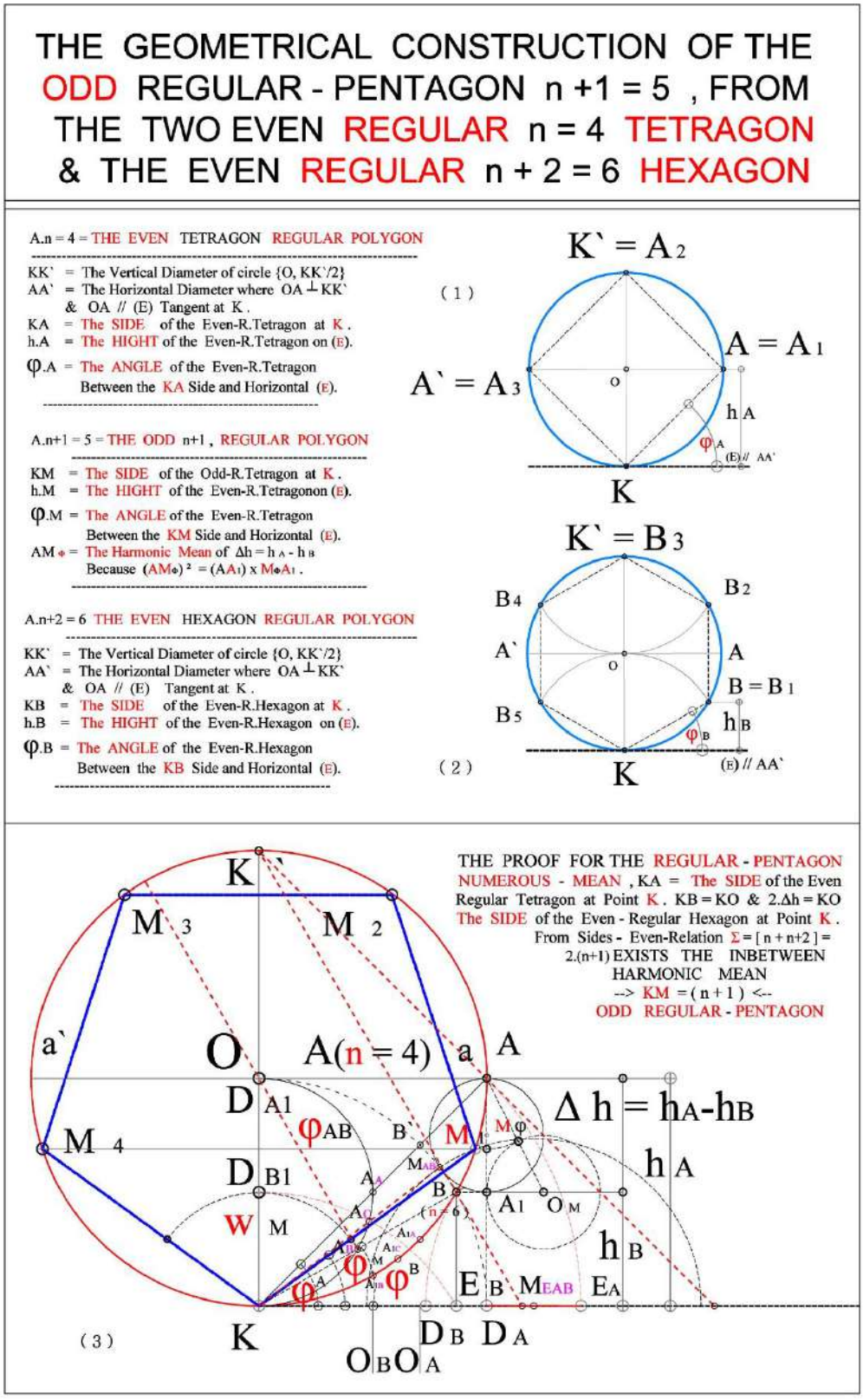


Figure-5-

In (1) is shown the Geometrical Construction of the **Even Tetragon** in Circle .
In (2) is shown the Geometrical Construction of the **Even Hexagon** in Circle .
In (3) is shown the Geometrical Construction of the **Odd Pentagon** in Circle .

D... THE TWIN – COUPLE TRIANGLES Method FOR THE CONSTRUCTION OF THE REGULAR --Odd – POLYGONS & PENTAGON .

Data : Fig.-6--1,2,3.4--

The circles and the Three Triangles are constructed using a Ruler and a Compass .

1… In (1) The Construction of the Golden Ratio **DM** on the Straight line Segment **KD** where DM = Φ.DM and , DM² = DM . MK ,

2… In (2) The Construction of the Regular Tetragon with Side KA = Ka`, and Hexagon with Side KB = KO

3… In (3) The Construction of the Sides in a Regular ODD Polygons $\mathbf{A_1\ A_{k+1}}$, $\mathbf{A_1\ A_{k+2}}$ with Diameter $|A_1OM_{1(k+1)}|$ where Point $|\mathbf{M_{1(k+1)}}|$ is the Extension of the Apothem $|A_1M_{(k+1)}|$ on main circle [O, OK] , on the Polygon Heights $\mathbf{h_{k+1}}$, $\mathbf{h_{k+2}}$.

4… In (4) and on the circles [O,OK] , [K, KA] , [K, KB] , are Drawn the Three Twin Coupled Triangles , **KBK` , KBE $_B$, KAE $_A$,** which solve the Odd - Problem .

D1... The Construction of The Regular Polygone And Pentagone : Fig --6-4--

STEP-**1** ⇒ Draw the circle (O , OK) and the two Perpendicular Diameters , **aa` ⊥ KK ` . And** the Tangent **K(E)** at Point K , **Using a Ruler and a Compass.**

STEP-**2** ⇒ Draw **KA** , **KB** the Two Segments such that are the Sides of an , **n** = 4 & **n+2 = 6 Even** Regular Polygone Tetragone , Hexagone , **Using a Ruler and a Compass.**

STEP-**3** ⇒ Draw circles (K, KA) , (K, KB) which intersect tangent **K(E)** at Points $\mathbf{E_A}$, $\mathbf{E_B}$ **And** the line-Sectors |**AE $_A$**| , |**BE $_B$**| . respectively , **Using a Ruler and a Compass.**

STEP-**4** ⇒ Draw the Perpendiculars **AD $_A$** , **BD $_B$** on Tangent **K(E)** , **And** the two circles with **Diameters** , **KA , KB** , **Using a Ruler and a Compass.**

STEP-**5** ⇒ Draw the circle with **Diameter KO=KB** , to Intersect the Sectors KA , KB at Points **A $_A$, A $_B$** →The Apothems **OA $_A$** , **OA $_B$** , which Produced at Points **A $_{1A}$** , **A $_{1B}$** , of the Circle (O, OK) → Apothem $\mathbf{A_C}$ on circle **of** Diameter |KO| is the Midpoint Between $|0A_{1.C}|$, $|0A_{2.C}|$ lines where Point $\mathbf{A_{2.C}}$ = The Golden - Ratio - Point on Sector $|A_{1.A}$, $A_{1.B}|$ & Points **B , B`** on [K,KB] , **Using Ruler and Compass.**

STEP-**6** ⇒ Project the line-Sector **KA $_C$** to circle (K , KB) , from Point **A $_C$** to Point $\mathbf{M_{AB}}$ On circle [K . KB] , **Using a Ruler and a Compass .**

STEP-**7** ⇒ Extend the line-Sector | **KA $_C$** | to circle (O , OK) Such that intersects Circle (O , OK) at Point $\mathbf{C_{n+1}}$, **Using a Ruler and a Compass .**

{ Chord KC $_{n+1}$ = KC is The Side of Pentagon in The IN-Between Even-Set }

***The following have been Proven* :**

a)... That Between Any **Two-Even-Sequence-Sets** of Points , A , B , of circle (O, OK) and Sides KA , KB , **EXISTS** a Chord **MC** which is **The Side of their In-Between Regular - Odd- Polygon** . with Difference of Heights **Δh** = $\mathbf{h_A - h_B}$, or of Projections

b)... That The Geometrical locus of Any Two Even-Sequence-Sets of Points A , B , **is** Between their **Directional-Triangle , from the constant Difference of their Height Projections , and On The Apothem Circle .**

c)... That , For The Geometrical Construction of the Regular Polygons are needed the Couple of Triangles KAK` , $KA.E_A$, $KB.E_B$ related to the **Circle of Apothems** on Diameter KO such **for n and 2n Polygons ,** and The Golden - Ratio - Point $\mathbf{A_{2.C}}$ on Sector $A_{1.A}$, $A_{1.B}$| for the Apothem $\mathbf{A_{.C}}$, combined with the main circle (O , OK) **.**

d)... That The Golden Ratio on the Height Difference $D_{A1} - D_{B1} = \Delta h = \mathbf{h_A - h_B}$, issues Also **AND** for the Regular Pentagon - Decagon Only .

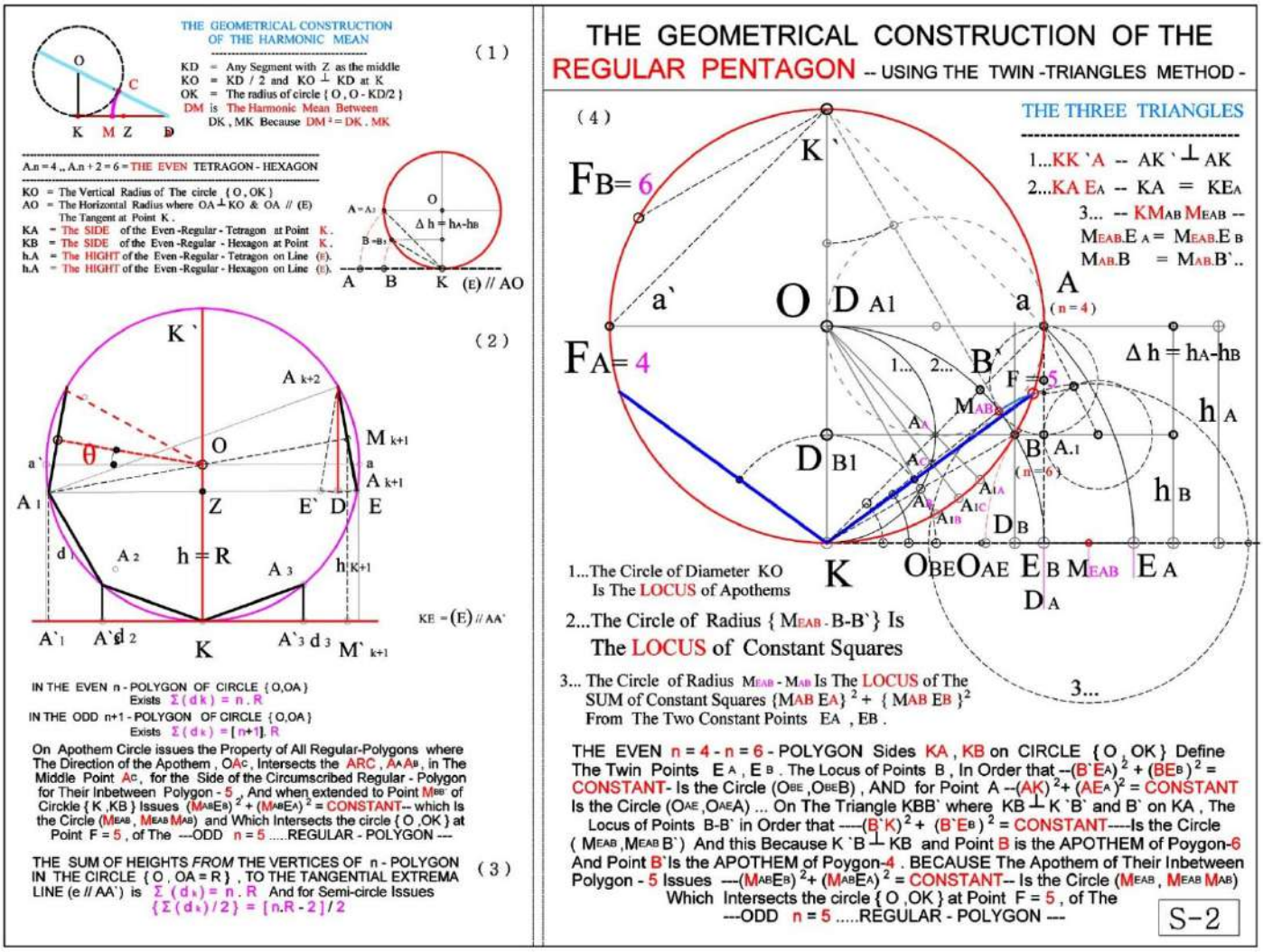


Figure-6-

In (1) is shown the Geometrical Construction of The Golden Ratio on a Difference DK = Δh .

In (2) is shown the Geometrical Construction of Tetragon= n and Hexagon= n + 2 in (O,OK) .

In (3) is shown the Geometrical Proof of the n = 2.k Even and n = 2k+1 Odd Polygons.

In (4) is shown → **The** Directional–Triangles KAE $_A$, KBE $_B$ **, The** Circle of **Apothems** (1) (DB_1 , $DB_1K = DB_1O$) , **The** Circle (K , |KB|=|KB`|) , of **Constant Different Squares** (2) **The** Circle of **Constant Sum Squares** (M_{EAB} , $M_{EAB}\,M_{AB}$) ... (3) , and All Points on the Main circle (O , OK) and on axis **.**

2-D.. THE PROOF FOR TWIN - COUPLE TRIANGLES METHOD FOR THE CONSTRUCTION OF THE REGULAR - ODD – POLYGONS ,

→ USING A RULER & A COMPSS ←

Data in [Fig- 7—3-].

2d-a.. The System of circle [O , OK] , of Diameters KK `⊥ aa`= 2R **,** KK `⊥ $K_{(E)}$ and Chords KA = $KE_A$ , KB = $KE_B$ , Represent The Fixed Twin - Couple of Triangles **KAK` , KA E_A , KA E_B .** where KA , KB , Is an [**n , n+2**] , Even Sequence Set . **The** Circle of Apothems with Diameter |KO| , **Denotes** at Point A_A , **The Foot of n-Apothem ,** And By Extending the Apothem OA_A to the Circle [O , OK] at Point $A_{1.A}$, then is Defined *According to Archimedes* the Vertex of the 2n-Sided Polygon With the same way By Extending the Apothem OA_B to the Circle [O , OK] at Point $A_{1.B}$, then is Defined the Vertex of the 2[n+2]-Sided Polygon for Even Points A ,B.

therefore → **On Apothem`s circle is detected the Apothem of n+1 Odd Polygon .**

For the Even Sequence Set [n , n+2] , n = Regular-**Tetragon**, n+2= Regular-**Hexagon** n+1= Regular-**Pentagon** , and n → A , n+2 → B , n+1 → C , then issues ,

$\mathbf{A_{1.A}}$ = The Vertex of the 2n-Sided Polygon = Octagon .

$\mathbf{A_{1.B}}$ = The Vertex of the 2n-Sided Polygon = Dodecagon .

$\mathbf{A_{1.C}}$ = The Foot of the 2n-Sided Polygon = Decagon .

$\mathbf{A_{2.C}}$ = The Golden - Ratio - Point on Sector → $|A_{1.A} , A_{1.B}|$

$\mathbf{A_C}$ = The Apothem`s-Foot of the n+1 , Sided Pentagon .

⇒ **The** Circle [K, KB] of the ***Constant-Difference-Squares*** intersects Tangent K(e) at Point E_B , while the Circle [K, KA] at Point E_A , Simultaneously The Circle [K, KB] **Denotes** Points B , B`, M_{AB} , as the Intersection of the circle and Side |KA| , & as the Intersection of the line-Sector $\mathbf{KA_C}$ Produced at Point M_{AB} of the Circle [K, KB] .

therefore → **Apothem`s-Foot $\mathbf{A_C}$ is transferred at Point $\mathbf{M_{AB}}$ which is On The Constant-Difference-Squares circle [K, KB]**

⇒ **The** Circle [O_B,O_BB] defines The **Constant - Sum - Squares** [$|BK|^2$ +$|BE_B|^2$]=$|O_BB|^2$. from Midpoint O_B , **And Circle [$\mathbf{M_{EAB}}$, $\mathbf{M_{EAB}}$B]** defines The **Constant-Sum – Squares** [$|BE_A|^2$+$|BE_B|^2$] = $|M_{EAB}B|^2$. in Circle [M_{EAB} ,M_{EAB}B] and $|M_{EAB}E_A|$=$|M_{EAB}E_B|$ on $|E_A- E_B|$ Since issues the Relation [$\frac{h_A - h_B}{\sin\varphi} = \frac{\sin\varphi}{(r_a - r_b)} = \lambda_\varphi$], **then sin φ** of the Regular ODD Polygon must Be satisfied on **Even-Polygons** , **Both** Difference of Heights $\mathbf{h_A - h_B}$, And Their Apothems $\mathbf{A_{1.A}}$, $\mathbf{A_{1.B}}$, **i.e.** On line Sector | $A_{1.A}$, $A_{1.B}$| Is Constructed with a Ruler & a Compass the Golden Ratio Point $\mathbf{A_{2.C}}$ Such that Denotes Between $|OA_{1.C}|$, $|OA_{2.C}|$ lines, The Point $\mathbf{A_C}$ on Apothem`s Circle **.** By extending $\mathbf{KA_C}$ on Circle [K , KB] is Defined Point M_{AB} of The Constant - Sum - Squares and on Circle [O , OK] Point $\mathbf{C_{n+1}}$, which is the Vertex of **The Even Sequence Set** [n , n+2] → **n+1= Regular – Pentagon .**

2d-b..THE GEOMETRICAL - PROOF :

a)..On the Isosceles Triangle KB E_B issues **The Constant Squares Difference** $|BK|^2-|BE_B|^2$, from Points K , E_B , **The Constant Sum of Squares** $|BK|^2 + |BE_B|^2$ from Midpoint O_{BE} of Distance $KE_B = KB$, and **The Angles** $K\widetilde{K`}B = \varphi_B = B\widetilde{K}E_B = K\widetilde{O}A_B$ **„ Because Above Properties exist for Point B „** ***Therefore*** **„ Point B exist On the Locus B-B `.**

b)..On the Circle of Apothems [The Circle of Diameter OK] are **The Foots A_B** , and when extended to circle [O, OK] as Points A_{1B} these are **The Vertices** of the 2n-Sided Regular Polygon. Because Apothems are Perpendicular to the Mid-Sides , Angles φ_{KOA_B} at Center Point O are equal to φ_B , or $\boldsymbol{\varphi_B = \varphi_{KOA_B}}$, Analogically The Vertices of the 2n-Sided Regular Polygons Belong to Even Polygons . ***Therefore*** By determining the Midpoint of **Arc** $|A_{1A}- A_{1B}|$, Is thus Determined **Point** $\mathbf{A_{1C}}$ .The Golden - Ratio - Point $\mathbf{A_{2C}}$ on Sector $|A_{1.A}, A_{1.B}|$ and lines $|OA_{1.C}|, |OA_{2.C}|$ define **The Foot** $\mathbf{A_C}$, & for 2n-Sided Inbetween Regular ODD Polygon **.** For n = 4 exists → Octagon at Point A_{1A} , n = 6 → Dodecagon at Point A_{1B} , and for n = 5 → **The Decagon at** Foot A_C on Circle of Apothems .

c).. By Extending the lines KA_A , KA_B , KA_C , with the Foots of Apothems A_A , A_B , A_C , to Points B`, B , M_{AB} , of circle [K , KB] , **Define Point** $\mathbf{M_{AB}}$ **of the Regular Decagon** . Because Points on circle [K , KB] occupy the Property of **Constant Sum of Squares** the Same issues for Point M_{AB} , **i.e. The Radius** $|M_{EAB}\ M_{AB}|$ **is Constant „„** ***Therefore*** **„„**

By extending $\mathbf{KA_C}$ on Circle [K,KB] is Defined Point M_{AB} of The Constant-Sum-Squares and on Circle [O , OK] Point $\mathbf{C_{n+1}}$ **,** which is the Vertex of **The Even Sequence Set** [n , n+2] → **n+1= Regular – Pentagon .** ….ο.ε.δ.

3-d-T.. THE REGULAR TRIGONE :

Chord KK` is the Side λ_A of the Regular Digone in circle (O , OK) with Point D_{A1} at Point K and is the Constant Projection of Point , and Chord $KA = KE_A$ is the Side λ_B of the Regular Tetragon in circle (O , OK) . The Circle of **Apothems** is Circle of Diameter OK and circle (O,OK) Archimedes **2n-Sided** Polygons **.** The Locus of the **Constant-Difference** of Squares $[|BK|^2 - |BE_A|^2]$ is the Mid-Perpendicular on KO Chord , which denotes Point B of the circle (O , OK). The Locus of the **Constant - Sum** of Squares $[|BK|^2 + |BE_A|^2]$ is The circle with Center the Mid-Point O_{AE} of KE_A and Radius $[O_{AE}\ B]$ which Intersects the main circle at Point B . From above Pont B is the Vertex of the 2n =2x3 = 6 In-Between Regular Hexagon , ***Therefore*** **,**

Point **B** is the Vertex of Regular-Trigone and **|K`B|** The Side of the Regular Trigone .

3-d-P.. THE REGULAR PENTAGON :

For the Regular Pentagon the central Angle $w_5 = [\frac{2}{5}].180 = 72^o$ and $\varphi_5 = 36^o$, therefore issues Angle $E\widetilde{A_{k+2}}K = 72^o - 36^o = 36^o = \varphi_5$, and so Side $[E\ A_{k+2}] = EK$. Besides the $\boldsymbol{\lambda_{2.N}}$ Polygon in the same circle is $\boldsymbol{\lambda_{2.N}} = \sqrt{2R^2 - R\sqrt{4R^2 - \lambda^2{}_N}} = [A_{k+1}\ K`] = [A_{k+2}\ K`]$ and , $A_{k+2}\ D \parallel K`A_{k+1}$, and issues → $[K\ E] = [E\ A_{k+2}] = [A_{k+1}\ A_{k+2}]$,

The Apothem K M_{k+1} = 2.A`O , The Sides [ K $A_{k+1}$] = [A` A_{k+2}] = 2.A`O , **so that to Be Possible the Construction of the 2-n Polygon** , where then equation (5) becomes [ $A_{k+1}$ $A_{k+2}$ ] ² = [ 2.A`O] ² - [A` $A_{k+1}$] * [ $A_{k+1}$ $A_{k+2}$ ] …… (**6**) , and By Placing [ $A_{k+1}$ $A_{k+2}$] = x , [A` A_{k+2}] = 2.R → x^2 = [2R] ² - 2R.x ← $\mathbf{x^2 + 2R.x - 4R^2 = 0}$ ...(7)

The second Degree Equation (7) occupies the following Roots .

$x_{1,2} = -\frac{2R}{2} \pm \frac{1}{2}\sqrt{4R^2 + 16.R^2} = -R \pm R.\sqrt{5}] = R.[\sqrt{5} - 1] = 2R.\Phi$,

The New Method Applied on Pentagon :

For the Even Sequence Set [n , n+2] , n = Regular-**Tetragon**, n+2= Regular-**Hexagon** n+1= Regular-**Pentagon** , and n → A , n+2 → B , n+1 → C , then issues ,

$\mathbf{A_{1.A}}$ = The Vertex of the 2n-Sided Polygon = Octagon .
$\mathbf{A_{1.B}}$ = The Vertex of the 2n-Sided Polygon = Dodecagon .
$\mathbf{A_{1.C}}$ = The Foot of the 2n-Sided Polygon = Decagon .
$\mathbf{A_{2.C}}$ = The Golden - Ratio - Point on Sector → $|A_{1.A}$, $A_{1.B}|$
$\mathbf{A_{C}}$ = The Apothem`s-Foot of the n+1 , Sided Pentagon .

⇒ **The** Circle [K, KB] of the ***Constant-Difference-Squares*** intersects Tangent K(e) at Point E_B , while the Circle [K, KA] at Point E_A , Simultaneously The Circle [K, KB] **Denotes** Points B , B`, M_{AB} , as the Intersection of the circle and Side |KA| , & as the Intersection of the line-Sector $\mathbf{KA_C}$ Produced at Point M_{AB} of the Circle [K, KB] .

therefore → **Apothem`s-Foot $\mathbf{A_C}$ is transferred at Point $\mathbf{M_{AB}}$ which is On The Constant-Difference-Squares circle [K, KB]**

⇒ **The** Circle [O_B,O_BB] defines The **Constant - Sum - Squares** [$|BK|^2$ +$|BE_B|^2$]=$|O_BB|^2$. from Midpoint O_B , **And Circle [$\mathbf{M_{EAB}}$, $\mathbf{M_{EAB}}$B]** defines The **Constant-Sum – Squares** [$|BE_A|^2$+$|BE_B|^2$] = $|M_{EAB}B|^2$. in Circle [M_{EAB} ,M_{EAB}B] and $|M_{EAB}E_A|$=$|M_{EAB}E_B|$ on $|E_A - E_B|$ Since issues the Relation [$\frac{h_A - h_B}{\sin\varphi} = \frac{\sin\varphi}{(r_a - r_b)} = \lambda_\varphi$], **then sin φ** of the Regular ODD Polygon Must Be satisfied on **Even-Polygons** , **Both** Difference of Heights $\mathbf{h_A - h_B}$, And Their Apothems $\mathbf{A_{1.A}}$, $\mathbf{A_{1.B}}$, **i.e.** On line Sector | $A_{1.A}$, $A_{1.B}$| Is Constructed with a Ruler & a Compass the Golden Ratio Point $\mathbf{A_{2.C}}$ Such that Denotes Between $|OA_{1.C}|$, $|OA_{2.C}|$ lines, The Point $\mathbf{A_C}$ on Apothem`s Circle **.** By extending $\mathbf{KA_C}$ on Circle [K , KB] is Defined Point M_{AB} of The Constant - Sum - Squares and on Circle [O , OK] Point $\mathbf{C_{n+1}}$, which is the Vertex of **The Even Sequence Set** [n , n+2] → **n+1= Regular – Pentagon .**

Remark :

Since Point **B** is on The Circle of **Apothems** and |BK| = |BO| , the line-Sector $|D_{B1}B|$ is The Locus of **Constant-Difference** Squares [$|BK|^2$ - $|BE_A|^2$] , And The Locus of the **Constant -Sum** Squares **.** When line-Sector $|D_{B1}B|$ is Produced , Intersects $|AD_A|$ at Point A_1 where issues $|AA_1|$ = h_A - h_B = Δh , ***Therefore on Pentagon ,***

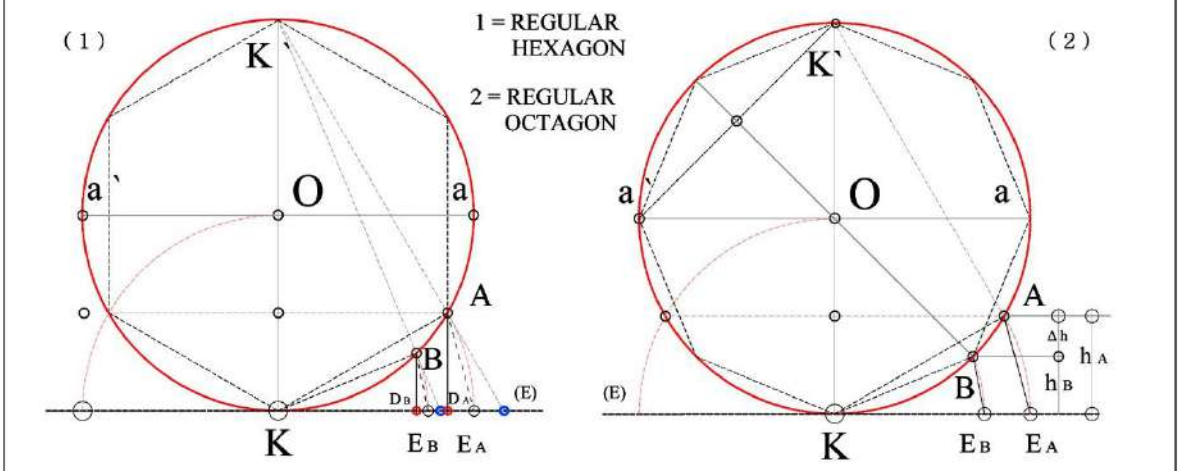
The Harmonic Mean issues as $\sin\varphi_a . \sin\varphi_b = \sin^2\varphi_o$, where $\sin\varphi = (\frac{h_\varphi}{\lambda_\varphi})$ on Difference of Heights [$\mathbf{h_A - h_B}$] , Between the Apothems [$\mathbf{r_a - r_b}$] ,

Since the Ratio of Heights is |R| : |R/2|=**2 : 1** , as issues in the **Golden Radio** Formations ,

therefore → **The Golden Radio on R , issues for the Regular-Decagon-Vertices .**

4-d-..THE REGULAR HEPTAGON : Data in [Fig--**7**--].

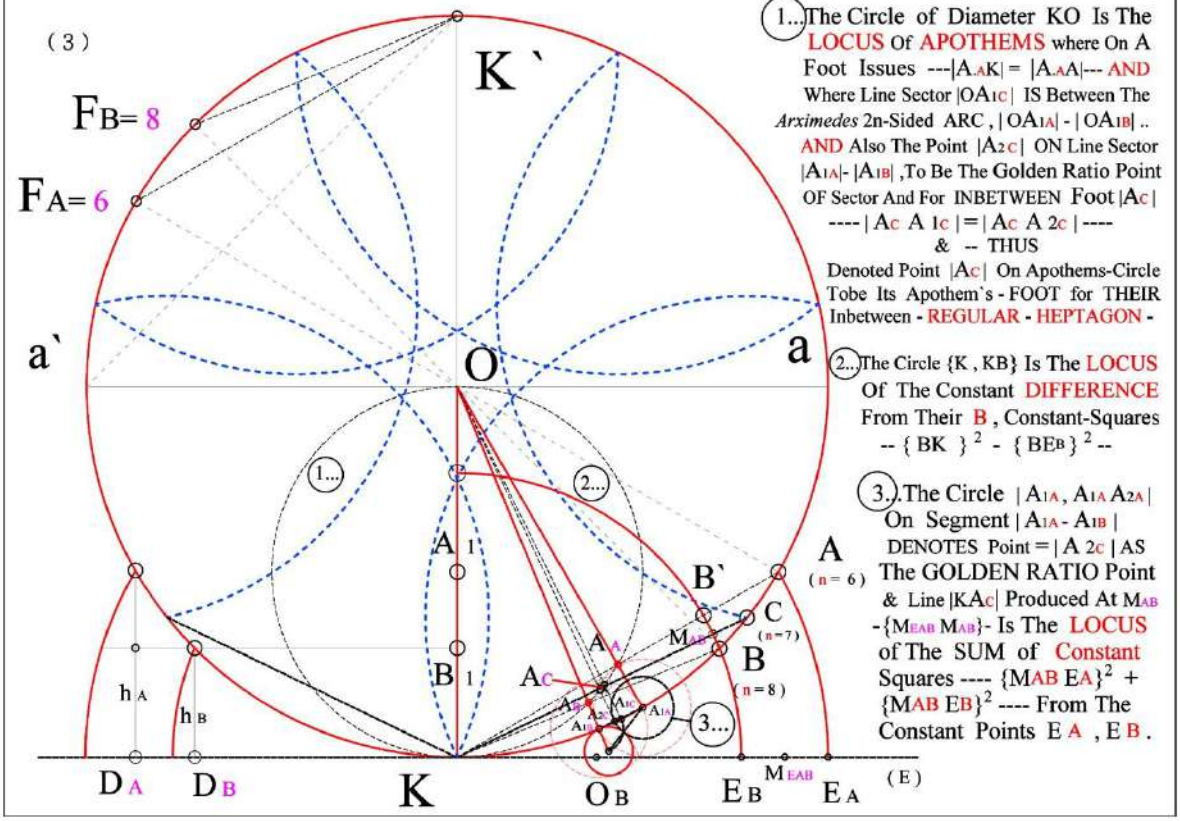
THE GEOMETRICAL CONSTRUCTION OF THE ODD REGULAR - HEPTAGON n +1 = 7 , FROM THE TWO EVEN REGULAR n = 6 HEXAGON AND THE EVEN REGULAR n + 2 = 8 OCTAGON



$KF_A = 6$ = The Side of Regular - Hexagon < $KA = KO = R$ >

$KF_B = 8$ = The Side of Regular - Octagon < $KB = R\sqrt{2-\sqrt{2}}$ >

$KF_C = 7$ = The Side of Regular - Heptagon < $KC = (n = 7)$ >



1...The Circle of Diameter KO Is The LOCUS Of APOTHEMS where On A Foot Issues ---|A.AK| = |A.AA|--- AND Where Line Sector |OA1C| IS Between The *Arximedes* 2n-Sided ARC , | OA1A| - | OA1B| .. AND Also The Point |A2C| ON Line Sector |A1A|- |A1B| ,To Be The Golden Ratio Point OF Sector And For INBETWEEN Foot |AC| ---- | AC A 1C | = | AC A 2C | ---- & -- THUS Denoted Point |AC| On Apothems-Circle Tobe Its Apothem`s - FOOT for THEIR Inbetween - REGULAR - HEPTAGON -

2...The Circle {K , KB} Is The LOCUS Of The Constant DIFFERENCE From Their B , Constant-Squares -- { BK } 2 - { BEB } 2 --

3...The Circle | A1A , A1A A2A | On Segment | A1A - A1B | DENOTES Point = | A 2C | AS The GOLDEN RATIO Point & Line |KAC| Produced At MAB -{MEAB MAB}- Is The LOCUS of The SUM of Constant Squares ---- $\{MAB\ EA\}^2$ + $\{MAB\ EB\}^2$ ---- From The Constant Points E A , E B .

Figure-7- The Prior and ALL Proofs are Applied on Regular Heptagon :

For the Even Sequence Set [n , n+2] , n = Regular-**Hexagon** , n+2= Regular -**Octagon**
n+1= Regular-**Heptagon** , and n → A , n+2 → B , n+1 → C , then issues for Heights ,

A = c.(1e) → **The Sum Σ , of Heights** h_n **, for All Polygons** ←

$\Sigma h_{n+2} - \Sigma h_n = (6+2)R - 6R =$ **2R** → **constant**

$\Sigma h_{n+2} + \Sigma h_n = (6+2)R + 6R =$ **(6+1) .2R** → **constant** meaning that ,

$\Sigma h_{n+2} + \Sigma h_n = (6+2)R + 6R =$ **(7) . 2R** → **constant** meaning that

Between the Two Numerical Even-Regular-Polygons [6 , 8] exists

The Only One (6+1) = 7 Odd – Regular -Heptagon . i.e.

In the System of Regular-Polygons the Even Vertices are Divided on the Diameters , while the Odd Vertices on the Foot of Height (Apothem **) are extended on Diameters .**

B = c.(1,d3) → **The Difference of the Squares $|AK|^2 - |AE_A|^2$ on Isosceles Triangle KAE_A** ←

On the Isosceles Triangle [AKE_A] issues $|AK|^2 - AE_A|^2 = 2.|KD_A - D_AE_A|.|KE_A|$, where O_A is the middle of KE_A , and then issues $|KA|^2 - |AE_A|^2 = 2|KA|.|O_AD_A|$ or , $2.|KA|.|O_AD_A|$, and issues → $\mathbf{|AK|^2 - |AE_A|^2 = 2.|AK|.|O_AD_A|}$ ←

i.e. The **Difference** of the Squares $|AK|^2 - |AE_A|^2$ on the Isosceles Triangle KAE_A, $KA=KE_A$ from Point **A ,** to the Two Constant Points K , E_A , is Constant and Equal to the Product of the Double Chord KA to the Distance of Midpoint to the Foot of A .The same for **B** The Difference of the Squares $|BK|^2 - |BE_B|^2$ on the Isosceles Triangle KBE_B, $KB=KE_B$ from Point **B ,** to the Two Constant Points K , E_B , is Constant and Equal to the Product of Twice the Chord KB multiplied by the Length between the Midpoint O_B and the Foot D_B of Point B . ***Therefore* ,**

The Difference of Squares is Constant for all Even - Sequence– Sets of Chords . And are Required The Three Circles in order to Remain Constant On E_A , E_B .

On Right angle Triangle KAK`, and ***Pythagoras-Relations*** issues $|KA|^2 = |KD_{A1}|.KK` = 2R.h_A$ $|KB|^2 = 2R.h_B$, By Subtraction $|KA|^2 - |KB|^2 = 2R.[h_A - h_B]$ or $\mathbf{\Delta h = h_A - h_B = [KA^2 - KB^2]/2R}$

And The Sum of Heights $\mathbf{[\, h_A + h_B \,] = [\, |KA|^2 + |KB|^2 \,[\, / 2R}$. …Conclusion →

Since the Points K , B , E_B , D_B , O_B , **ARE Fix & Constant** for **Triangle KBE_B ≡ KBD_A ,** and **The Geometrical locus of Points E_B Exist also in the Prior KBD_A**
Therefore In Even - Sequence-Set [6 - 8] , **Triangle KBD_A** is **Fix & Constant** .
In the same Locus AD_A and from the Side of the Regular n = 6-Polygon Exists and is Calculated the Side of the Regular 2n = 12-Polygon , and in the Case of Inbetween , n+1= 7 Regular Heptagon Exists and is Calculated the Side of the Regular 2.| n+1 | = 14 Regular Decatetragon . At the same time ,

The **Geometrical locus** of Points E_A , which difference of squares $AK^2 - AE_A{}^2 = k^2$ from the Two constant Points K , E_A , is **the Constant Height h_A** equal to $|AD_A|$ at the Constant Point D_A , which Distances from the midpoint O_A of KE_A are as

→ $\mathbf{[O_BD_B] = k^2 / 2.KD_A = [BK^2 - BD_A{}^2] / 2.KD_A = constant}$ ←…..Conclusion

Difference & Sum of The Squares Must BOTH Be on those three Circles which are Required in Order to Remain Both Constant , from The Two Fixed Points E_A , E_A.

Since issues the Relation $[\frac{h_A - h_B}{\sin\varphi} = \frac{\sin\varphi}{(r_a - r_b)} = \lambda_\varphi]$, **then** $\sin\varphi$ of the Regular ODD Polygon must Be satisfied on **Even-Polygons** , **Both** Difference of Heights $h_A - h_B$, And Their Apothems $A_{1.A}$, $A_{1.B}$, **i.e.** On line Sector $|A_{1.A}, A_{1.B}|$ Is Constructed with a Ruler & a Compass the Golden Ratio Point $A_{2.C}$ Such that Denotes Between $|OA_{1.C}|$, $|OA_{2.C}|$ lines, The Point $A_{.C}$ on Apothem`s Circle . By extending $KA_{.C}$ on Circle [K , KB] is Defined Point M_{AB} of The Constant - Sum - Squares and on Circle [O , OK] Point C_{n+1} , which is the Vertex of **The Even Sequence Set** [n , n+2] → **n+1= Regular – Heptagon .**

C= 1-c3.. → The Sum of Squares $[|AK|^2 + |BK|^2] = 2R. [h_A + h_B]$ ←

when Applied on an Even Sequence Set of Chords [n , n + 2] then issues ,

On the Isosceles Triangle → $|\frac{AD_A}{AE_A}| = 2.|\frac{AD_A}{KA.}|.|\frac{KD_A}{KA.}|$ ← OR $|KA|^2 = 2.|KD_A|.|AE_A|$

The same for KB , KC , → $|KB|^2 = 2.|KD_B|.|BE_B$ **&** $|KC|^2 = 2.|KD_C|.|CE_C|$ ←

2-**c- THE PROOF FOR CONSTANT Sum Squares in The Directional-Triangles .**

By applying The Median Theorem for Triangles to the Directional Triangles then is Revealed the Locus of the Sum Squares .

On Triangle KBE_B , issues $|BK|^2 + |BE_B|^2 = 2.|O_{BE}B|^2 + \frac{|KE_B|^2}{2.}$ = **Constant-s²** , and since $KE_B = |KB|$ and $|O_{BE}B| = |KB| / 2$, then Becomes → $|O_{BE}B|^2 = s^2/2 - \frac{|KE_B|^2}{4.}$ ← or

Radius $|O_{BE}B| = \sqrt{s^2/2 - \frac{|KE_B|^2}{4.}} = \frac{1}{2}\sqrt{2.s^2 - |KB|^2}$ = **Constant** . .(e-1) **i.e.**

Point B on Circle $|O_{BE}, O_{BE}B|$ formulates Triangle KBE_B , such that The Squares from B To the Two Constant Points K , E_B , is Constant as (e-1)

And by Changing the Two Constant Points to E_B , E_A , their New Sum is , $[BE_A|^2 + BE_B|^2] = s^2$, and their Center O_{BE} is their New Midpoint M_{EAB} ,

***Therefore* , All Points of this Circle Occupy**

The Constant **Radius** → $|M_{EAB}B| = \frac{1}{2}\sqrt{2.s^2 - |KB|^2}$ = **Constant** .... .(e-2) **OR** , **The Locus** of Any Point **X** of the Plane **For Constant Sum-Squares** from Constant Edge Points E_B , E_A , of Line - Sector , **Is The Circle** with **Center** the Mid-Point $|M_{EAB}|$ of The Sector , **And Radius** The Diamesus $|XM_{EAB}|$ of Triangle $|XE_B E_A|$.

4C.. → The How On Apothem`s circle is detected the Apothem of n+1 Odd Polygon ←

Apothem is the Distance of the **Center O** to the Midpoint A_B of **Sector KB** , where $KA_B = A_B B$, and for any other Sector KA , $KA_A = A_A A$. When Apothem is extended to The Circle [O, OK] Intersects this circle at Point A_{1B} , A_{1A} , **i.e.** The Points A_B , A_A are on The Locus of Apothems → { ***The Circle of Diameter KO*** } & according to Archimedes , [4] Elements Book 1. The Points A_{1B} , A_{1A} are **The Vertices of the 2n – Regular Polygons** { 2n=Dodecagon , 2(n+2)=Decaexagon } **on The Circle [O , OK]** where issues , **n.2R = constant , And Since** ⇒

$\Sigma h_{n+2} - \Sigma h_n = (6+2)R - 6R =$ **1 .2R**

$\Sigma h_{n+2} + \Sigma h_n = (6+2)R + 6R =$ **(6+1) .2R**

$\Sigma h_{n+2} + \Sigma h_n = (6+2)R + 6R =$ **(7) . 2R** then ,

Using a Compass we Draw the Equal circles $[A_{1A}\ ,\ A_{1A}\ A_A]$, $[A_{1B}\ ,\ A_{1A}\ A_A]$, which Cut each other at two Points , and by **Using a Ruler** on The two Points Cut the Chord $|\mathbf{A_{1A}\ A_{1B}}|$ **, at Midpoint** $|\mathbf{A_{1C}}|$. Since issues Relation $[\ \frac{\mathbf{h_A - h_B}}{\sin\boldsymbol{\varphi}} = \frac{\sin\boldsymbol{\varphi}}{(\mathbf{r_a - r_b})} = \lambda_{\varphi}\ ]$, **Then** $\sin\boldsymbol{\varphi}$ of the Regular ODD Polygon must Be satisfied on **Even-Polygons** , **Both** Difference of Heights $\mathbf{h_A - h_B}$, And Their Apothems $\mathbf{A_{1.A}}$, $\mathbf{A_{1.B}}$, **i.e.** On line Sector $|A_{1.A}\ ,\ A_{1.B}|$ Is Constructed with a Ruler & a Compass The Golden Ratio Point $\mathbf{A_{2.C}}$ Such that Denotes Between $|OA_{1.C}|$, $|OA_{2.C}|$ lines , The Point $\mathbf{A_{.C}}$ on Apothem`s Circle **.** By extending $\mathbf{KA_{.C}}$ on Circle [K , KB] is Defined Point M_{AB} of The Constant - Sum Squares and on Circle [O , OK] Point $\mathbf{C_{n+1}}$ **,** which is the Vertex of **The Even Set** [n , n+2] → **n+1** = **The Vertex of the Regular Decatetragon , And Chord** $\mathbf{KC_{n+1=7}}$ **The Side of The Regular Heptagon** . …… ο.ε.δ.

5C.. → The How and Where Between Any Two Numerical Even -Regular-Polygons (n , n +2) exists The Only One (n+1) Odd – Regular - Polygon . [b2=d] ..

For the commonly Divergence angle , $\boldsymbol{\varphi}$, equation (1) becomes $\mathbf{h_{\varphi}}$ between $h_A - h_B$,

From $\Delta h = h_A - h_B = \lambda_a \sin\varphi_a - \lambda_b . \sin\varphi_b$, and for the In-between is

$$\mathbf{\Delta h = h_A - h_B = (\lambda_a - \lambda_b) . \sin\varphi = \lambda_{\varphi} . \sin\varphi} \quad \ldots\ldots (2)$$

or , $h_A - h_B = (r_a . \sin\varphi - r_b . \sin\varphi) . \sin\varphi = (r_a - r_b) . \sin^2\varphi$ and ,

$\mathbf{h_A - h_B} = (r_a - r_b) . \sin^2\varphi = \Delta h = \lambda_{\varphi} . \sin\varphi$ or , $\frac{\mathbf{h_A - h_B}}{\sin\boldsymbol{\varphi}} = \frac{\sin\boldsymbol{\varphi}}{(\mathbf{r_a - r_b})} = \lambda_{\varphi}$ ...(**3**)

That is , $\mathbf{\sin\varphi} = (\frac{\mathbf{h_{\varphi}}}{\boldsymbol{\lambda_{\varphi}}})$, is the **Harmonic - Mean** between $[\ \mathbf{h_A - h_B}\ ]$, $[\ \mathbf{r_a - r_b}\ ]$, **And** since $[\mathbf{r_a}] = \frac{1}{2}\sqrt{4R^2 - \lambda^2_a}$, **Also** $[\mathbf{r_b}] = \frac{1}{2}\sqrt{4R^2 - \lambda^2_b}$, **And** Harmonic–Mean on $(\lambda_a - \lambda_b)$.

i.e. The Archimedes **2n-Polygons , IS Needed at FIRST , To Be Applied on The Vertices** $\mathbf{A_{1B}}$, $\mathbf{A_{1A}}$ **,** to Denote Vertex $\mathbf{A_{1C}}$ On Circle [O,OK] , **& The Harmonic–Mean THEN** to Denote Point $\mathbf{A_{2C}}$, On line-Sector $[\mathbf{A_{1B} - A_{1A}}]$ **.**

6C.. → The How & Why Gravitational Forces are Acting on Geometrical Polygons ← From Kepler`s Second Law , Equal Areas are Swept Out in Equal Intervals of Time , Because the Vector $\overleftrightarrow{L}$ of Angular Momentum $\overline{\overline{B}}$ and mass , m , are Both Constant , the Areal-Velocity $\frac{dA}{dt} = \frac{L}{2.m}$ is also Constant in Both Magnitude and Direction, ***Therefore*** , **Equal Areas of Geometry-Ellipse-Traces are Swept Out in Equal Intervals of Time , The Origination of Hydrogen Energy-cave H : [106]**

→ **From Kepler** third law Energy-Closed-Space equation and Newton`s Laws of motion

Constant $k = v^2 . r = (w r)^2 . r = [\ \frac{\mathbf{2\pi}}{\mathbf{T}}\mathbf{r}\ ]^2 . r = \frac{\mathbf{4\pi^2 r^2}}{\mathbf{T^2}} . r = \frac{\mathbf{4\pi^2 r^3}}{\mathbf{T^2}} = 4\pi^2 . \frac{r^3}{T^2} = 4\pi^2 . r^3 . \mathbf{f^2_p}$ ...(k)

Because (k) is constant , $r^3 . f^2_p$, is also a Constant multiplication of cave **r** , and the frequency f , and the , Work ≡ motion is conserved in cave ,**r**, as the , **n** , frequencies

→ In Planck`s-length the Period of Oscillation is $T = \frac{4\pi r}{\sigma\,(1+\sqrt{5})} = 4{,}192.10^{-8}$ s , and **Frequency** $\mathbf{f_P = \frac{1}{T} \equiv \frac{\sigma.(1+\sqrt{5})}{4\pi r} = 2{,}3857265.10^{7}\,Hz}$, which is the minimum in Planck-cave .

The extreme for stresses is $\sigma 1{,}2 = \sigma 1\ /2 \pm (\ ½\ ).\sqrt{\sigma 1^2 + 4.\sigma 1^2} = \sigma_1 .[1 \pm (\sqrt{5})] / 2 = \boldsymbol{\sigma}\ \Phi$,

velocity **v** = w .r = $\frac{2\pi}{T} = 2\pi r$. f = $\mathbf{[\frac{\sigma}{2}].(1+\sqrt{5})}$, frequency $\mathbf{f = \frac{(1+\sqrt{5}\,]).\,\sigma}{4\pi r}}$, Period $T = \frac{4\pi r}{\sigma(1+\sqrt{5})}$

→ **From** Orbit vibration $f_n = \frac{(1+\sqrt{5})\sigma}{4\pi r} = \frac{\sigma}{2\pi r}\,\Phi$, $f^2{}_n = \frac{\sigma^2}{4\pi^2 r^2}\,\Phi^2 = \frac{1}{g\,.a^3}$ and $ga^3\Phi^2 = \frac{4\pi^2 a^2}{\sigma^2}$, or

$a = \frac{1}{g}\,[\frac{2\pi}{\sigma\,\Phi}]^{\,2}$ and from Work W = 2E = B w =J.w² , or 2E = 2πf and $\mathbf{\overline{B}f = \frac{E}{\pi}}$ and Energy

in Planck-scale-cave $2E = \overline{B}f_n = [\frac{(1+\sqrt{5})\sigma}{4\pi r}].\overline{B} = [\frac{\sigma}{2\pi r}].\overline{B}\ \Phi$, or $\mathbf{E = [\Phi\,\frac{\sigma}{4\pi r}].\overline{B}}$ i.e.

Energy in Planck-cave-Particles is dependent on their Spin only , and for Electron $r = a_e$

$E = [\Phi\,\frac{\sigma}{4\pi r}].\overline{B} = 1{,}6180339\ [(\frac{1}{4\pi.1{,}6840307.10^{-17}}].\ 2{,}845976.\ 10^{-17} = \frac{21{,}76.10^{-19}}{1{,}6.10^{\,19}.} = 13{,}6$ eV

$f_N = n\,\frac{(1+\sqrt{5})\sigma}{2\pi r} = \frac{nB}{\pi^2 r4}$, and for a Damping-cave → r(t) = r(t + w) ← as Planck`s scale is

with **min-Damping = 1** and **Unit-Energy-Quantity $\mathbf{W_u}$,** (the critical-energy-unit in

min , r) is this Unit-Stress-Gravity g , as $k = E = \frac{T^2}{a^3} = g = \frac{1}{f^2.a^3}$, i.e. →

Stress g , ***when is entering into the minimum cave*** a , of a minimum Surface , ***then from the Period of Rotation*** T on the Perimeter, ***is created in Surface the minimum Quantity of Energy-cave*** **and is the Hydrogen-Atom ,**where $\mathbf{gf^2 \equiv}$ the **Energy-Part** embodied with stress g , and cave a^3 is the **Space-Part** , in 3-DOF Space as →

$T^2 = g\,a^3 = 9{,}808238.[\ 2\ ,1145016.10^{-11}\ ]^{\,3} = 9\ ,2728158.10^{-32}$ s , and Period

$T = 3{,}04513.10^{-16}$ s , or frequency $f = 3{,}2839982.10^{15}$ / s . From equation **E = h f** =

$6.62607.10^{-34}\ .\ 3{,}2839982.10^{15} = 2{,}175999.\ 10^{-18}$ J / $(1{,}6022.\ 10^{-19})$ = **13,59999 eV**

Above Quantized Energy of **13,6 eV** ***corresponds to Hydrogen-Atom-cave .***

It was shown that ***in Conservative Systems*** of a Central-Force , the Total energy **E** is conserved and at Periapsis , energy $E = \frac{G.Mm}{2\,a}$ and $\mathbf{e} = \sqrt{1 + 2EL^2/G^2M^2m^3}$

and for e = 0 then → $\mathbf{E} = -\ \frac{G^2M^2\,m^3}{2\,L^2}$, i.e. energy is always Negative .

From Hydrogen cave issues $k = E = \frac{T^2}{a^3} = g = [\frac{4\pi^2}{GM}]$ therefore $GM = \frac{4\pi^2 a}{g}$ and

from Total-energy $\mathbf{E} = -\ \frac{GM\,m}{2\,a}$, Rotational-Momentum $\mathbf{L} = \sqrt{(1 - e^2).\,GMm^2.a}$

eccentricity e = 0 , G Mm = - 2a.E , and then → $L^2 = GMm^2.a = -\ 2aE[a] = -\ 2a^2E$.

i.e. Equation $L^2 = -\ 2a^2.E$, denotes that Angular -Momentum L ,
in Orbit - Rims is always Negative and equal to , $L = -\ a\sqrt{2aE}$.

So , Hydrogen-Atom , is the minimum-Energy-cave In Planck`s-Length , occupying the minimum **Negative-Energy** in Rims , *the Space* ,where Quantum-Energy-Rims are the Plane Traces-Conductors in where can roll the ⊖ charged Electrons .

→ **In Hydrogen-cave** , the Resonance **Frequency** of an **Electron** ⊖ , **and a Proton** ⊕ **,** happens from the Stationary Unit-Cave of a System of Kepler second Planetary-law as equation , $4\,\pi^2\,m\,f^2{}_o = k$, and from constant law of Areas $1 = k\ .\mathbf{f^2_o\ a^3}$. Their common Constant-Energy k , is → $k = 4\,\pi^2\,m\,f^2{}_p =$

$= \frac{1}{f^2_p\ a^3}$ or , $f^4_p = \frac{1}{4\pi^2 m a^3}$ and $f_p = \sqrt[4]{\frac{1}{4\pi^2 m.a^3}}$. A loop in Planck`s length $2a = L_P = \pi.1{,}616199.10^{-35}$ m and frequency $f = \frac{c}{\lambda}$, where then $f^4_p = \frac{1}{4\pi^2 m a^3} = [\frac{c}{\lambda}]^4 = [\frac{1}{\lambda}]^4.\{c^4 = \frac{1}{[4\pi a^3]m\pi a}\}$,or for Unit Intensity $I = \frac{Energy}{Area} = \frac{E = hf}{A = 4\pi a^3} = \frac{1KW}{A} = \frac{1}{[4\pi a^3]} = \frac{1}{\lambda^4}\{mc^4.\pi.a\} = \frac{1}{\lambda^4}\{1\}.80{,}760865.10^{32}.(3{,}14).0{,}8080995.10^{-35}$ $= 0{,}5865.\{\frac{1}{\lambda^4}\}$, or Energy-Intension → $\mathbf{I = 0{,}5865\ .\ \{\frac{1}{\lambda^4}\}}$ ← which is the Rayleigh Scattering , **i.e.**

The Quanta of Energy , Passing through a Unit-Surface or **Volume** ,
which becomes from the Kepler`s Second and third law Mechanism ,
is **Inversely Proportional** to the **fourth Power** of the wavelength $\boldsymbol{\lambda}$.

→ **In** Outer Electromagnetic fields , frequency ,**w**, is that of inner motion → $\mathbf{w} = \frac{2\pi}{T} = 2\pi.f_N \equiv [\frac{1+\sqrt{5}}{2}]\frac{\sigma}{2\pi r}$ $= \frac{\sigma.\Phi}{r}$ **,** which continuous to follow the ***Growth-Golden-Ratio-Pattern*** . Photon occupies **such** the Property of Material Point **as** the Growth-Pattern . With this way Photon`s frequency → $f_P = 1 / T_P$, Kicks-Start everything found on its-way . From Resonance constant $k = \frac{1}{f^2_n.a^3}$ → or $1 = k\ .f^2_n.\ a^3$ ← and from The Electric-Magnetic Field of 3-Conductors Results on **2-Anti-Parallel - Strands**. Which Response to the two Apothems OA_{1A} , OA_{2A} , of the ODD Regular Space Polygons .

7C..→ Kepler`s laws and Newton`s Lagrange laws ← for Orbits , then the cave-Semi-major-axis is

$\mathbf{a} = \sqrt[3]{T^2/k} = \sqrt[3]{\frac{1}{g.f^2}} = \sqrt[3]{\frac{4.\pi^2.r^2}{g.\sigma^2.\Phi^2}} = \sqrt[3]{\frac{[4\pi^2].x\ \ r^2}{[g.\Phi^2].x\ \sigma^2}}$, and $\mathbf{f_R} = \frac{w_R}{2\pi} = \sqrt[2]{\frac{k=1}{g.a^3}} = \sqrt[2]{\frac{\sigma}{g.\Phi^2.a^3}} \equiv \sqrt{\frac{k}{md}}$ **eV** …(a)

i.e. Is the Resonance-frequency f_R , in **Photons loop** , and since being the Energy-Space Quanta , it follows the **Growth – Golden Ratio-Pattern** , when travels in all Cosmos.

From Momentum ≡ Spin ≡$\bar{\mathbf{B}}$≡ r m v = $\mathbf{[\frac{\pi.r^2}{4}].\bar{c}\ x\ \bar{r}}$ where $\mathbf{\pm\ [\frac{\pi.r^2}{4}]}$ = The **Quarter circle.**

The **Quarter circle , + + ,** consist the + Spin ≡ $\bar{\mathbf{B}}$ which is the Gravity Force , and the

Quarter circle , - - , consist the -Spin ≡ $\bar{\mathbf{B}}$ which is the Gravity Force from Total ,+, -,-,+,

When r → 0 Or when $\bar{c}$ doesn`t act on the **Quarter circle** then happens **Black-Hole.**

In [108] is Determined the when Newton's Gravitational Force $\bar{\mathbf{G}}$, Stops Acting
Thrusting , on The Light velocity Vector $\bar{\mathbf{c}}$, and happen the Black-Holes .

→ Frequency in Material-Point is → $\mathbf{f_n} = (\frac{n\sigma}{8\ r^2}).\bar{B} \equiv [\frac{1+\sqrt{5}}{2}]\frac{\sigma}{2\pi r} \equiv [\frac{n\sigma}{8\ r^2}].\bar{B} \equiv [\frac{1+\sqrt{5}}{2}]\frac{Stress}{Perimeter}$ $\equiv [\frac{1+\sqrt{5}}{2}]\frac{Force\ [N]}{Area*L[m^3]} \equiv \frac{E}{h}$, then it occupies the Property of Golden-ratio Pattern Φ , From Resonance constant $k = \frac{1}{f^2_n.a^3}$ → or $1 = k\ .f^2_n.\ a^3$ ← & from Euler`s Rigid Body Dynamics work

→ $W = 2L = \bar{B}\ .\ \bar{w} = J\ .\ w^2 \equiv h.\ \mathbf{f_n}$ ←

From Kepler-Orbit-Vibration the equation of inverse Period → $f_n = \frac{(1+\sqrt{5})\sigma}{4\pi r} = [\frac{\sigma}{2\pi r}]\ \Phi$,

$f^2_n = \frac{\sigma^2}{4\pi^2 r^2}\Phi^2 = \frac{1}{g.a^3}$, and $ga^3\Phi^2 = \frac{4\pi^2 a^2}{\sigma^2}$, or $a = \frac{1}{g}[\frac{2\pi}{\sigma\ \Phi}]^2$ and from Work in Orbits

$W = 2E = B\ w = J.w^2$, or **2E = 2π f B** i.e. → **Total – Spin * Frequency** ≡ $\bar{\mathbf{B}}\ \mathbf{f} = \frac{E}{\pi}$, and

Energy ≡ motion in Planck-scale-cave $r = L_P$, is $2E = \bar{B}f_n = [\frac{(1+\sqrt{5})\sigma}{4\pi r}].\bar{B} = [\frac{\sigma}{2\pi r}].\bar{B}\ \Phi$, or

$\mathbf{E = [\Phi\frac{\sigma}{4\pi r}]\ .\ \bar{B} \equiv \frac{|B|^2}{2\ \pi^2 r^4} \equiv \frac{|B|^2}{2\ [\pi.r^2]^2}}$ ... (4) **i.e.** The Energy E , is Distributed into the Area of **2 - circles** $[\pi.r^2]$ **, And** for the Electric-Magnetic Field of 3-Conductors in Cubes , Results on **2-Anti - Parallel – Strands** , which Response for Polygons to the **Two Apothems** OA_{1C} , OA_{2C} , of the **ODD** Regular Space Polygons .o.ε.δ.

7C..→ The How & Why Forces act on Energy Conductors and Geometry-Surfaces. [106]

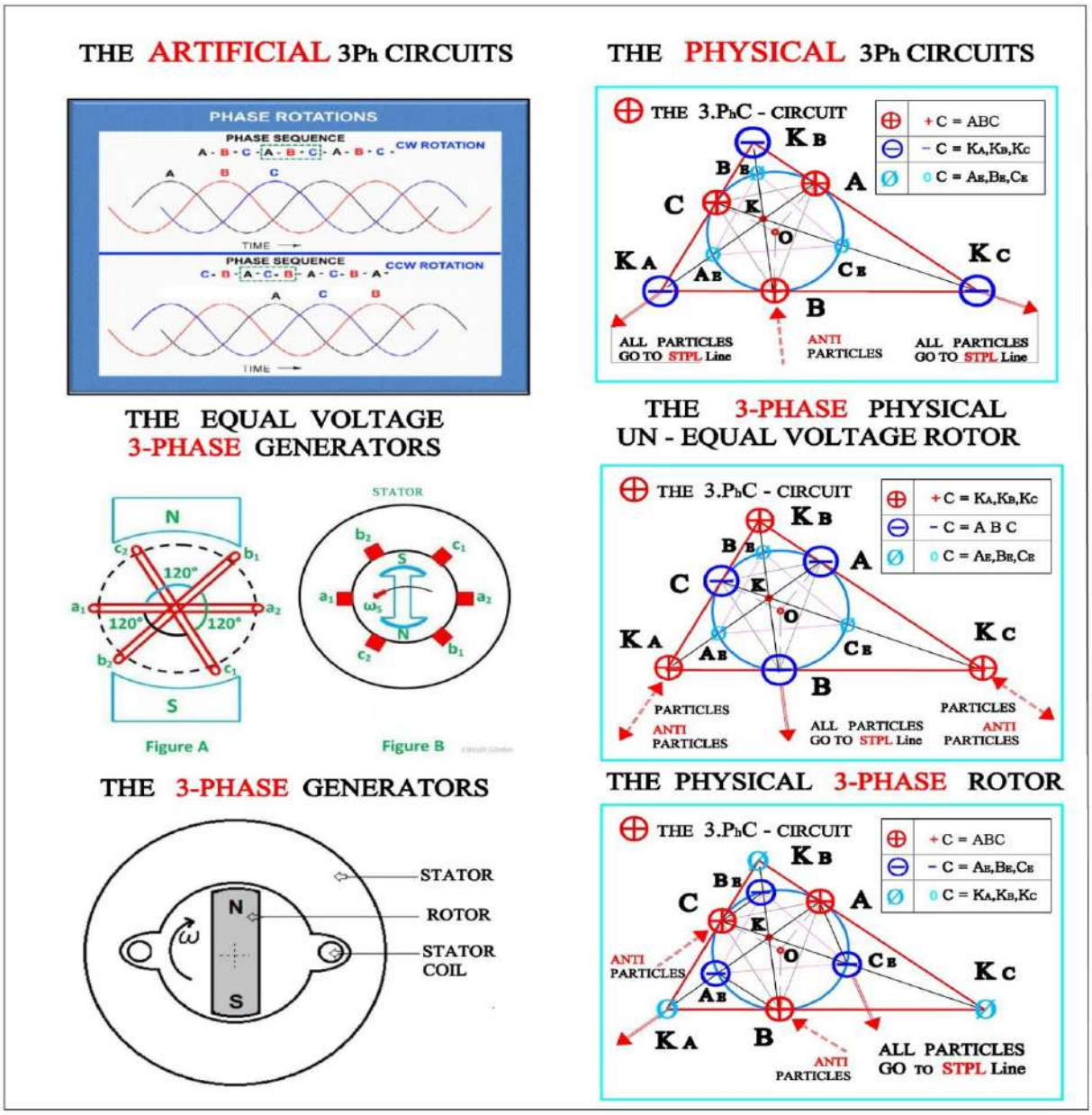


Figure –7a- 1).. In The Left-column exist the Three-Phase E.M.Fs in a 3 Phase Circuit in a 3 – Phase System of **Artificial - Generator** .

2.. In The Right-column Exist the Three-Phase E.M.Fs in 3-Phase Circuits and a Circular 3- Phase **PHYSICAL - Generator** Shakes Off this Quantum Energy.

3.. In the Symmetrical 3 -Phase System , The Rotor defines the Equal speed in an Artificial-Generator , while Light-velocity , **c** , defines velocity between , + and -

4.. In Figure -8- , **STPL of** → D_A , P_A D_B , P_B D_C , P_C ← **Points** become from the Un-equal voltages analogous to , AK_A , BK_B , CK_C , while in Figure -11- , STPL Line tends to Infinite , ∞ , with different orientation Parallel lines .

8C.. → The Action of Gravitational Force G , On Polygons Surfaces . [109]

The Gravitational force $\Rightarrow$ $\mathbf{G = g\ k = k_E\ g = g.k_R\ g_R = \bar{p},g_c = \sigma = \frac{Force}{Area} = \frac{g.Mass}{Area}}$,

In [75-B] ,The Permeable-Resonance-Path for M-Point is → The Resonance-frequencies $\mathbf{f_R}$ [S≡ $\mathbf{f_{1=n}}$,$\mathbf{f_2}$, $\mathbf{f_{R=}w^2}$] with $\mathbf{f_n}$= [n $\frac{(1+\sqrt{5})\sigma}{4\pi r} = \frac{n\sigma.\bar{B}}{8\,r^2}$] related to Spin $\bar{\mathbf{B}}$, Stress $\boldsymbol{\sigma}$ and cave r .

Since frequency in Material-Point is →

$\mathbf{f_n}$ = ($\frac{n\sigma}{8\,r^2}$) .$\bar{\mathbf{B}}$ ≡ [$\frac{1+\sqrt{5}}{2}$] $\frac{\sigma}{2\pi r}$ ≡ [$\frac{n\sigma}{8\,r^2}$].$\bar{B}$ ≡ [$\frac{1+\sqrt{5}}{2}$]$\frac{\text{Stress}}{\text{Perimeter}}$ ≡ [$\frac{1+\sqrt{5}}{2}$]$\frac{\text{Force [N]}}{\text{Area}*\text{L}[m^3]}$ ≡ $\frac{\mathbf{E}}{\mathbf{h}}$, then

it occupies the Property of the Golden-ratio Pattern Φ , and equation defines that the Material Point of frequency $\mathbf{f_n}$, when collide with another Material Point , or with any other Particle or Particles then Produces another monad as → 1 ≡ A New Quaternion and the first continuous to be of the same Identity , frequency $\mathbf{f_n}$, as before and from Euler`s , Rigid Body Dynamics work is → W = 2L = $\bar{B}$. $\bar{w}$ = J . w² ≡ h. $\mathbf{f_n}$ ←

from Energy-loop relation 1 = c. r³. $\mathbf{f_P}^2$ → f_P=$\sqrt{1/cr^3}$, from f = E / h then f = $\frac{\mathbf{E}}{\mathbf{h}}$ = $\sqrt{1/cr^3}$

or → E = h.$\sqrt{1/c.\,r^3}$, and $\frac{E^2}{h^2} = \frac{1}{cr^3}$ → E ² = $\frac{h^2}{c\,r^3}$, an Energy relation between the **, c , r** .

This relation is used for Black- Holes Energy where r , result to zero , [71]

The Permissible - Permeable - Resonance - PATHS is as below for the MEANS ,

1) Solids → The Normal-mode-Vibration - System { - w² [M] + [K] (X) } = 0]
2) Liquids → The Cauchy Stress-Tensor as Momentum equation { ∇.σ = -∇p +∇.τ }
3) Gazes → The combined Avogadro`s Pressure-law { PV = n RT = n.mv²/3 }
4) Crystals → The Cauchy Ellipsoid-Stress-tensor where { E⊥B⊥r ≡ $\sigma_1 \perp \sigma_2 \perp \sigma_3$ }
5) Molecules → The Lattice-Crystal-Arrangement with their Chemical Bonds relation
6) Atoms → The Energy-Rims with Chemical Bonds , Ionic , Covalent , relation
7) Particles → The Resultance of the One-Dimensional-Collision $\bar{v}_{ij}$= $\bar{v}_j$ - $\bar{v}_i$ = $\bar{w}_{ij}$. $\bar{r}_{ij}$
8) M-Points → The Resonance-frequencies $\mathbf{f_R}$ [S ≡ $f_{1=\,n}$,f_2, f_3,$f_{R=}w^2$] = $\mathbf{f_n}$ = [⊕ ↔ ⊖]

= n $\frac{(1+\sqrt{5})\sigma}{4\pi r} = \frac{n.\sigma.\bar{B}}{8\,r^2}$ → i.e. the Golden-ratio-Pattern , which is related to Spin.

9) Cave-Orbit→ The Relation c.L_s = L_v , Light-velocity 3.10^8 m/s*1.10^{-42} m cave =3.10^{-34} m²/s are the Cave-Energy-Plane-Rims in Atom`s , Planet orbits.

9C..→In Figure -13-The Electric-Magnetic Field of 3-Conductors **as Regular Tetraahedron** , Results on **2-Anti-Parallel-Strands** and this Because a vector of Amplitude **A** , **Rotating** at constant Angular Speed , **w** , can be represented as a Complex quantity **z** in the Argand Diagram as $\mathbf{z = A.e^{iwt}}$ = A cos wt + i. A sin wt = x +i. y of Wave-form where x ≡ **The Real** and y =**The Imaginary Part** of the Wave ≡ **(s + $\bar{\mathbf{v}}$ ∇i**).

The Stability of→{ ⊕4-**Space** [D A B C]} Regular-***Tetrahedron*** , **IN** {⊖ 8-**Anti- Space *Cube*** [$DA_1B\,D_1$, $C\,C_1AB_1$] } , and The →{ ⊕ 8 - **Space** [DA_1BD_1 , CC_1AB_1] }- ***Cube*** **INTO The** {⊖ 16-**Anti-Space** [1 ,2 ,3 ,4 ,5 , 6, 7, 8-9 ,10 ,11, 12 , 13,14 ,15,16]} ***Sphere Cube Vertices.***

Point [**16** ≡|⇉|**Z**|] consists The **Navel - String Gate** in {Z-(ΔD , ABC) , Δ[T1T2T3]}Stationary Bases in Dual Photon → $\bar{v}$ [$\frac{\sigma\Phi}{2\pi r}$+$\frac{\sigma}{2\pi r}$] ≡ $\bar{v}$.[$\boxed{\bar{f}_n}$ + f_n] ← ,

5-d-.. THE REGULAR ENNEAGON :

For the Even Sequence Set [n , n+2] , n = Regular-**Octagon**, n+2= Regular-**Decagon** n+1= Regular-**Enneagon** , and n → A , n+2 → B , n+1 → C , then issues ,

$\mathbf{A_{1.A}}$ = The Vertex of the 2n-Sided Polygon = Decaexagon .
$\mathbf{A_{1.B}}$ = The Vertex of the 2n-Sided Polygon = Ecosagonon .
$\mathbf{A_{1.C}}$ = The Foot of the 2n-Sided Polygon = Decaoctagon .
$\mathbf{A_{2.C}}$ = The Golden - Ratio - Point on Sector → | $A_{1.A}$, $A_{1.B}$ |
$\mathbf{A_{C}}$ = The Apothem`s-Foot of the n+1 , Sided Enneagon .

⇒ **The** Circle [K, KB] of the ***Constant-Difference-Squares*** intersects Tangent K(e) at Point E_B , while the Circle [K, KA] at Point E_A , Simultaneously The Circle [K, KB] **Denotes** Points B , B`, M_{AB} , as the Intersection of the circle and Side |KA| , & as the Intersection of the line-Sector $\mathbf{KA_C}$ Produced at Point M_{AB} of the Circle [K, KB] .

therefore → **Apothem`s-Foot $\mathbf{A_C}$ is transferred at Point $\mathbf{M_{AB}}$ which is On The Constant-Difference-Squares circle [K, KB]**

⇒ **The** Circle [O_B,O_BB] defines The **Constant - Sum - Squares** [|BK|² +|BE_B|²]=|O_BB| ². from Midpoint O_B , **And Circle [$\mathbf{M_{EAB}}$, $\mathbf{M_{EAB}}$B]** defines The **Constant-Sum – Squares** [|BE_A|²+|BE_B|²] = |M_{EAB}B|². in Circle [M_{EAB} ,M_{EAB}B] and |$M_{EAB}E_A$|=|$M_{EAB}E_B$| on |E_A- E_B| Since issues the Relation [$\frac{h_A - h_B}{\sin\varphi} = \frac{\sin\varphi}{(r_a - r_b)} = \lambda_\varphi$], **then sin φ** of the Regular ODD Polygon must Be satisfied on **Even-Polygons** , **Both** Difference of Heights $\mathbf{h_A - h_B}$, And Their Apothems $\mathbf{A_{1.A}}$, $\mathbf{A_{1.B}}$, **i.e.** On line Sector | $A_{1.A}$, $A_{1.B}$| Is Constructed with a Ruler & a Compass the Golden Ratio Point $\mathbf{A_{2.C}}$ Such that Denotes Between |$OA_{1.C}$| , |$OA_{2.C}$| lines, The Point $\mathbf{A_{.C}}$ on Apothem`s Circle **.** By extending $\mathbf{KA_{.C}}$ on Circle [K , KB] is Defined Point M_{AB} of The Constant - Sum - Squares and on Circle [O , OK] Point $\mathbf{C_{n+1}}$, which is The Vertex of **The Even Sequence Set** [n , n+2] → **n+1= Regular – Pentagon .**

Remark :

Since Point **B** is on The Circle of **Apothems** and |BK| = |BO| , the line-Sector |D_{B1}B| is The Locus of **Constant-Difference** Squares [|BK|² - |BE_A|²] , And The Locus of the **Constant -Sum** Squares **.** When line-Sector |D_{B1}B| is Produced , Intersects |AD_A| at Point A_1 where issues |AA_1| = h_A - h_B = Δh , ***Therefore on Enneagon ,***
The Harmonic Mean issues as $\sin\varphi_a . \sin\varphi_b = \sin^2\varphi_o$, where $\sin\varphi = (\frac{h_\varphi}{\lambda_\varphi})$
o The Electric-Magnetic Field of 3-Conductors Results on **2-Anti-Parallel - Strands**.

⇒ **Since** the Total Energy in Areas or Polygons is → $\mathbf{E} = [\Phi\frac{\sigma}{4\pi r}].\bar{\mathbf{B}} \equiv \frac{|\mathbf{B}|^2}{2\pi^2 r^4} \equiv \frac{|\mathbf{B}|^2}{2[\pi.r^2]^2}$ ←

Then the Constant - Sum - Squares [|BK|² +|BE_B|²]= |O_BB| ² , Is transferred onto These **Two Strands of Apothems** which Response on the two Apothems OA_{1A} , OA_{2A} , of the ODD Regular Space Polygons . and are such for the Difference of Heights [$\mathbf{h_A - h_B}$] , as for the Difference of Apothems [$\mathbf{r_a - r_b}$] , ο.ε.δ.

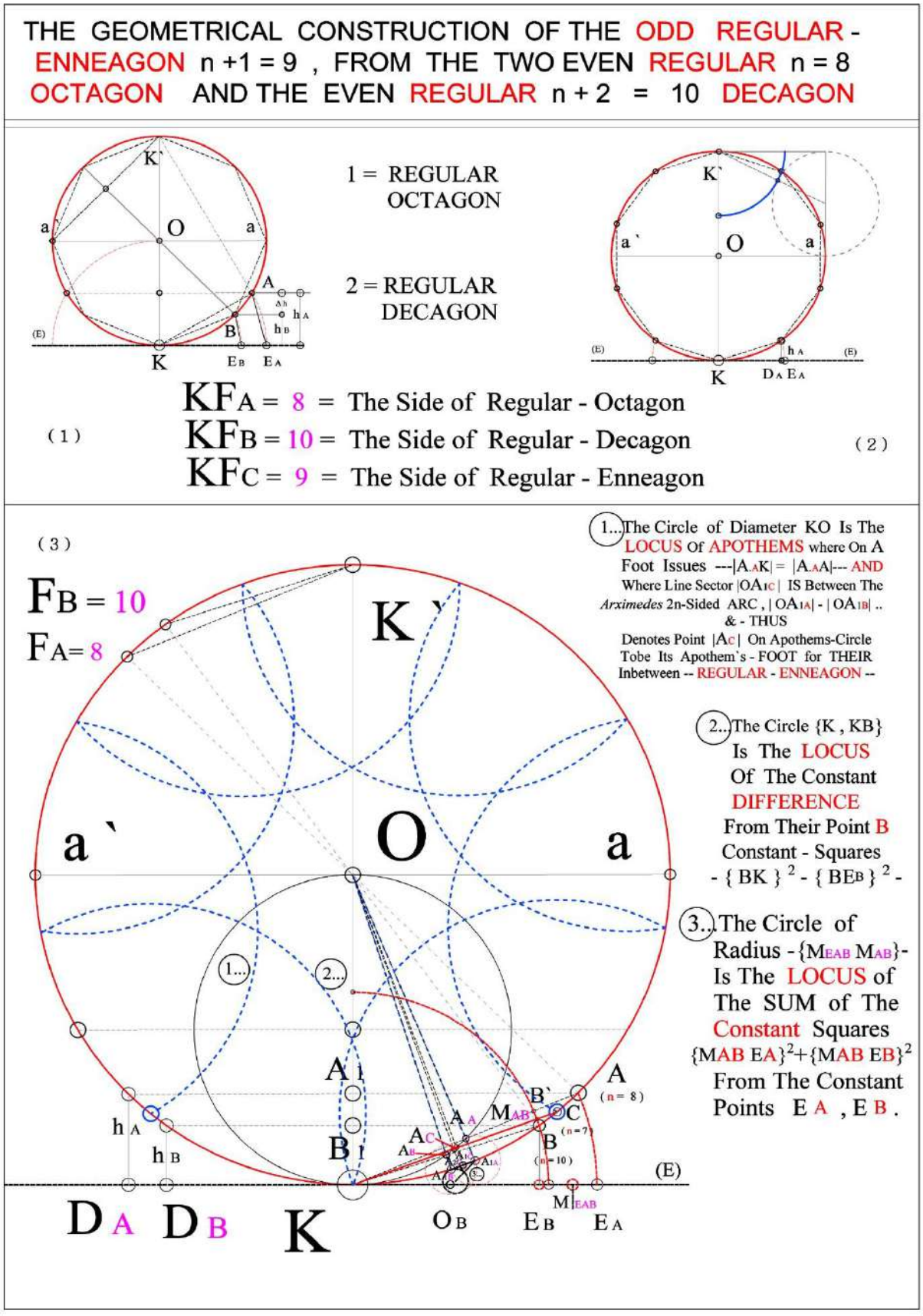


Figure-8- The Regular Enneagon :

6-d-.. THE REGULAR DECAEPTAGON :

Data in [Fig--8--].

The System of the circle [O , OK] , of Diameters KK`⊥ aa`= 2R **,** KK `⊥ $K_{(E)}$ and Chords KA = $KE_A$ , KB = $KE_B$ , Represent The Fixed Twin - Couple of Triangles **KAK ` , KA E_A , KA E_B .** where KA , KB , Is an [**n , n+2**] , Even Sequence Set . **The** Circle of Apothems with Diameter |KO| , **Denotes** at Point A_A , **The Foot of n-Apothem ,** And By Extending the Apothem OA_A to the Circle [O , OK] at Point $A_{1.A}$, then is Defined *According to Archimedes* the Vertex of the 2n-Sided Polygon With the same way By Extending the Apothem OA_B to the Circle [O , OK] at Point $A_{1.B}$, then is Defined the Vertex of the 2[n+2]-Sided Polygon for Even Points A ,B.

therefore → **On Apothem`s circle is detected the Apothem of n+1 Odd Polygon .**

For Even Sequence Set [n , n+2] , n = Regular-**Decaexagon** , n+2= Regular-**Decaoctagon** n+1= Regular-**Decaeptagon** , and n → A , n+2 → B , n+1 → C , then issues ,

$\mathbf{A_{1.A}}$ = The Vertex of the 2n-Sided Polygon = Triantadiogonion .

$\mathbf{A_{1.B}}$ = The Vertex of the 2n-Sided Polygon = Triantaexagonion .

$\mathbf{A_{1.C}}$ = The Foot of the 2n-Sided Polygon = Triantatetragonion .

$\mathbf{A_{2.C}}$ = The Golden - Ratio - Point on Sector → | $A_{1.A}$, $A_{1.B}$ |

$\mathbf{A_{C}}$ = The Apothem`s-Foot of the n+1, Sided Decaeptagon .

⇒ **The** Circle [K, KB] of the ***Constant-Difference-Squares*** intersects Tangent K(e) at Point E_B , while the Circle [K, KA] at Point E_A , Simultaneously The Circle [K, KB] **Denotes** Points B , B`, M_{AB} , as the Intersection of the circle and Side |KA| , & as the Intersection of the line-Sector **KA_C** Produced at Point M_{AB} of the Circle [K, KB] .

therefore → **Apothem`s-Foot A_C is transferred at Point M_{AB} which is On The Constant-Difference-Squares circle [K, KB]**

⇒ **The** Circle [O_B,O_BB] defines The **Constant - Sum - Squares** [$|BK|^2$ +$|BE_B|^2$]=$|O_BB|^2$. from Midpoint O_B , **And Circle [M_{EAB} , M_{EAB}B]** defines The **Constant-Sum – Squares** [$|BE_A|^2$+$|BE_B|^2$] = $|M_{EAB}B|^2$. in Circle [M_{EAB} ,M_{EAB}B] and $|M_{EAB}E_A|$=$|M_{EAB}E_B|$ on $|E_A - E_B|$ Since issues the Relation [$\frac{h_A - h_B}{\sin\varphi} = \frac{\sin\varphi}{(r_a - r_b)} = \lambda_\varphi$], **then** $\sin\varphi$ of the Regular ODD Polygon must Be satisfied on **Even-Polygons** , **Both** Difference of Heights $\mathbf{h_A - h_B}$, And Their Apothems $\mathbf{A_{1.A}}$, $\mathbf{A_{1.B}}$, **i.e.** On line Sector | $A_{1.A}$, $A_{1.B}$| Is Constructed with a Ruler & a Compass the Golden Ratio Point $\mathbf{A_{2.C}}$ Such that Denotes Between $|OA_{1.C}|$, $|OA_{2.C}|$ lines, The Point $\mathbf{A_{.C}}$ on Apothem`s Circle **.** By extending **$KA_{.C}$** on Circle [K , KB] is Defined Point M_{AB} of The Constant - Sum - Squares and on Circle [O , OK] Point $\mathbf{C_{n+1}}$ **,** which is The Vertex of **The Even Sequence Set** [n , n+2] → **n+1= Regular – Pentagon .**

Remark :

Since Point **B** is on The Circle of **Apothems** and |BK| = |BO| , the line-Sector $|D_{B1}B|$ is The Locus of **Constant-Difference** Squares [$|BK|^2$ - $|BE_A|^2$] , And The Locus of the **Constant -Sum** Squares **.** When line-Sector $|D_{B1}B|$ is Produced , Intersects $|AD_A|$ at Point A_1 where issues $|AA_1| = h_A - h_B = \Delta h$, ***Therefore on Enneagon ,***
The Harmonic Mean issues as $\sin\varphi_a . \sin\varphi_b = \sin^2\varphi_o$, where $\sin\varphi = (\frac{h_\varphi}{\lambda_\varphi})$
on Difference of Heights [$\mathbf{h_A - h_B}$] , Between the Apothems [$\mathbf{r_a - r_b}$] , o.ε.δ.

General Remarks regarding The Regular Polygons :

1.... **The Even Sequence Set** [n , n+2] , **n** for Tetragon and **Side KA = R,$\sqrt{3}$** , **n+2** for Hexagon and Side **KB = R** , The Difference of Squares is $3R^2 - R^2 = \mathbf{2.R^2}$, and The Sum of Squares $4.R^2$, **i.e.** an Ratio **1 : 2** , as issues in **Golden Radio** Formation
therefore → **The Golden Radio on R** , **issues for Regular-Decagon-Vertices** .

2… **The Method Solves** the Problem of The Regular Polygons in Plane Using a **Circle with Two Perpendicular axis KK` , aa` ,** and An Even-Sequence-Set of Chords. This Process allows The Rotation of The System through the Two axis separately and Thus is Possible **The Spatial Construction of the Regular Polygons** .
In a Sphere with Infinite great circles within The Sphere is Possible By Rotation on and Through the Two Perpendicular axis , to Design all Sequence-Sets of The Even Regular - Polygons with their Inbetween Odd Polygon into The Spatial Geometry **.**

3…**The Precise Definition of Points** on lines and Circles enables the Programming of the Method to Use The Coordinate Systems (x . y , z) , r , φ , (i , j , k) For Algebrical Solutions and many Others .

4… **Because The Spaces–Anti-Spaces** Exist simultaneously on their Great Circles of The Spheres , Then All Informations can Immediately Transferred to Each - Other **.**

5....**The Gravitational force G and the Stresses σ , in Nature-Spatial-Polygons :** [77]
From A-7 , In Planck`s cave , The ***Least-Unit-Energy-cave*** is that of **Hydrogen-Cave** H , and Issues $g . r^3 . \mathbf{f^2_p} = 1$, or $g = [\frac{1}{\mathbf{r^3.f_p{}^2}}] = [\frac{T^2_P}{r^3}]$. The Earth constant $k_E = 6{,}8116.10^{-12}$, and **The Gravitational-Constant-Force G , becomes as Stress** ≡ Force/Area ≡ **g** , as equation $G = g\ k_E = g .[g_E\ k_E] = [\frac{T^2_P}{a^3}].[g_L\ k_L] = [\frac{c.r^3}{a^3}].[g_L\ k_L] = 9{,}8076925 * 6{,}8116.10^{-12} \equiv 6{,}68056.10^{-11}\ \frac{m^3}{Ns^2}$, and **Effects on gravity** → **g** ≡ 9,8076925 **as Stress** $\frac{Kg}{cm^2}$ [73]
The Beyond-Planck-length Force **F = σ . A** = The Glue-Bond ≡ Stress x Area ≡ $[\frac{2\pi f.r}{\Phi}].A = w\ r.[\frac{\mathbf{A}}{\mathbf{\Phi}}] = \bar{v}\ [\frac{\mathbf{A}}{\mathbf{\Phi}}]$, and becomes a moving Storage $\frac{\mathbf{A}}{\mathbf{\Phi}}$ travelling with velocity **n** times that of light-velocity **c** , as equation $\bar{v} = n\ c$.

5-1.. For The Beyond-Planck-Length Force issues → $\mathbf{F = \Phi^3.\boxed{[\oplus\ \sigma\ \ominus]} = \sigma\ x\ \Phi^3}$ where ,
σ ≡ $\boxed{[\oplus\ \sigma\ \ominus]}$ **≡ The Stress of Cave** as the **Quantized distance r** → $\boxed{[\oplus\leftarrow \mathbf{r} \rightarrow\ominus]}$
and $\Phi = \frac{(1+\sqrt{5})}{2}$ is the mould of Quantization , ***a kind of Impedance in Electricity*** , and
in Material-Points frequency $\mathbf{f} = \frac{c}{2\pi r} = 2{,}93949410^{42}$ H , and $\sigma = \frac{\mathbf{c}}{\mathbf{\Phi}}$.

5-2.. For the In Planck-Length- G-Force as Stress $\boldsymbol{\sigma} = \frac{2\pi rf}{(n)\Phi} = \frac{2\pi rf}{1.\Phi} = \frac{2\pi.1{,}616199.10^{-35}.2{,}93949410^{42}}{1{,}6180339}$

= 1,84456315.10^{8} t/m2 = 1,84456315.10^{11} Kg/m2 , and the Angular velocity , w, from Total Planck Cave is |w| = $\frac{\sigma}{2r}$ [1+$\sqrt{5}$] = ($\frac{1,852816510^{11}}{2.1,616199.10^{-35}}$).1,6180339 = 9.274599.$10^{46}$, rad/sec and

5-3.. For the Outer-Planck-Length- G-Force as Stress $\boldsymbol{\sigma = \sigma_{Pl}}$ = 1,84456315. 10^{-11} Kg/m2 , and **,** $\mathbf{G = \sigma \times \Phi^3 \equiv [1,84456315.10^{-11}] . [3,6180339887] = 6,673692.10^{-11}\ \frac{m^3}{Ns^2}}$, *Nevertheless* For Earth-System mass M_E = 5,9723.10^{24} Kg , and for the Area → Radius 6378,137 Km = 6 , 378.10^{6} m , then Earth-constant $\mathbf{k_E = \frac{[6,378.10^6]^2}{5,9723.10^{24}}}$ = 6,8115518.10^{-12} and Gravitational Force $\mathbf{G = g.k_E \equiv g\ .[g_L\ k_L] \equiv [\frac{T^2_P}{a^3}].[g_L\ k_L] \equiv 9,80769.6,8115518.10^{-12} \equiv 6,68056.10^{-11}\ \frac{m^3}{Ns^2}}$

6a..All Stresses Become from a Force and a Surface as equation $\sigma = \frac{F}{A}$, and in the case of Gravitational constant G and a cave , r , then → $\sigma = \frac{G}{4\pi r^2} = \frac{G}{\pi(2r)^2} = \frac{G}{\pi s^2}$ **,** or **a vector** |$\bar{s}$|. Above relation means that Force **G** needs a Vector-Surface $\boldsymbol{\pi.s^2}$ to be spread as Stress $\boldsymbol{\sigma}$ which is the case of Constant-light-velocity **c** as ⇉ , or ⇈ $c_{E \perp M}$. [96] The **Vector** |$\bar{s}$| **transfers the Axial-Rotational motion to circulate Polygonal-Plane-motion** .

6b.. The case of a vector **s** , is the **Linear-Stress** while of an Plane is the **Surface-Stress** and consequently of a Volume is a **Space-Stress** , as this was referred before for **G** Force , i.e. $G \equiv \sigma.\Phi^3 \equiv \Phi^2$. [{$\sigma\ \Phi$} ≡ $2\pi f_P\ r \equiv w\ r \equiv \bar{v} \equiv m\ a = \bar{c} = [\frac{2.B}{\pi r^3}]$ are all Energy-Vectors .

3c.. Since Stresses follow equation $\boldsymbol{\sigma = \frac{F}{A} = \frac{2\pi rf}{\Phi}}$, conclusively since all **Forces and Areas** are every-where and are related to their-cave **r** , through their frequency , **f** , **therefore** it is the **mean of motion for every-Information . in all Nature** .

7..The Origination of Spin and Stresses σ : [64]

It is an Application to Material-Points [⊕↔⊖] , by considering the Positive-constituent with angular velocity $\bar{w} = \bar{v} / r = \frac{\sigma}{2r}$ [1+$\sqrt{5}$] ...(1) [70] and for an angle 45° from , **x** ,axis, where then the Ellipsoid of Angular-velocity is Perpendicular to the Plane of motion .The Moment of Inertia to , **z** , axis is that of Sphere equal to $J_3 = \frac{\pi r^4}{2}$ which is the same in all Principal axes , and exists , $J = J_1 = J_2 = J_3 = \frac{\pi r^4}{2}$, therefore Angular-Kinetic–Energy ≡ Angular-velocity-Ellipsoid and then becomes , $J_1 w_1^2 + J_2 w_2^2 + J_3 w_3^2 = 2E$ …....(2) This Energy-Ellipsoid as $\bar{B}$ ≡ Spin is → $\boldsymbol{w_1^2 + w_2^2 + w_3^2 = \frac{2E}{J^2} = \frac{4\,E}{(\pi\, r^4)^2}}$. From B = J w

Angular-momentum ≡ Spin ≡ $\boldsymbol{\bar{B} \equiv [\pi\sigma.r^3\ (1+\sqrt{5})\ /4]}$ …..(3) In Figure -6- and for the center , K , of ⊕ sphere , issues $\bar{v}_K = [\ \bar{w}.\bar{r}_K] = [\ \frac{\sigma[1+\sqrt{5}]}{2r}2r] = \sigma\ [1+\sqrt{5}]$ and $\bar{B}=\bar{S}=$ = [$\bar{r}$. m$\bar{v}$] = [r.m. σ(1+√5)] and for m = 1 then $\bar{B}$ = [r σ (1+√5)] .The Interchangable Ellipsoids of Angular velocity [70-P49] , and Momentum for the same Moment of Inertia is $J_1 = J_2 = J_3 = J_0$,and Angular Velocity $w_1 = w_2 = w_3 = w$, the Momentum $B_1 = B_2 = B_3 = B$ becomes 3J w^2 = **C** and $3B^2/J$ = **C** and since for circle $J = \frac{\pi r^4}{2}$ then $\frac{3\pi r^4}{2}w^2$ = **C** = ($\frac{3\pi r^4}{2}$) w^2 = ($\frac{3\pi r^2}{2}$)(rw)²=($\frac{3\pi r^2}{2}$)[σ (1 + √5)] ² =3πr²σ.[3+√5] → ***The Ellipsoid of Angular-velocity***,$\bar{\mathbf{w}}$, and $\mathbf{3B^2/J} = \frac{3(rmv)^2}{J} = \frac{3(rv)^2}{J} = \frac{3r^2.\sigma^2[3+\sqrt{5}]}{4}[\frac{2}{\pi r^4}] = \frac{3.\sigma^2[3+\sqrt{5}]}{2\pi r^2}$ → ***The Momentum-Ellipsoid,*** $\bar{\mathbf{B}}$.

The Angular-momentum In Planck`s-Length ≡ Spin ≡| $\bar{B}$ |≡ $\boldsymbol{\pi^2\ r^4\ f = \frac{(\pi\Phi r^3).\sigma}{2}}$ ...(3)

The value of **|B|** = [2 **.** 8,79455.10^{-35}**.1.** 1,6180339] = **2,845976. 10^{-34}** {Kg/m/s}. For Planck-Length $\mathbf{r_P = 1,61623.10^{-35}\sqrt{3}\pi}$ = 8,79455. 10^{-35} m , **velocity** | $\bar{v}$ |= $\frac{\sigma}{r}$ [1+ $\sqrt{5}$] and from (3) then → **Planck-cave-Stress** $\boldsymbol{\sigma = \frac{\bar{B}}{r.\Phi} = 1}$, **Total-Energy 2E = [J w^2]** , From | $\bar{v}$ |= $\frac{\sigma}{r}$ [1+ $\sqrt{5}$] = **c** = 3. 10^{8} m/s then , $\boldsymbol{\sigma = \frac{3.10^8}{3,679551.10^{34}}}$ = **8,1477332.10^{-27}**Kg/m² |**w**| = $\frac{\sigma}{2r}$ [1+ $\sqrt{5}$] = ($\frac{8,1477332.10^{-27}}{2.8.7945510^{-35}}$).3,2360675 = 1,499.$10^{8}$, or **|w| = 1, 5.10^{8}** rad / sec ,

For $\sigma = 8{,}147733.10^{-27}$ Kg/m² and $\bar{B} = 5{,}691952\ .10^{-34}$J then , Period $\mathbf{T} = \frac{2\pi}{w} = \frac{2\pi r}{v} = \frac{4\pi r}{\sigma\,(1+\sqrt{5})} = \frac{4\pi.8.79455.10^{-35}}{\sigma\,(1+\sqrt{5})} = \frac{3{,}4151}{\sigma}10^{-34}$ s ,or **Period T=** $[\frac{\mathbf{4{,}191584}}{\mathbf{10^{9}}}]$ s , and frequency $f = \frac{1}{T}$

i.e. **Planck-frequency** $\mathbf{f_1} = 2{,}38573294.10^{34}$ Hz . From above issues ,

7a).. The Spin of cave , **r**, is Equal to the Angular-momentum-Vector → **Spin ≡** $|\bar{B}| \equiv \mathbf{r\ \sigma\ \Phi}$ which contains and is the Golden-Radio-frequency Φ as Pressure , σ , in cave r .

7b).. In Planck`s-length [for light velocity **c** =3.10^8 m/s] , **velocity** is $|\bar{c}| = \frac{\sigma}{r}\ [1+\sqrt{5}\ ]$ m/s and in cave $\mathbf{r_P = 8,794}10^{-35}$ m , the **Pressure** $\boldsymbol{\sigma} = \frac{\mathbf{r.c}}{[1+\sqrt{5}\,]} = 8{,}147733.10^{-27}$ Kg/m².

7c).. In Planck`s-length the Period of Oscillation is $T = \frac{4\pi r}{\sigma\,(1+\sqrt{5})} = 4{,}192.10^{-8}$ s , and the **Frequency** $\mathbf{f_P} = \frac{1}{T} \equiv \frac{\sigma.(1+\sqrt{5})}{4\pi r} = \mathbf{2{,}3857265.10^{7}Hz}$, which is the minimum in Planck-cave .

The extreme for stresses $\boldsymbol{\sigma 1,2} = \sigma 1\ /2 \pm (\ ½\ ).\sqrt{\sigma 1^2 + 4.\sigma 1^2} = \sigma_1.[1 \pm (\sqrt{5})]\ /\ 2 \equiv \boldsymbol{\sigma}\ \Phi$,

velocity **v** = $(w = \frac{2\pi}{T})r = 2\pi r\ .f = [\frac{\sigma}{2}].(1+\sqrt{5})$,frequency $\mathbf{f} = \frac{(1+\sqrt{5}\,)\,.\,\sigma}{4\pi r}$, Period $T = \frac{4\pi r}{\sigma(1+\sqrt{5})}$

7d).. From Kepler-Orbit-Vibration the equation of inverse Period → $f_n = \frac{(1+\sqrt{5})\sigma}{4\pi r} = [\frac{\sigma}{2\pi r}]\ \Phi$,

$f^{2}_{n} = \frac{\sigma^2}{4\pi^2 r^2}\Phi^2 = \frac{1}{g.a^3}$, and $ga^3\Phi^2 = \frac{4\pi^2 a^2}{\sigma^2}$, or $a = \frac{1}{g}[\frac{2\pi}{\sigma\,\Phi}]^{2}$ and from Work in Orbits

$W = 2E = B\ w = J.w^2$, or $\mathbf{2E = 2\pi\ f\ B}$ i.e. → **Total - Spin*Frequency** $\equiv \bar{\mathbf{B}}\ \mathbf{f} = \frac{\mathbf{E}}{\boldsymbol{\pi}}$, and

Energy ≡ motion in Planck-scale-cave r = L_P , is $2E = \bar{B}f_n = [\frac{(1+\sqrt{5})\sigma}{4\pi r}].\bar{B} = [\frac{\sigma}{2\pi r}].\bar{B}\ \Phi$, or

$$\mathbf{E = [\ \Phi\ \frac{\sigma}{4\pi r}\ ]\ .\ \bar{B}\ \equiv\ \frac{|\,B\,|^{2}}{2\,\pi^2 r^4}} \quad \ldots\ldots (4) \qquad \textbf{i.e.}$$

Energy of Particles In-Beyond Planck-cave is dependent on their Spin only , and for the Electron with cave $r = a_e = 1{,}6819781.10^{-17}$m , and Principal-first-Stress σ = 1 , then ,

$\mathbf{E} = [\boldsymbol{\Phi}\frac{\boldsymbol{\sigma}}{\mathbf{4\pi r}}].\bar{\mathbf{B}} = 1{,}6180339\ [(\frac{1}{4\pi.1{,}6819781.10^{-17}}].\ 2{,}845976.\ 10^{-34} = \frac{2{,}17872.10^{-18}}{1{,}602.10^{-19}.} = \mathbf{13{,}6\ eV}$

All above equations define → The Ubiquity of Golden-Ratio-Φ and Stress σ of caves ← in motions with ***Angular - Momentum , Stresses*** , ***Frequency*** or ***Velocity*** in nature [64-A-B-C] , AND

The Gravitational-Force as Stress $\boldsymbol{\sigma = 2G.\pi r f\ /\ \Phi}$ **→ Is The Holding Energy in Polygons**

7-d... The Stability of the Cosmic Particles Conductors and Atoms–Nucleus . [109-111]

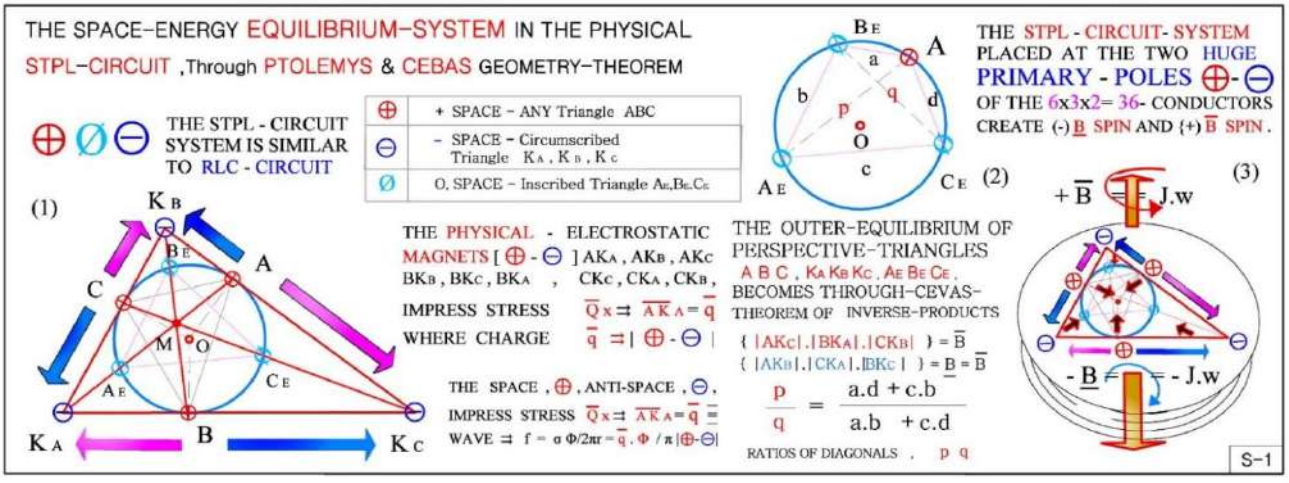


Figure -9- : **The Stability** of → { ⊕ - **Space** [A B C] } , {⊖ - **Anti-Space** [K_A K_B K_C]} , of **The Sub-Space** [A_E , B_E , C_E]}, and **Neutral-Space** {[⊕↔⊖] ≡ ds ≡ [⊕←ds→⊖]}.
SINCE , Energy of The first Wheel-Rim or Orbital is distributed to the Couple of (2e) Electrons of the Orbital , And Then this is a Cycloid - Anti-Cycloid - Stability System .

This Still Systems of **Space –Anti space** Regular Spatial Polygons **DOES NOT** mean Matter–Antimatter But The State of Equilibrium of Space and Anti Space .

→ The **State of Equilibrium in Caves ≡ The Ceba`s Energy triangle** . ← **Remark : The Stability of Spaces–Anti-spaces** Exists from the Simultaneous Actions on Their **Regular Sets of Polygons** , The **Vector** $|\bar{s}|$ **transfers the Axial-Rotational motion to circulate Polygonal -Plane-motion , and Informations can Immediately Be Transferred** to Each Other through **Their Common Spatial Paths which are the Infinite great-circles into their Common Sphere** .[100]. The Conductors formulate In-Plane the Triangle , which is the most Steady Figure , and in Space **The Regular Tetrahedron which is the first Spatial Figure** .

E... THE TWIN – COUPLE TRIANGLES Method FOR CONSTRUCTING THE REGULAR --- Odd –-- POLYGONS . IN SPATIAL -- GEOMETRY .

1e... The Spatial Regular Polygones .

Regular petrials

Name	Petrial tetrahedron	Petrial cube	Petrial octahedron	Petrial dodecahedron	Petrial icosahedron
Symbol	$\{3,3\}^{\pi}$	$\{4,3\}^{\pi}$	$\{3,4\}^{\pi}$	$\{5,3\}^{\pi}$	$\{3,5\}^{\pi}$
(v,e,f), χ	(4,6,3), $\chi = 1$	(8,12,4), $\chi = 0$	(6,12,4), $\chi = -2$	(20,30,6), $\chi = -4$	(12,30,6), $\chi = -12$
Faces	3 skew squares	4 skew hexagons		6 skew decagons	
Image					
Animation					
Related figures	$\{4,3\}_3 = \{4,3\}/2 = \{4,3\}_{(2,0)}$	$\{6,3\}_3 = \{6,3\}_{(2,0)}$	$\{6,4\}_3 = \{6,4\}_{(4,0)}$	$\{10,3\}_5$	$\{10,5\}_3$

Figure-10

In Chemistry , Atoms follow the Spatial Geometry where .

In an Octava of Electrons Protons are Nested in the **In-Sphere** ≡ n =1-122 Regular Triangles , The In-Sphere is Nested In the **Regular Tetrahedron** , which is Dual with the 3-Diagonals of The Cube , And the Cube is Nested in The **Ex-Sphere** which are the Orbits of Electrons .

2e... The Geometrical & Physical Notion of the Spatial Regular Polygons :

The 4-Geometrical Spaces → In-Sphere , Tetrahedron , Cube , Ex-Sphere ← consist the

Basic Geometrical Moulds . relation for Plane Work , which is The Quantization of Geometry Shapes into the Plane – Stores [The Surfaces] of Anti-Space , and consist The Unification of Geometry–monads with those of Energy monads , and which were analyzed and have been fully described , by markos 20/8/2017

The **Vectors of Amplitude** ON the 4 -Spaces consist the **E-Form-Mould** . and is constructed **in Space as** → The Inscribed and the Circumscribed Sphere as **Regular Tetra Pleuron** , and **in Anti-Space** → The Inscribed and the Circumscribed to the Regular **Tetra Hedron**, **Regular Penta Hedron** ← **in Sub-Space** as The Inscribed and the Circumscribed to the circle **Regular Hexa Hedron** ← Continually , Space Regular Polygons Into the Anti-space Regular Polygons.

The Plane & Spatial Regular Polygons in Mexico Asteraceae - Plants .





Figure -11-

For the Even Sequence Set [n , n+2] , where
n = Regular-**Hexagon** , n+2=Regular-**Octagon**
n+1= Regular-**Heptagon** , and
n → A , n+2 → B , n+1 → C , then issues
The Regular-Heptagon Calices ,

For the Even Sequence Set [n , n+2] , where
n = Regular-**Octagon** , n+2=Regular-**Decagon**
n+1= Regular-**Enneagon** , and
n → A , n+2 → B , n+1 → C , then issues
The Regular-Enneagon Calyces ,



Remarks :

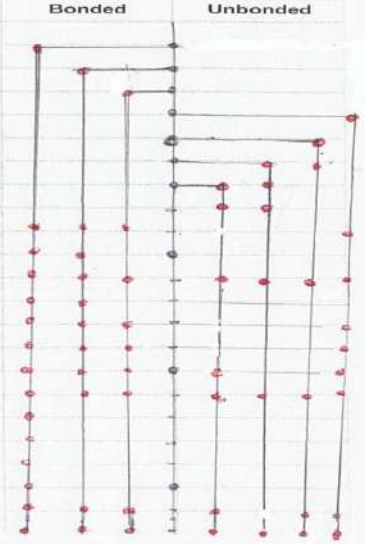
1…The Entire System of the Three-circles , [O,OK] , [K, KA] , [K, KB] and Three-Triangles |KBK`| , $|KBE_B|$, $|KAE_A|$, **Works Collectively** to keep the Difference + Sum of Squares [c(1b)].
2…2.[n+1] Regular Triantatetragon **must Be** *According to Archimedes* in Midpoint $\mathbf{A_{1C}}$ of Arc or Chord $|A_{1A}\text{-}A_{1B}|$ ***Therefore*** **, Line-Chord** $\mathbf{|OA_{1C}|}$ Intersects The Circle of Apothems at Point $\mathbf{A_C}$
3…Since Extension of Line-Sector $\mathbf{|KA_C|}$ on **Constant-Difference** Squares Circle [K,KB] is at Point $\mathbf{M_{AB}}$, **Then** it is on the Locus of the **Constant-Sum** Squares from Mid-Point M_{EAB} ..
4… The Gravitational - Force as Stress $\boldsymbol{\sigma = 2G.\pi rf / \Phi}$ **, IS** The Holding Energy in **Polygons**
The G-Force is Stress $\boldsymbol{\sigma} = \frac{2\pi rf}{(n)\Phi} = \frac{2\pi rf}{1.\Phi}$, and is the **Energy** of The first **Wheel-Rim** or **Orbital**
& Thus distributed to the Even Sequence Sets of **Orbital in Polygons** , [Figure-11-]

Compound

Description	The 7-Acids in Asteraceae Plants , 1=Hinevonin , 2=Hymenoxol , 3=Hymenolahe , 4=Monoethalamine , 5=Hymenoxynin , 6= Genariol , 7=Carnaerol
Formula	$NC_{21}C_{21}C_{15}C_{15}C_{10}C_{10}C_2H_{34}H_{22}H_{20}H_{16}H_{16}H_{14}H_7O_9O_8O_5O_4OOO$
Total Number of Elements	257
Stiffness Factor	942480

Properties

#	Number	Symbol	Mass	Total Mass	Pins	Sockets	Bonded	Unbonded
1	21	C	12	252	84	84		
2	21	C	12	252	84	84		
3	15	C	12	180	60	60		
4	15	C	12	180	60	60		
5	9	O	16	144	18	18		
6	8	O	16	128	16	16		
7	10	C	12	120	40	40		
8	10	C	12	120	40	40		
9	5	O	16	80	10	10		
10	4	O	16	64	8	8		
11	34	H	1	34	34	34		
12	2	C	12	24	8	8		
13	22	H	1	22	22	22		
14	20	H	1	20	20	20		
15	18	H	1	18	18	18		
16	18	H	1	18	18	18		
17	1	O	16	16	2	2		
18	1	O	16	16	2	2		
19	1	O	16	16	2	2		
20	1	N	14	14	3	3		
21	14	H	1	14	14	14		
22	7	H	1	7	7	7		



Bond - Mode

C_{84}^{84} C_{84}^{84} C_{60}^{60} C_{60}^{60} O_{18}^{18} O_{16}^{16} C_{40}^{40} C_{40}^{40} O_{10}^{10} O_{8}^{8} H_{34}^{34} C_{8}^{8} H_{22}^{22}

Matrices

Mass Matrix

$$m \times \begin{vmatrix} 252 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 252 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 180 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

Imprint -1-

The Asteraceae Plants occupy the Even Sequence Set [n , n+2] , where n = the Regular **Hexagon** and , n+2=the Regular- **Octagon** , wherewhen n+1= the Regular-**Heptagon** , Plants also occupy the Even Sequence Set [n , n+2] , where n = the Regular **Octagon** and , n+2=the Regular- **Decagon** , wherewhen n+1= the Regular - **Enneagon** ,

THE STIFFNESS - FINAL ENERGY - WAVEFORM SIGNAL

From modes

$W_1 = 16.45832 \times 10^{15}$ Hz	$U_1 = 1.22966 \times 10^5$ m/s	$\lambda_1 = 0.469439 \times 10^{-10}$ m	$A_1 = 0.074714 \times 10^{-10}$ m
$W_2 = 12.94255 \times 10^{15}$ Hz	$U_2 = 1.090441 \times 10^5$ m/s	$\lambda_2 = 0.529373 \times 10^{-10}$ m	$A_2 = 0.084252 \times 10^{-10}$ m
$W_3 = 9.835727 \times 10^{15}$ Hz	$U_3 = 1.124759 \times 10^5$ m/s	$\lambda_3 = 0.71851 \times 10^{-10}$ m	$A_3 = 0.114354 \times 10^{-10}$ m
$W_4 = 13.067232 \times 10^{15}$ Hz	$U_4 = 1.296427 \times 10^5$ m/s	$\lambda_4 = 0.623368 \times 10^{-10}$ m	$A_4 = 0.099212 \times 10^{-10}$ m
$W_5 = 6.782438 \times 10^{15}$ Hz	$U_5 = 1.044249 \times 10^5$ m/s	$\lambda_5 = 0.967382 \times 10^{-10}$ m	$A_5 = 0.153964 \times 10^{-10}$ m
$W_6 = 4.131685 \times 10^{15}$ Hz	$U_6 = 0.864472 \times 10^5$ m/s	$\lambda_6 = 1.314631 \times 10^{-10}$ m	$A_6 = 0.20923 \times 10^{-10}$ m
$W_7 = 8.030838 \times 10^{15}$ Hz	$U_7 = 1.24475 \times 10^5$ m/s	$\lambda_7 = 0.973871 \times 10^{-10}$ m	$A_7 = 0.154996 \times 10^{-10}$ m
$W_8 = 10.930415 \times 10^{15}$ Hz	$U_8 = 1.452179 \times 10^5$ m/s	$\lambda_8 = 0.834763 \times 10^{-10}$ m	$A_8 = 0.132857 \times 10^{-10}$ m
$W_9 = 5.210897 \times 10^{15}$ Hz	$U_9 = 1.228015 \times 10^5$ m/s	$\lambda_9 = 1.480714 \times 10^{-10}$ m	$A_9 = 0.235663 \times 10^{-10}$ m
$W_{10} = 1.970038 \times 10^{15}$ Hz	$U_{10} = 0.844189 \times 10^5$ m/s	$\lambda_{10} = 2.692434 \times 10^{-10}$ m	$A_{10} = 0.428514 \times 10^{-10}$ m
$W_{11} = 3.543857 \times 10^{15}$ Hz	$U_{11} = 1.553428 \times 10^5$ m/s	$\lambda_{11} = 2.754196 \times 10^{-10}$ m	$A_{11} = 0.438344 \times 10^{-10}$ m
$W_{12} = 2.084823 \times 10^{15}$ Hz	$U_{12} = 1.418148 \times 10^5$ m/s	$\lambda_{12} = 4.273977 \times 10^{-10}$ m	$A_{12} = 0.680225 \times 10^{-10}$ m
$W_{13} = 3.24442 \times 10^{15}$ Hz	$U_{13} = 1.847778 \times 10^5$ m/s	$\lambda_{13} = 3.578429 \times 10^{-10}$ m	$A_{13} = 0.569525 \times 10^{-10}$ m
$W_{14} = 3.090339 \times 10^{15}$ Hz	$U_{14} = 1.891388 \times 10^5$ m/s	$\lambda_{14} = 3.845513 \times 10^{-10}$ m	$A_{14} = 0.612032 \times 10^{-10}$ m
$W_{15} = 2.962436 \times 10^{15}$ Hz	$U_{15} = 1.952004 \times 10^5$ m/s	$\lambda_{15} = 4.140107 \times 10^{-10}$ m	$A_{15} = 0.658919 \times 10^{-10}$ m
$W_{16} = 2.455004 \times 10^{15}$ Hz	$U_{16} = 1.776979 \times 10^5$ m/s	$\lambda_{16} = 4.54789 \times 10^{-10}$ m	$A_{16} = 0.723819 \times 10^{-10}$ m
$W_{17} = 2.179892 \times 10^{15}$ Hz	$U_{17} = 1.776029 \times 10^5$ m/s	$\lambda_{17} = 5.119115 \times 10^{-10}$ m	$A_{17} = 0.814732 \times 10^{-10}$ m
$W_{18} = 2.179892 \times 10^{15}$ Hz	$U_{18} = 1.776029 \times 10^5$ m/s	$\lambda_{18} = 5.119115 \times 10^{-10}$ m	$A_{18} = 0.814732 \times 10^{-10}$ m
$W_{19} = 1.878872 \times 10^{15}$ Hz	$U_{19} = 1.64885 \times 10^5$ m/s	$\lambda_{19} = 5.513962 \times 10^{-10}$ m	$A_{19} = 0.877574 \times 10^{-10}$ m
$W_{20} = 0.891324 \times 10^{15}$ Hz	$U_{20} = 1.214077 \times 10^5$ m/s	$\lambda_{20} = 8.558362 \times 10^{-10}$ m	$A_{20} = 1.362106 \times 10^{-10}$ m
$W_{21} = 2.43617 \times 10^{15}$ Hz	$U_{21} = 2.007161 \times 10^5$ m/s	$\lambda_{21} = 5.176718 \times 10^{-10}$ m	$A_{21} = 0.8239 \times 10^{-10}$ m
$W_{22} = 1.441038 \times 10^{15}$ Hz	$U_{22} = 2.183138 \times 10^5$ m/s	$\lambda_{22} = 9.518876 \times 10^{-10}$ m	$A_{22} = 1.514976 \times 10^{-10}$ m

Circular - Frequency	= W_R	=	58.874103 x 10^{15} Hz
Resonance - Energy	= E_R	=	38.75098313296261 eV
Resultant - Velocity	= U_R	=	33.624122 x 10^5 m/s
Resultant - λ	= λ_R	=	3.588447 x 10^{-10} m
Re Helical - r = A_R	= r_R	=	0.5711190532 x 10^{-10} m
Bands UL - Amplitude	= A_{RB}	=	0.28556 x 10^{-10} m
Resultant - Potential	= V_{RP}	=	1241.7 x 10^{-20} Volt
LC - Circuit Potential	= V_{LC}	=	215.201094 x 10^{-6} Volt
Intensity - Current	= I_C	=	365.527596 x 10^{-3} Ampere
Vaporation -Temperature	= T_V	=	792.886 Kelvin
Magnetic - Field	= M_F	=	0.254077 x 10^{-6} Tesla
LC - Circuit - Power	= P_{LC}	=	786619.3842 x 10^{-10} Watt
T.Modulated - Power	= P_{TRM}	=	1573238.7684 x 10^{-10} Watt
SideBands - Power	= P_{SBM}	=	393309.6921 x 10^{-10} Watt

σ_1	= U_1/φ	=	0.759972 x 10^5 N/mm²	
Δw_1	= $W_R - W_1$	=	42.415783 x 10^{15} Hz	min.Amplitude Modulation
Σw_1	= $W_R + W_1$	=	75.332423 x 10^{15} Hz	max.Amplitude Modulation
fw_1	= $\Delta W_1/2\pi$	=	6.750682 x 10^{15} Hz	con.Frequency Modulation
E dF_1	= h x fw_1	=	27.91810341 eV	
k_1	= $\Delta w_1 / \Sigma w_1$	=	0.563048173	

Imprint-2-

From The Electronic - Program < **Programming The Atoms And Compounds** >
The Chemical Construction of the 7-9 , Acids in Asteraceae Plants creates L.C- Chemical Coupling in the Even Sequence Regular Sets of the Elements in 7-9, Bonds as their Acids .

The 7-Acids in Asteraceae Plants , 1=Hinevonin , 2=Hymenoxol , 3=Hymenolahe , 4=Monoethalamine , 5=Hymenoxynin , 6= Genariol , 7=Carnaerol

LC - Chemical Coupling

Resonance - Frequency	=	W_0	=	58.874103 x 10^{15} Hz
Energy	=	Q_0	=	6.2086 x 10^{-18} J
LC - Circuit-Coupling	=	LC_{LC}	=	16.985 x 10^{-18} Farad/s
Wavelength - λ	=	λ_{2L}	=	3.588447 x 10^{-10} m
Wave Amplitude	=	A_{wave}	=	0.5985 x 10^{-25} m
Inductance	=	L	=	1 x 10^{-19} Hz
Capacity	=	C	=	2.885037 x 10^{-15} Farad
Resonance-Voltage	=	V_R	=	2152.010936 x 10^{-6} Volt
Voltage across Inductor	=	V_L	=	3.6552 x 10^{-20} eV
Power of LC-System	=	P_{CL}	=	133.61 x 10^{-22} Watt
Loudness of Resonance-Power	=	P_{RL}	=	167.6992 Decibel
Maximum Flowing Current	=	I_{max}	=	365.527596 x 10^{-3} Ampere
Capacity Discharged Period	=	T_s	=	0.2668 x 10^{-16} s
Radiation - Thermal	=	T_K	=	90.909149 Kelvin
Radiation - Frequency	=	f_{RAD}	=	-189.417212 x 10^{10} Hz
Conversion - Temperature	=	T_V	=	0.7821 Kelvin
Radius In Cleft	=	r_{LC}	=	3.787767 x 10^{-10} m
Diameter of Compound	=	d_{com}	=	8.656995 A°

2-e... The Spatial Regular Polygones in Atoms - Structure .

The Gravitational - Force as Stress $\sigma = 2G.\pi rf / \Phi$, **IS** The Holding Energy in **Polygons**

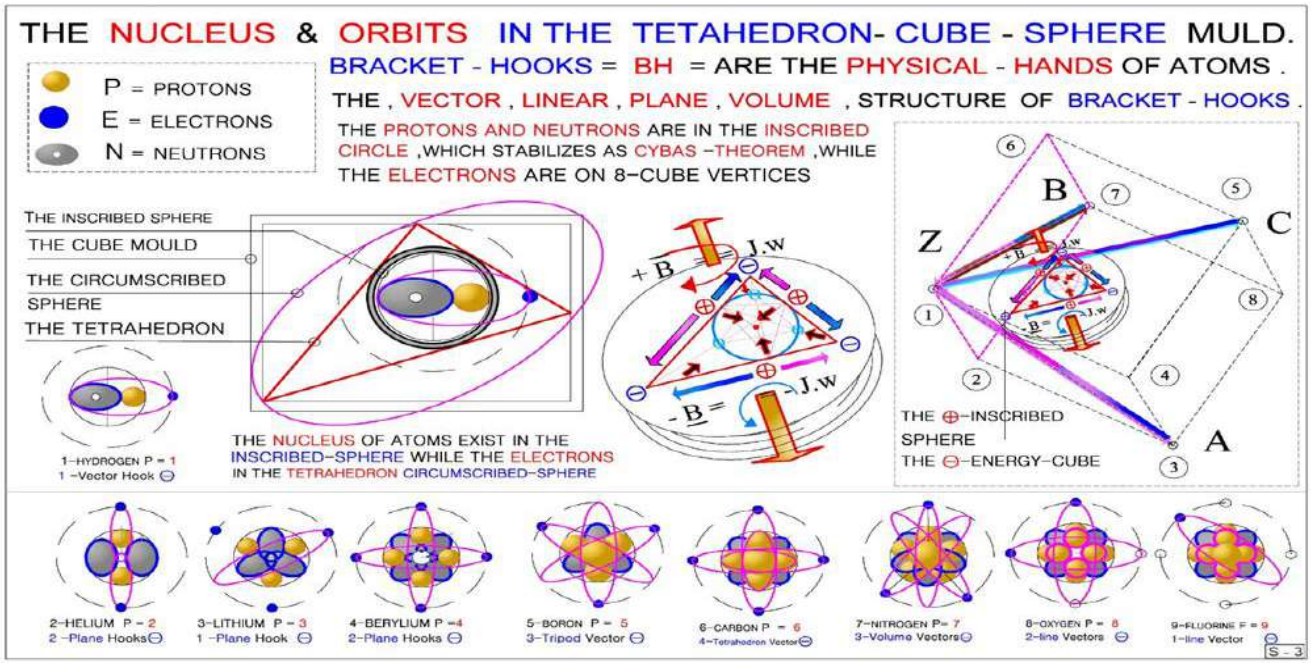


Figure - 12- : The Active-Conductors of Elements and **The Space , Anti-Space formation** .

3e-a... The Conservation of Energy in Caves and in Atoms Compounds .

THE ATOMS – ACTIONS : $[[A]] \rightarrow \{[B] \pm \mathbf{E_n}\} \equiv C \equiv \mathbf{E_n} + \{[A] \pm [B]\}, \rightarrow \mathbf{E_n} = \mathbf{E_C} + \mathbf{E_A} \leftarrow$

[[A]] = [Atom OR Compound] ≡ The Signalling.(Wc) → Carrier - Wave

[[B]] = Any[A-OR-C] ≡ Any Signalling.(w) Wave

[[C]] = The New **M**odulated [A] **S**ignal $\equiv f\frac{w+wc}{2} + f\frac{w-wc}{2} \equiv$ [MAS]

ANTIDOTE = The **D**emodulated [MAS] $\equiv f\frac{w+2wc}{2} + f\frac{w-2wc}{2} \equiv$ [DMAS]

The Atoms Action (1) ... [A] → {[B]±$\mathbf{E_N}$} ≡ [C] ≡ ± $\mathbf{E_N}$ + {[B] + [A]} , , [A] ≡ [C] +{ $\mathbf{X_N}$}

[A] = The Initial , [C] = The Final , [X_N] = {[B] ± [E_N]} ≡ The Antidote .

The Atoms Action (2) …. [A] → [± E_N] ≡ [C] ≡≡≡ ± $\mathbf{E_n}$ **+ [A]** , , , , [A] ≡ [C] + [E_N]

[A] = The Initial , [C} = The Final , [E_N] ≡ The Antidote , The **Circular-Signal-Action** is as below,

[I = CARRIER] →[F=MODULATOR]→[I+F = MODULATED]

→[I+F]+M ↔ [DM=ANTIDOTES]**=DEMODULATION]**

In Caves , the Communications – Frequencies are the **Resonances which consist the Master-Keys** of all caves .

In Fig-12- The Active-Conductors of Elements and The Space , Anti-Space formation . and **ZA , ZB , ZC ,** are the three **Conductors** On the Three Diagonals of Cube with 1 , 2 , 3 , 4 , 5 , 6 , 7, 8 , Vertices . At Point Z is Acting a **Complex-Vector** $\bar{Z}$ and which transform the Vibration to the three conductors in **Space Pattern** . It was shown [61] that The Stresses of Complex-Vectors are Spread in Surfaces Only for Still states.

The number of Moulds of **Stationary-States** are → $\mathbf{2}^{n=1-4\ Points} \equiv 2 , 4 , 8 , 16 \equiv$ **4–Types** ←

[**2**] = For the **P**roton = Nucleus , [4] = Tetrahedron , [8] = Cube = The Neutral-caves , [16]=

The Double Number of Vertices for the 8-Stationary Neutral-caves & called **TEES-Pattern**

→{ The Sphere – IN TETRAHEDRON – IN Cube , IN External Sphere }←

The Active-Conductors of Elements and The Space , Anti-Space formation :

Point , in E-geometry is considered nothing , without Position and direction while in , MG ≡ ***Material - Geometry*** [1 Positive ⊕ and 1 Negative ⊖] . From Nothing (**i.e.** the Point Z) to Existence (i.e. to Be another Spherical Point P) issues the **Zero Virtual work law** , where zero Work is the equilibrium of Two Equal and Opposite forces on Points and is [⊕ - d - ⊖] . Thus Space [S] is **Z**-Point ⊕ , Anti-space **P**-Point ⊖ , and d = **ZP** = The Conductor of Points . Since , ***The Position and the Number of Points*** create the Spaces so the Only Three elements **are** → {⊕ , [⊕↔⊖] , ⊖} ≡ [+, 0 , -] ← **for Material Geometry exist linearly** in moulds .

The Stresses of Complex-Vectors **Act** on a Surface , **A circle of Sphere** , between the Sub Units **Axially** and **Rotationally** following ***Ptolemy`s and Cebas Theorem*** of Equilibrium**.** The Space Anti-Space and , Neutral-Space Conductors are self-Sustaining .

Remarks : ***The Work of Elastic Deformation*** .

Elastic Deformation requires (1) That every New Stress that is applied to a New infinite Element is always added to the existing one .(2).. Be always Unity Work of Deformation .

When a Body contains Potential Energy ,

During any Unloading it gives off Mechanical - Work , Since the Points of Application of the GRADUALLY - withdrawn Loads are Displaced.

If the Unloading is ABRUPT , Then Kinetic Energy is converted into Potential and after is Kinetic , converted Back into Kinetic and so on . **That is** , the Body Performs an Oscillating motion until the Mechanical Energy is Extinguished due to a State of Equilibrium or Alters .

3-e… THE TETRAHEDRON IN CUBE IN EX-SPHERE MECHANISM . [109-111]

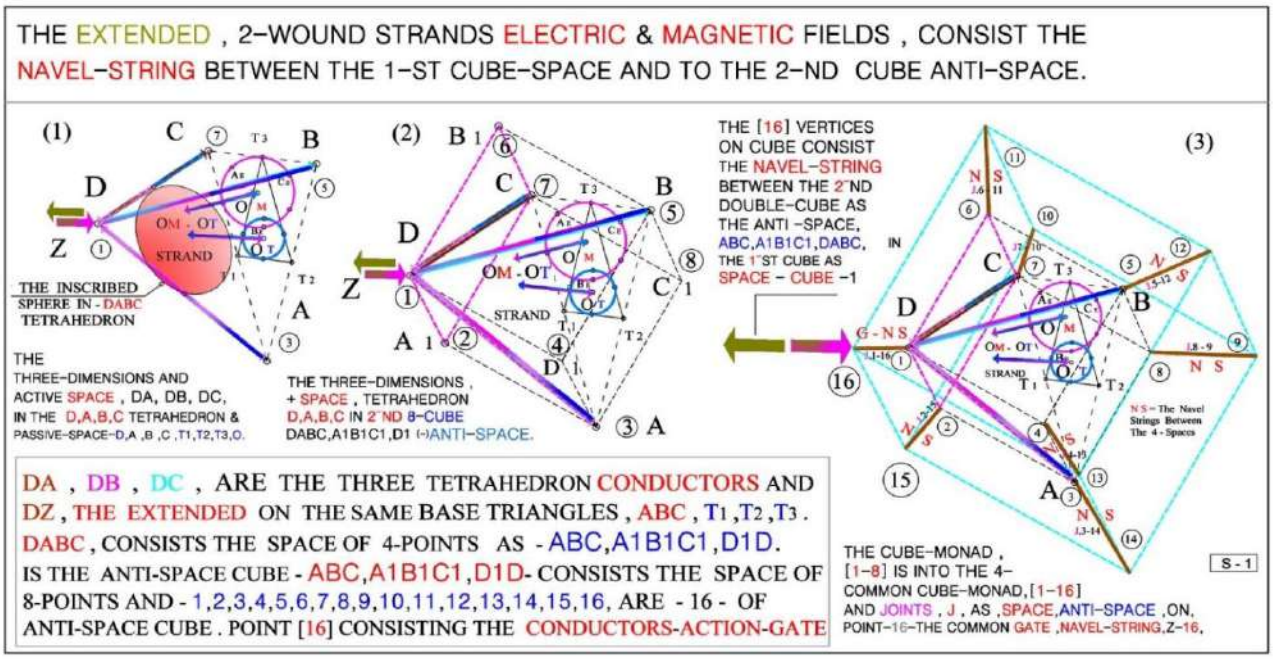


Figure -13-The Electric-Magnetic Field of 3-Conductors Results on **2-Anti-Parallel - Strands**. Which Response to the two Apothems $0A_{1A}$, $0A_{2A}$, of the ODD Regular Space Polygons . **The Stability** of→{ ⊕4-**Space** [D A B C]} Regular-***Tetrahedron*** , **IN** {⊖ 8-**Anti- Space *Cube*** [$DA_1B\,D_1$, $C\,C_1AB_1$] } , and The →{ ⊕ 8 - **Space** [DA_1BD_1 , CC_1AB_1] }- ***Cube*** **INTO The** {⊖ 16-**Anti-Space** [1 ,2 ,3 ,4 ,5 , 6, 7, 8-9 ,10 ,11, 12 , 13,14 ,15,16]} ***Sphere Cube Vertices.*** Point [**16** ≡|⇉|**Z**|] consists a **Navel-String** Gate in {Z-(ΔD,ABC),Δ[T1T2T3]}Stationary Bases In Dual Photon $\bar{v}\,[\frac{\sigma\Phi}{2\pi r}+\frac{\sigma}{2\pi r}] \equiv \bar{v}.[\,\boxed{\bar{f}_n} + f_n\,]$, The **Colours** are the **Still-Sub-Units** , or the Mode Shapes for Eigenvectors of the Photon - Particle Storage → $[\bar{v}\,.\boxed{\bar{f}_n}\,]$ ← and which exists as Frequencies ,Violet , Blue , Green , with their Complementary Yellow , Orange , Red ← [93]. The Electric and Magnetic Field of 4-Conductors ZA, ZB, ZC, ZD , On 2-Anti Parallel Strands . The Electromagnetic-Resultant-Vectors ZD Acts on $\overleftrightarrow{0_M \Uparrow\Downarrow Z}$, $\overleftrightarrow{0_T \Uparrow\Downarrow Z'}$ Base ABC with **Complementary** Vectors , at 0_M , 0_T centers of Rotation $ZD0_M$, $ZD0_T$. Because of Different Torsion on ZD axis , $ZD0_M \neq Z0_M$, The Vectors-Foot transport to 0_T of $ZD0_T$ New axis **. DNA** follows this way of balancing Sustaining on **RNA**-structure

4-e-a... The Matter Antimatter in 3-D Origination Mechanism [⊕↔⊖] [103]

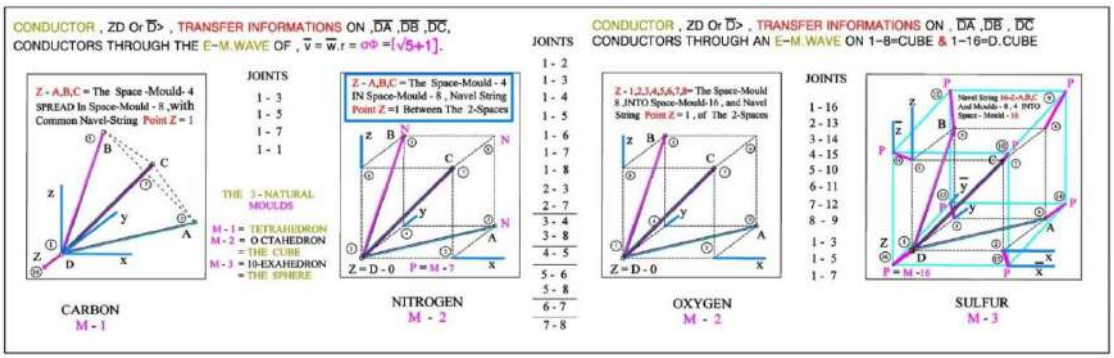


Figure – 13-a - : The 4-Dimentional Spinning , **Tetrahedron - Cube–Spheres-Mould .**
The **Stationary-States** are → $2^{n=1-4\ \text{Points}} \equiv$ **2 , 4 , 8 , 16** ≡ **4–Types** ←

4-e... The In–SPHERE In-TETRAHEDRON In-CUBE In-EX.SPHERE MOULD .[103]

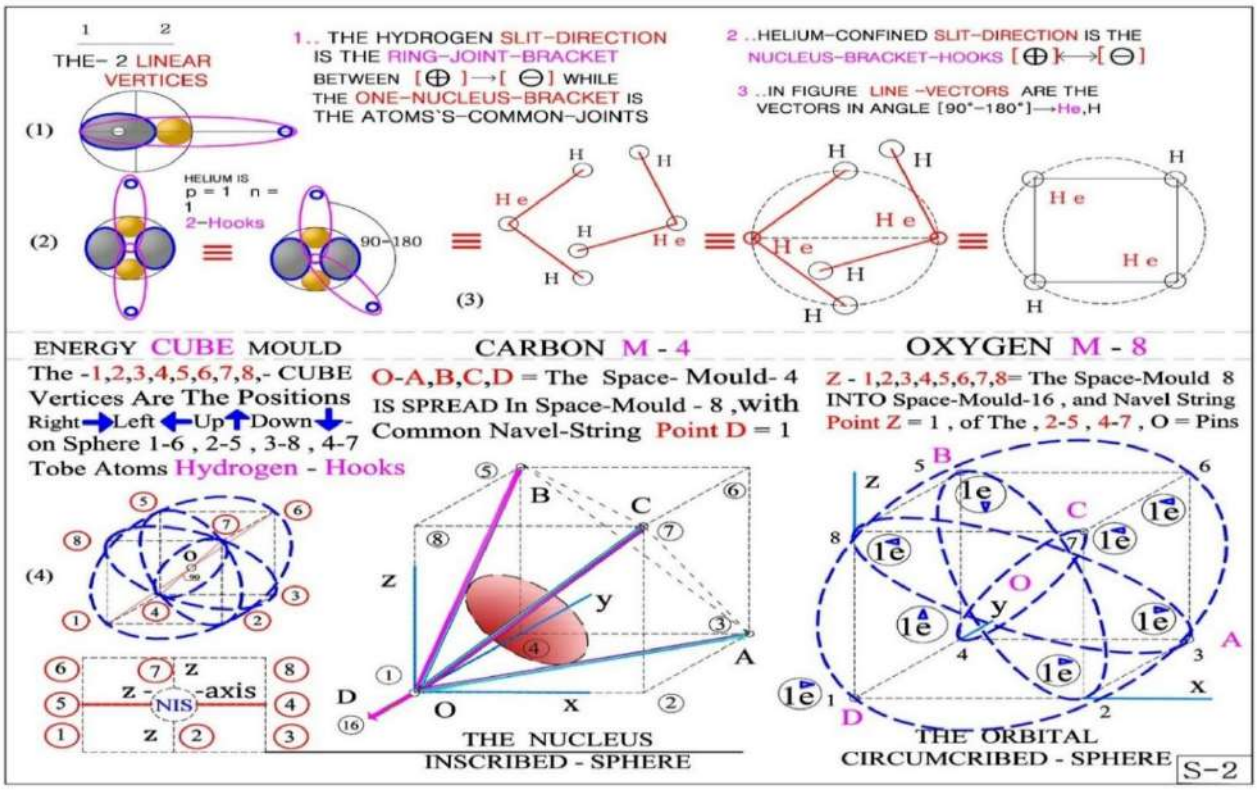


Figure – 14.. :The Hydrogen , Carbon , Oxygen , Atoms-Conductors **Into** the **Sphere-Cube-Mould** . Deka Hexahedron , Octahedron ≡ Cube , Tetrahedron ≡ 4-Points , Triangle . **The Nucleus** ≡ (In-Sphere) , **Tetrahedron** ≡ (D - A.B.C) **,** **Cube** ≡ (O,2,A,4,B,6,C,8) , **Orbitals** ≡ (Ex-Sphere) **, Mould .**

When **Vector ≡ 2-Points** ≡ The Closed-Conductor , 1-Point ≡ The Opened -Conductor ≡ The Bracket-Hook . Oxygen is the Element which fills 8 - 2 = 6 vertices of Cube as EL →

$$\begin{matrix} 1e & 0e & 1e \\ 1e & \overleftrightarrow{2} & 0e \\ 1e & 1e & 1e \end{matrix} \equiv \left(\pm\tfrac{2}{2}\right)$$, where ⓪ is the first , **Zero-Acting Cube** filled with , 1e ,

Therefore is the only Steady Element which occupies the first Energy-Level ①

with 2 Electron-Positions as ① = 2e = $\overleftrightarrow{2}$ Positions at $\vec{1}$, $\overleftarrow{1}$

The Electron Being in the Hydrogen Precesses Because of the different axis of Rotation and Nutation`s , from the continuous and immense-communication to the Effect of gravity **g** . Since Electron is continually oscillating , with the Nutation-frequency $\mathbf{f_N}$ **,** so Produces a continuous and Oscillating Magnetic-field –M- which in turn is the source of an oscillating Electric-field – E - which implies the **Regeneration of each other** , i.e. it is a **Propagating Electromagnetic-Wave** where **E = B c** , and with a Quantum-Energy $E = h f_N$ or $E = 2\mu.B$, This Energy E , consists the Hydrogen Bracket **Hook [HBH]** , or $[\overrightarrow{BH}]$ and it is the only **free monad** which can **Bond** to all Energy Structures **.**

The **Spheres – Cube - Structure of Atoms** In Sphere Center –O- allows **The Stability** of the System , Per–Two electrons in Dipole Positions , 1 ↔ 6 , 2 ↔ 5 , 3 ↔ 8 , 4 ↔7 , consisting

The-Positive Dipole-$[\overrightarrow{BH}]$, to Be able to move → Into-In- Outside it ← and allow the Atoms - Bonding as this in Energy levels ***.The Cube-Structure of Atoms and Orbits in the Out-Sphere*** formulates + $[\overrightarrow{BH}]$ to those **Dipole Positions which lack of one electron** and so can **PLUG -In** the **LOOP** of the others . The Atoms with Zero Stable State are **,**

The Hydrogen Bracket-Hook **Into the Carbon** encloses the Inscribed Sphere for the Nucleus , **Into the Oxygen** for the Orbitals , **Into the Sulfur** enclosed into the Circumscribed Sphere for the Atoms-Conductors Sustains **.**

The First 4-Dimentional Spinning ,**Tetrahedron-Cube–Sphere-Mould are ,** 9c--2

1… O-1-3-5-7 - → **The First Solid Tetrahedron Structure in Nature** ,

2… O-1-2-3-4-5-6-7-8 → **The First Cube with max. Positions e = 2.n² = 8** ,

3… R - [O , A , B , C] → NUCLEUS , **IS IN the First Inscribed Sphere into Tetrahedron.**

4… R-O-1, 2, 3, 4, 5, 6, 7, 8 → ORBITS **ARE IN Enveloped Cube-Sphere of Tetrahedron.**

The Deka Hexahedron , Octahedron ≡ Cubes , Tetrahedron ≡ 4-Points , Triangle ≡ 3-Points , Vector ≡ 2-Points≡The Closed-Conductor , 1-Point≡The Opened -Conductor≡The Bracket-Hook .

The Hydrogen **Nucleus** is kept in the Inscribed to the ***Tetrahedron-Cube-System*** where exist the Protons and Neutrons , while The Electrons of Hydrogen **Orbits** are kept into the Circumscribed, to the ***Tetrahedron – Cube - System*** . Both Equilibrium by following the Menelaus - Ceba`s Principles in The Regular Triangles , Hexagons , Octagons . Decaexagones ... [70]

Any Two Perpendicular **Energy-Vectors** , $|\mathbf{Q} \rightrightarrows \overrightarrow{AK_A} = \leftrightarrow|$, $|\mathbf{Q} \uparrow\uparrow \overrightarrow{AK_A} = \updownarrow|$ of this Propagating Energy **,** FORM The **Energy-Square** , $|\leftrightarrow|.|\updownarrow| \equiv |\mathbf{Q} \rightrightarrows|.|\mathbf{Q} \uparrow\uparrow| \equiv q_0$ **,** on Conductor AK_A and **Then** Energy-Circular-Prism = $|AK_A| .\{\mathbf{Q} \rightrightarrows \overrightarrow{AK_A}\}.\{\mathbf{Q} \uparrow\uparrow \overrightarrow{AK_A}\}$ $\equiv [\overline{\mathbf{Q}}_x \overline{\mathbf{Q}}_y].|AK_A| = |\leftrightarrow|.|\updownarrow|.|AK_A|$, **i.e.**

An Energy-Circular-Cone which is a Moving Energy -Volume , and **Because** from Cauchy equations of Stresses are of three Dimensions , **Then The Energy Stresses** remains Flat ONLY- WHEN the Plane-Section is a circle [$\mathbf{E_2}$] and The Square becomes circle as The , ***Anti-Quadrature***..

Because from Mechanics , the Hydrogen **Nucleus** is kept in the Inscribed to the ***Tetrahedron-Cube-System*** where exist the Protons and Neutrons , while The Electrons of Hydrogen **Orbits** are kept into the Circumscribed , to the ***Tetrahedron–Cube-System*** so . Both Equilibrium by following the Menelaus – Ceba`s Principles .

Oxygen is the Element which fills 8 - 2 = 6 vertices of Cube and for Neon 10 -2 = 8 , as

$$\mathbf{Ox} \rightarrow \begin{Bmatrix} 1e & 0| & 0/ \\ 1e & \overrightarrow{2}| & 1e \\ 1e/ & 1e| & 1e \end{Bmatrix} \equiv \begin{pmatrix} +2 \\ -2 \end{pmatrix} \quad , \quad ② \equiv \mathbf{Ne}\ \overrightarrow{2}_{1x8} = \overrightarrow{2} + 8 = 10 \quad \rightarrow \begin{Bmatrix} 1 & 1 & 1 \\ 1 & \overrightarrow{2} & 1 \\ 1 & 1 & 1 \end{Bmatrix},$$

The Possible Orbits of Oxygen are Two as , → 1e /-- $\overrightarrow{2}$/-- / 0 ← and → 1e |-- $\overrightarrow{2}$|-- 0 | ← for all the 8-Cube Positions . ⓪ is the first , **Zero-Acting Cube** and is filled with 1e , therefore is the Only Steady Element which occupies the first Energy-Level ① with 2 Electron-Positions as ① = 2e Positions at $\vec{1}$, $\overleftarrow{1}$ **.** In the same way ① is the first **One-Acting Cube** and is filled with 2e, and so on , The Electron being in the Hydrogen -cave Precesses because of the different axis of Rotation and Nutation`s , from the continuous and immense-communication to the gravity **g** . Since Electron is continually oscillating with the Nutation-frequency $\mathbf{f_N}$ so it

Produces a continuous and oscillating Magnetic-field –**M**- which in turn is the source of an oscillating Electric - field – **E** - which implies the **Regeneration of each other** , **i.e.** it is a **Propagating Electromagnetic-Wave** where **E = B c** and with a quantum-Energy E = h f_N or E = 2μ.B , This Energy **E** consists the Hydrogen Bracket **Hook HBH** , or $[\overrightarrow{BH}]$ and it is the only **free monad** which can **Bond** to all Energy Structures .

The **Sphere-Cube-Tetrahedron Structure of Atoms** In Sphere Center –O- allows to issue The **Ceba`s Theorem for 6-Stresses** on the Inscribed Tetrahedron which Secure **The Stability** for the Nucleus-System , and for the Outer Electron-System the Stability is Obtained Per–Two electrons the Dipole Positions 1 ↔ 6 , 2 ↔ 5 , 3 ↔ 8 , 4 ↔7 , consisting The-Positive Dipole-$[\overrightarrow{BH}]$, to be able to move → Into-In- Outside it ← and allow the Atoms-Bonding as this is in Energy levels **.**

The Cube-Structure of Atoms and Orbits in the Out-Sphere formulates the $+[\overrightarrow{BH}]$ to those **Dipole Positions which lack of one electron** and so can **PLUG-In** the **LOOP** of the others.

Conclusion :

1… The Forces of Atom`s NUCLEUS , are Holded from → The Inscribed Sphere In The Tetrahedron , which follows the Ptolemy`s and Ceba`s Vectors Structure .

2… The Forces of Atom`s ORBITALS , are Holded from → The Into the Cube , Circumscribed–Sphere Complex Vectors Structure .

3… The Forces of Atom`s POSITIONS , are Holded from → The on Cube Inscribed Tetrahedron Complex Vectors Structure .

4… The Stability of the {TCIC} Mechanism follows the Ptolemy`s & Ceba`s Complex Vectors Structure [62]

The Action of the Complex–Vectors create the Waves of Motion ≡ **Energy Waves** which are closed curves and Transported . The **Pattern** of Disturbances with the Informations Propagates from the One-Edge-Point to the other Edge-Point of Conductors , **OR** is on the [Sub-Units] .

Coulomb force $[\oplus \rightarrow \ominus] \equiv F_{[+ \leftrightarrow -]} = \frac{[\oplus \leftrightarrow \ominus]}{r^2} = \frac{[\mathbf{2\,\sigma}]}{\mathbf{r^2}} = F = \boldsymbol{\sigma}.A = [\frac{2\pi rf}{\Phi}].A = w\ r.[\frac{\mathbf{A}}{\boldsymbol{\Phi}}] = v\ .[\frac{\mathbf{A}}{\boldsymbol{\Phi}}]$, and **Acts** on a Surface , A , between the Sub-Units **Axially** and **Rotationally** following Ptolemy`s and Ceba`s Theorem of Equilibrium **.**

Remarks : *The Work of Elastic Deformation* *On the* **{ TCIS }** ***Mechanism*** :

Every New Stress that is Applied to D - Element is always added to the existing And the Unity Work of Deformation is transformed to three Tetrahedron Sides.

When a **SIGNAL** ***Strikes the*** **TCIS – Cube`s** ***Mechanism*** **.**

The Signal is **Energy-Wave** ***carrying motion*** . Unloading Happens on the three **Still-Spaces in Tetrahedron - Sides** which withdrawn Loads are Displaced On the Anti-Parallel-Strand . If the Unloading is ABRUPT, then Kinetic Energy of 1-Strand is converted in the 2-Strand as Potential and so on . That is , CUBES Perform an Oscillating motion until the Mechanical Energy is extinguished to their Vertices , which are the stores of Information .

Anti-Parallel-Strand is the ***Body containing the Potential Energy ,*** During any Unloading on CUBES , it gives off **Mechanical - Work** , since the Points of application of the Gradually - withdrawn Loads are Displaced . The Stability of Kinetic – Potential Energy is **continuously** . **Potential** . From equation $F = \boldsymbol{\sigma}.A = [\frac{2\pi rf}{\Phi}].A = w\ r.[\frac{\mathbf{A}}{\boldsymbol{\Phi}}] = v\ .[\frac{\mathbf{A}}{\boldsymbol{\Phi}}]$ and G = Gravitational = $\boldsymbol{\sigma}.A = [\frac{2\pi rf}{\Phi}].A = w\ r.[\frac{\mathbf{A}}{\boldsymbol{\Phi}}] = c\ .[\frac{\mathbf{A}}{\boldsymbol{\Phi}}]$ is seen that any loaded **Conductor of Area A ,** is related to the **Golden Ratio Pattern** Φ , either in **Polygons-Plane or Apothem`s -Vector** , and **The Holding Energy in Polygons is → The Gravitational-Force which is as Stress $\boldsymbol{\sigma = 2G.\pi rf\ /\ \Phi}$.**

⇒ **Since** the Total Energy in Areas or Polygons is → $\mathbf{E = [\,\Phi \frac{\sigma}{4\pi r}\,].\overline{B} \equiv \frac{|\,B\,|^{\,2}}{2\,\pi^2 r^4} \equiv \frac{|\,B\,|^{\,2}}{2\,[\pi.r^2]\,[\pi.r^2]}} \leftarrow$

Then the Constant - Sum - Squares $[|BK|^2 + |BE_B|^2] = |0_B B|^2$, Is transferred onto These

Two Strands of Apothems which Response on the two Apothems $OA_{1A} \equiv DO_M$, $OA_{2A} \equiv DO_T$, [Fig-13] , of the ODD Regular Space Polygons . and are such for the Difference of Heights [$\mathbf{h_A - h_B}$] , as for the Difference of Apothems [$\mathbf{r_a - r_b}$] , For Total Energy E ,

5-e... The Figure-eight Knot & The Figure n = 1 ≈ ∞ , Knots Regular - Polygons : [114]

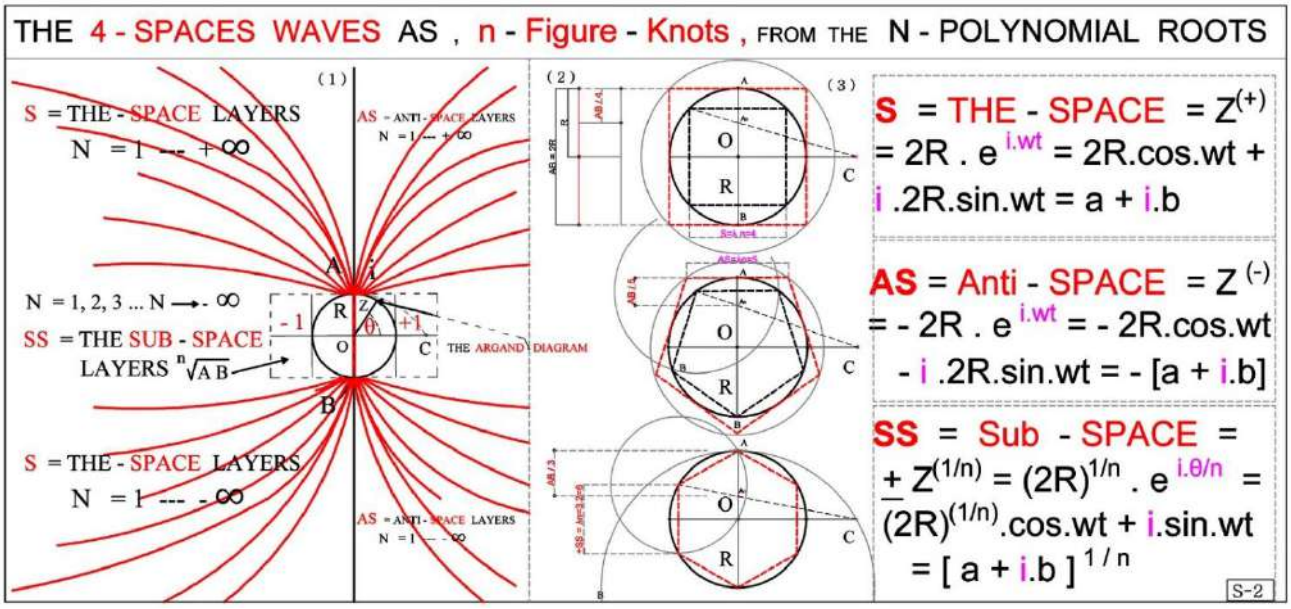


Figure - 15- : The Geometrical **Construction of the 4-Spaces** & the **Spaces Waves** of PNS .

In Argand Diagram AB = 2R → x = -1 , O , +1 , y = i , exist the **Vector of Amplitude** $\bar{\mathbf{z}} \equiv \mathbf{x + i \, . \, y}$, with angle $\boldsymbol{\theta}^{o} = [\, Z - OC \,]^{o}$, $[\mathbf{i} = \sqrt{-1}\,]$.**x** . The **Wave's " Strength "** and **" direction "**, where **x** is a number that represents the magnitude or length of the vector **z** , and from Definition of the **Complex Vector Space ,** is the **Set of Vectors of length from** The **n** Vertices which form the **n** Polygon Sides , or the Norm of the Complex-Vectors .

The Energy in Polygons Becomes from the **Angular-Momentum which is the Spin of their**

Apothems as relation → $\bar{\mathbf{B}} = \frac{2L}{\bar{w}} = \frac{2L}{2\pi f} = [\frac{2L}{2\pi}].[\frac{1}{f}] = \frac{\text{Constant}}{2\pi}\,[\frac{1}{f_m}]$ = **SPIN** ...[70] ←

In (1) , (2) , are the 4-Geometrical Spaces consisting The **Geometrical - Mould** .

In (2) , are the **Vectors of Amplitude** ON the 4 -Spaces consisting the **E-Form-Mould** .

It is constructed **in Space** → The Inscribed and the Circumscribed to the circle of diameter AB , The → **Regular Tetragon** ←

It is constructed **in Anti-Space** → The Inscribed and the Circumscribed to the circle of Diameter AB , The → **Regular Pentagon** ←

It is constructed **in Sub-Space** → The Inscribed and the Circumscribed to the circle of Diameter AB , The → **Regular Hexagon** ←

In (3) , are the **Norm** (magnitudes) of The **4 -Vectors** consisting the **Strength - Mould** .

In Fig-3 The time-harmonic function $\bar{\mathbf{z}}$ (t)≡**Wave** , x is the amplitude , w the angular frequency , and Phase of $\bar{\mathbf{z}}$ (t) to Be Determined by the number **n** , **n+1** of Vertices .

The **complex – Vector** ± $\bar{\mathbf{z}}$ (t) uses the **Norm** $|\bar{z}| = \mathbf{2.R}$, and the number **n** , of the Space which Determines the Number of Knots and Sides of **Regular Polygons** as ,

1…The Space is consisted of 4 masses [99] which Produce the **Space - Wave** = 2R.$e^{i.n\omega t}$,

2…The Anti-Space is consisted of 5 masses Producing the **Anti-Space–Wave**= - 2R.$e^{i.n\omega t}$

3…Sub-Space is consisted of 6 masses Producing the **Sub-Space–Wave** = $\pm 2R^{1/n}.\ e^{i.\theta/n}$

Remarks :

1.. The Real Shear -Stress that exists in the **Force–Carriers** Becomes from the Static Theory of Equivalent Tension (Moore's cycle) issues (1) = $\sigma = \sqrt{(\sigma 1 - \sigma 2)^2 + 4\,\tau_{12}}$, and $\sigma 1,2 = (\sigma 1+\sigma 2)/2 \pm (½)\sqrt{(\sigma 1 - \sigma 2)^2 + 4\,\tau_{yz}{}^2}$, instead of the classical case which is Stress $\sigma = 2.\tau_{max} = \sqrt{\sigma_1^2 + 4.\tau^2}$, . (2) . Because nature always follows the **uniform Tension** , so it follows the (1) case. where exist the equation $\sigma = \sqrt{(\sigma 1 - \sigma 2)^2 + 4\tau_{12}}$,and Stress **σ** 1,2 $= (\sigma 1+\sigma 2)/2 \pm (½)\sqrt{(\sigma 1 - \sigma 2)^2 + 4\tau_{yz}{}^2} = \frac{\sigma}{2}[\sqrt{5}+1] =$ **σ.[** $\frac{\sqrt{5}+1}{2}$ **]** $\Rightarrow$ ***Golden ratio for Stresses.***

2.. **The Bernoulli Astute** about Stresses – Linear - Distribution is equally True for Bending too.

3.. In the case of concentrated Stresses (such as Closed or Open Conductors) , **The Doubling Of the main Stresses** is as the Dublication of the Cube [126] and Trisection of angle [128] , where a **Right Angle Directional -Triangle** was of the relation 1 : 2 =30^{o} / 60^{o}, **in The case of the Isosceles Directional -Triangle** B $B_1 B_2$, is needed the relation 1 : 2 = 36^{o} / 72^{o}, and Observation is True , ***And this Because of the Above Golden ratio Pattern .***

4.. This Structure of **Space-Antispace** , transfers Information to each other , through their **Common Sub-space** which are in Sphere the Infinite great circles . Thus The G=-Force as Stress **σ** = $\frac{2\pi rf}{(n)\Phi} = \frac{2\pi rf}{1.\Phi}$ is the **Energy** as frequency in the **r** , Even Sequence Regular Set of their [n , n +2] Sets-Elements.

6-e-... The Complex - Force -Vectors from Golden-Ratio Stresses . [92 -114]

The Holding Energy in Polygons is The Gravitational-Force which is as Stress σ = 2G.πrf / Φ .

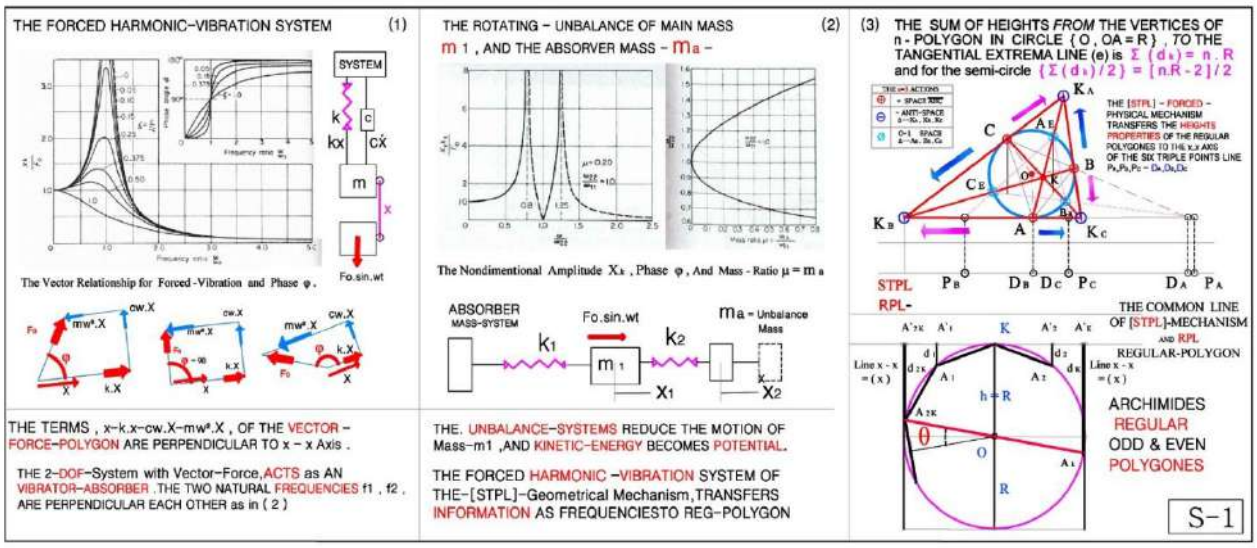


Figure -16- The **Complex-Force-Vector is F_0 .[cos(w t) + sin(w t)]= F_0 .$e^{i.[w\,t]}$ in Polygons**
And is shown the How Energy as Stresses Equilibrium in Regular-Polygons-Conductors

The Geometrical Construction of All - Even & Odd Regular Polygons .

In-1- The Terms = (m ẍ , cẋ , k x) , of **Vector Force Polygon** are Perpendicular to y-y axis. **In Case of Complex -Force -Vector** the Polygon becomes an **Absorber** which Reduces the motion from Kinetic Energy to **Potential Energy** (a Magnetic Field) .

In-2- The 2-DOF System with **Vector Force** such that , $w^2_{11} = \frac{k_1}{m_1}$, $w^2_{22} = \frac{k_2}{m_2}$, **acts** as **an Vibrator Absorber** , and reduces the motion of the mass m_1 to zero .The two **Natural Frequencies** , f_1 , f_2 , are Perpendicular [$\perp$] **each other** as the Diagrams. This happens for every action on the **8.[a , b , c , d] Physical - Mechanisms** .

In-3- The{ STPL } - Six Triple Points Line Mechanism is formed from **Three Tangents** from A ,B ,C Spaces on 2R - Circle **.** Since **Pascal-Desargues** lines meet on Markos STPL Mechanism lines ,**Then transfer the Properties & Information** (f_1 , f_2 , ,, f_n) From The **Complex-Vector-Absorber** = $\overline{\mathbf{2R}}$, →**To The Vector-Force-Polygons** ←

Remarks :

1.. **The n-Knots** Figures is any N-DOF System , which **Characteristic equation** results to an Algebraic equation of 3-n Degree , with main equation of motion [λ.M + K] X = 0

2.. **The n-Knots** Figures exist in Figures where the **Vector - Force - Polygons** have the same number **n** , as the in Figures . In (3) The Circularly- **Charged Breakages** are the three- **3** elements [**Σ = n . h**] ⇒ [$s^2 = \oplus$ **,** $2s^2 = \varnothing$, **-** $s^2 = \ominus$] which formulate the **3**-Knots Figures. This **{STPL}-Mechanism** of **3** elements (n=3) determinates the Mould which denotes the number **n** , of the Spaces for **Elementary-Particles Vector.**

3.. **The case of {TCIC}≡ [Σ = n . h]** ⇒ 8 x ① which is **the Physical Mechanism** , gives the **n-masses** of Atom`s &Compounds Determining the **Atoms Complex-Vector-Response**.

4.. **The case of Dublication of Energy** as Total Energy 2.E= **Spin** = $\check{S} = \vec{w}.\vec{B} = \frac{\vec{c}}{r}.\vec{B} = \vec{c}\,|\frac{\vec{B}}{r}|$ **Results to** $\vec{c} = \bar{c}_P = |\frac{\overrightarrow{2.r.\check{E}}}{\check{B}}|$ & The Centripetal velocity $\bar{c}_T \equiv \overrightarrow{RO} \equiv |\bar{v}| = \frac{\sigma}{2}[1+\sqrt{5}] \equiv \sigma.\Phi$,

From Golden ratio relation $\Phi^2 = \Phi + 1$, is shown the Reproduction of Energy Systems ;

From equation G = F = **σ** .A = $[\frac{2\pi rf}{\Phi}]$.A = wr.$[\frac{\mathbf{A}}{\mathbf{\Phi}}]$ = v.$[\frac{\mathbf{A}}{\mathbf{\Phi}}]$ is seen that any loaded **Area A** or **Conductor** $\vec{v}$ **,** is related to the **Golden Ratio Pattern** Φ , either in **Plane or in Vector** . When Energy is in concentrated Stresses (such as Closed or Open Conductors) , then **The Doubling of the main Stresses** is as the Dublication of the Cube [126] , and The **Trisectioning of the main Stresses** is as the Trisection of angle [128] , where

THE NEEDED MOULDS

a) is an **Isosceles Right Angle Directional -Triangle** which **Hypotenuse** lies on the Diameter of the circle with relation of Base-Angles 1 : 2 = 45° / 90° , & it is the **Material-Mould for the Squaring The circle** & Energy-circle [124].

b) is an **Isosceles Directional - Triangle** which **Base is a Chord** of the circle with relation of Base - Angles 2 : 1 = 72° / 36° , and it is the , **Material-Mould for Constructing the Regular Polygons of the circle .** & The Polygons of the Energy-circle [130].

c) is an **Right Angle Directional - Triangle** which **Hypotenuse** lies on the

Diameter of the circle with relation of Base - Angles 1 : 2 = 30ᵉ / 60ᵉ ,
and it is the **Material-Mould for (1) The Dublication of the Cube ←**
(2) The Trisection of any Angle ←

& **(1) The Dublication of the Energy - Cube** [126] .
(2) The Trisection of Energy - Octava . [128] .

The Golden ratio Φ on Segment AB , is at Point **C** Between A and B , while on the Material Point [⊕↻↺⊖] is on Principal Stress **σ** as frequency $f_n \equiv [\frac{1+\sqrt{5}}{2}] \frac{\sigma}{2\pi r} \equiv [\frac{n\sigma}{8\,r^2}].\bar{B} \equiv \frac{E}{h}$

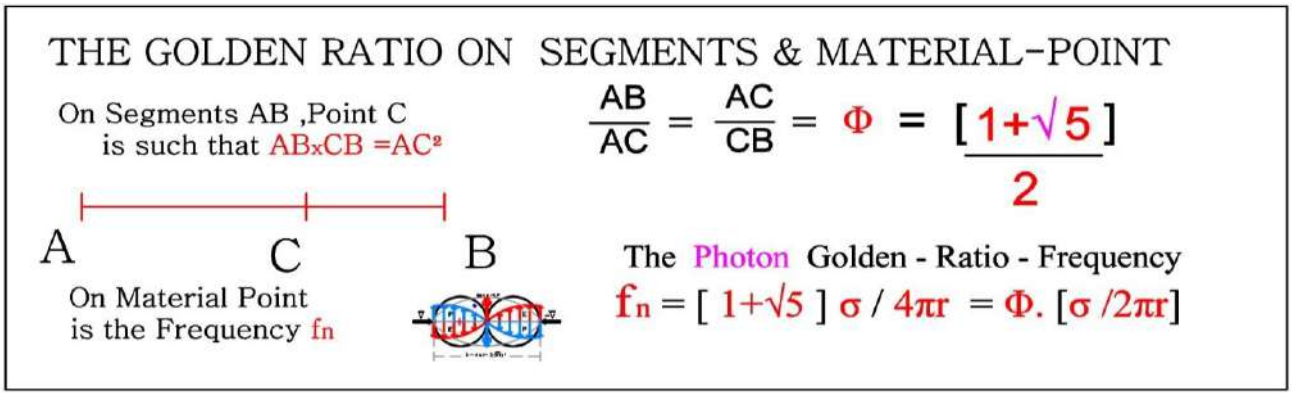
i.e. is the **Frequency f_n** or **Stress σ in Material - Point** and in cave 10^{-62} m &
also **as The-Golden-ratio Pattern** IN & Under **- Planck`s-level** 10^{-35} m .
The Work ≡ motion , in Material-Point is Produced from the frequencies f_N , &

f_N [S≡ $f_{1=n}$,f_2, f_3,$f_{R=}w^2$] = $n\frac{(1+\sqrt{5})\sigma}{4\pi r} = \frac{n\sigma.\bar{B}}{8\,r^2}$ ← The meter of Stationary motion , [125].

5.. **In this Document , which is The NEW Method for Constructing All Polygons in Geometry seems also in Spatial Geometry the same way which is Based on Three Triangles &Three Circles {1=Apothems , 2=Constant–Differences-Squares 3=Constant-Sum-Squares } where this Mechanism of these Triangles-Circles is in Grometry Drawn Step by Step with < A Ruler and a Compass only > , while in NATURE with Gravitational force G , where G-Force as Stress $\sigma = \frac{2\pi rf}{(n)\Phi} = \frac{2\pi rf}{1.\Phi}$ is the Energy of The first Wheel-Rim and which is distributed to the Even Sequence Sets of Orbital in Polygons with frequency f , in caves The-Polygons. In Mechanism and onto Any Even-Sequence --(n , n+2)-- of Polygons , The Inbetween Odd -(n+1)- is Detected on the Two Circles and Defined by the Third one on the main circle . The Resolution of Constructibility with Ruler and a Compass for All the Regular Polygons was achieved with , the Clear Step by Step Geometrical Analysis and Proofs , while is Highlighted The Erroneous Solutions at the time .** Markos 20 /10 /2026 .

7-e-... The Spin of Particles IS The Golden Ratio Φ & Stresses σ , in Regular-Polygons .

THE E – GEOMETRY & THE ENERGY CAVES [100]



The Golden-Ratio Property is continuously increasing by its self , ***a Self-Growing Property of frequency f_n of Material-point***. Equation (7) is also a very Special Property of the Golden ratio because , *according to Euclid* , A straight line AB is said to have been cut in Extreme and Mean ratio when as the whole line is to the greater segment AB / AC , so is the greater to the lesser

AC / CB , **and according to Markos ,** Since **frequency in Material-Point becomes** from Equilibrium of → **The Two-Opposite-Rotational-Energies** [±] ← as relation

$\overline{B}\,\overline{w} = L = \frac{1}{2} J_1 w_1^{\,2} + \frac{1}{2} J_2 w_2^{\,2} + \frac{1}{2} J_3 w_3^{\,2}$, or Angular-momentum B ≡ ± Spin S as ,

$\overline{B} = \frac{J.w}{2} = \frac{\pi r^4}{4}[2\pi f] = \frac{\pi^2 r^4}{2}[f] \equiv [\mathbf{Constant*f}] \equiv \frac{2L}{\overline{w}} = \frac{2L}{2\pi f} = [\frac{L}{\pi}].[\frac{1}{f}] = \frac{4r.L}{(1+\sqrt{5})\sigma}$, ***or***

$\overline{\mathbf{B}} = \frac{4r.L}{(1+\sqrt{5}).\sigma} = \frac{2r.[L=hf]}{(1+\sqrt{5}).\sigma} = \frac{hf}{2\pi f} = [\frac{h}{2\pi}] \equiv$ **SPIN** , where L = h f = Constant

and Frequency related to Φ is → $\mathbf{f_n} = [\frac{2}{\pi^2 r^4}].\overline{B} \equiv [\frac{1+\sqrt{5}}{2}]\frac{\sigma}{2\pi r} \equiv [\frac{\Phi\,\sigma}{2\pi\, r}]$ ……(7a)

The frequency ***which Occupies the Property of the Golden-Ratio-Pattern*** Φ , and equation (7-7a) defines that Material Point of frequency $\mathbf{f_n}$, *when collide with another* Material Point , *or with another Particle or Particles ,* then M-P Produces another monad as the ONE → **1** ≡ ***New Quaternion*** and the first continuous to be of the same Identity , ***frequency*** $\mathbf{f_n}$, as before

and from Euler`s , *rigid body dynamics work* $W = 2L = \overline{B}\,.\,\overline{w} = J\,.\,w^2 \equiv h.\mathbf{f_n} \equiv [\frac{\Phi\,\sigma}{2\pi\, r}]$ ←

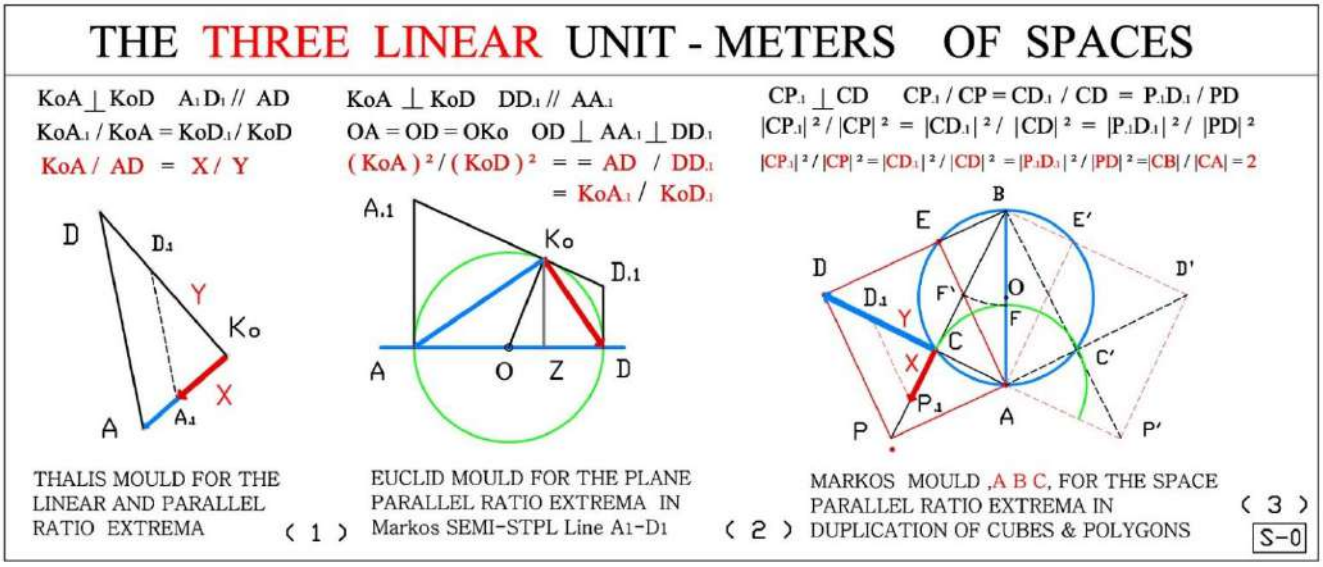


Figure – 17- : The Physical explanation of **The Golden-Ratio Φ on Vector $\overleftrightarrow{\mathbf{AB}}$,** in all Universe :
In figure , AB line-Vector is divided by Point C such that $AC = \frac{AB}{2}[\sqrt{5}+1] \equiv \mathbf{\Phi}$ **..**....(1)

Figure – 19 - : The Thales , Euclid , Markos Mould , for the **Linear–Plane-Space Constructions**.

In.1- , The Extrema Ratio Meters , Linear Segments $K_O A \perp K_O D$, and X : Y where $X = K_O A_1$ - $Y = K_O D_1$ Become $\mathbf{X : Y = K_O D : K_O A}$.

In.2- , The Diamesus $O K_O = OA = OD$ and $O D_1$, $O A_1$ are Diameters of the two circles where , $K_O A_1 = A A_1$, $K_O D_1 = D D_1$, the linear Ratio $\mathbf{K_O A / K_O D = A A_1 / D D_1}$, and Surface Ratio $\mathbf{[K_O A]^2 / [K_O D]^2 = A A_1 / D D_1 = \{K_O A_1 / K_O D_1\}}$ ….[124]

In.3-, The **MOULD** of The 2-Right Triangles **PAD , PBD** on where valids the Dublication $\mathbf{[CP]^3 = 2.[CD]^3}$ **.** on the Right angle Triangle , **ABC , where CB = 2.CA , and for Any Segment X , to Be Dublicated to Y . .**…. [126]

In triangle , ABC . Issues CB / CA = 2 and for **CA = R → CB = 2.R** , then **CA / CB = 2** . In Even Sequence Set [n , n+2] , for **n=** *Tetragon* and **Side KA= R** $\sqrt{2}$, **n+2** = *Hexagon* and Side **KB = R** , ***Issues*** a)..The Difference of Squares is $\mathbf{[KA]^2 - [KB]^2} = 2.R^2 - R^2 =$ **Constant**

And b)..The Sum of Squares **[KA]² + [KB]²** = 2.R² + R² = 3. R² = **Constant , i.e.** The Sides KA , KB, of Set [n , n+2] and Ratio 1 : 2 **, Follow the in Golden Radio Triangle ABC Ratio.**

The Holding Energy in Moulds is The Gravitational-Force which is as Stress σ = 2G.πrf / Φ .

F..THE GALOIS – CAUSS CONSTRUCTIBILITY OF THE REGULAR–POLYGONS

Gaus Galois Constructibility of Regular Polygons

AI Overview



A regular n-sided polygon can be constructed with a compass and straightedge if and only if n is the product of a power of 2 and any number of distinct Fermat primes ($2^{2^k}+1$). Carl Friedrich Gauss proved the sufficiency in 1796, and Pierre-Laurent Wantzel proved necessity using modern algebraic field theory. Wikipedia +2

The Gauss-Wantzel Theorem

- **Constructibility condition:** $n = 2^k p_1 p_2 \cdots p_m$.
- **Fermat primes (p_i):** Prime numbers of the form $p = 2^{2^k} + 1$.
- **Known Fermat primes:** 3, 5, 17, 257, and 65537.
- **Examples of constructible polygons:** Triangle ($n = 3$), pentagon ($n = 5$), heptadecagon ($n = 17$), and 15-gon (3×5).
- **Examples of non-constructible polygons:** Heptagon ($n = 7$), nonagon ($n = 9$), and undodecagon ($n = 11$). Wikipedia +4

Connection to Field Extensions and Galois Theory

- **Cyclotomic fields:** Constructing a regular n-gon is equivalent to building a primitive n-th root of unity ζ_n in the complex plane.
- **Degree of extension:** The extension field $\mathbb{Q}(\zeta_n)$ over $\mathbb{Q}$ has a degree given by Euler's totient function $\varphi(n)$.
- **Power of 2 requirement:** Ruler-and-compass operations restrict field extension degrees to powers of 2. Thus, $[\mathbb{Q}(\zeta_n):\ \mathbb{Q}] = \varphi(n) = 2^k$.
- **Galois group symmetry:** The Galois group $\mathrm{Gal}(\mathbb{Q}(\zeta_n)/\mathbb{Q})$ is abelian and isomorphic to $(\mathbb{Z}/n\mathbb{Z})^\times$, breaking down cleanly via nested square roots only when $\varphi(n)$ is a power of 2. YouTube · Richard E Bo... +1

The Judgment regarding The Correct or Not of The Constructibility , is left

Exclusively to The Dialectic Logic of Nature And to No other Logic or Reason
→ Markos Georgallides 26 / 09 / 2026 ←

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Article [124] is Dedicated to the **National Technical University of Athens** , NTUA , from where I Graduated and Acquired the Path Knowledge of the Truths of Nature..

Το Άρθρο [124] είναι Αφιερωμένο στο Εθνικό Μετσόβιο Πολυτεχνείο (ΕΜΠ) , από όπου Αποφοίτησα και Απέκτησα την Ατραπό της Γνώσης των Αληθειών της Φύσης.

Markos Georgallides comes from Cyprus and currently he resides in the City Larnaca , Cyprus after being expelled from his Home Town **Famagusta - Varosha ,** by The Barbaric Turks in August 1974 . He works as a consultant Civil and Architect Engineer having his own Business . He is also The author of numerous scholarly Articles focusing on **Euclidean** and **Material Geometry** , and mathematical To Physics and Mechanics related Subjects **.** He obtained his degree from the Athens , National Technical , Polytechnic University [NATUA] Athens- Greece , and subsequently studied in Germany , Math Theory of Photoelasticity **.**