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Representation of group algebra of 16 order non-abelian groups

By:

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Abstract

Group algebras of non-abelian groups of order 16 are represented in terms of block circulant matrices. These are nine groups listed according to the property of semidirect and the numbers of generators involved.

Keywords:

Circulant Matrix, Group Algebra, Semidirect Product, Non-abelian Group.

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I) Preliminaries

Let G be a group, assume that H is a normal subgroup of G, K is a subgroup of G, $H \cap K = \{1\}$, and $G = HK$. Suppose that K acts on H by automorphisms of H, then there exists a homomorphism $\emptyset: K \to Aut(H)$. Assume the action is by conjugation, then for $k \in K$ and $h \in H$ we have $kh = \emptyset(k)(h) = khk^{-1}$. G is an internal semidirect product of H and K by $\emptyset$, it is denoted by $G = H \rtimes_{\emptyset} K$[5].

We have nine non-abelian groups of order 16[3], which are as follows

i) $G_1 = \langle \alpha, \beta : \alpha^8 = \beta^4 = 1, \alpha^4 = \beta^2, \beta\alpha = \alpha^{-1}\beta \rangle$

ii) $G_2 = \langle \alpha, \beta : \alpha^8 = \beta^2 = 1, \beta\alpha = \alpha^3\beta \rangle$

iii) $G_3 = \langle \alpha, \beta : \alpha^8 = \beta^2 = 1, \beta\alpha = \alpha^5\beta \rangle$

iv) $G_4 = D_8 = \langle \alpha, \beta : \alpha^8 = \beta^2 = 1, \beta\alpha = \alpha^{-1}\beta \rangle$

v) $G_6 = (C_2 \times C_2) \rtimes C_4 = \langle \alpha, \beta : \alpha^4 = \beta^4 = 1, \beta\alpha = \alpha^{-1}\beta^{-1} \rangle$

vi) $G_5 = C_4 \rtimes C_4 = \langle \alpha, \beta : \alpha^4 = \beta^4 = 1, \beta\alpha = \alpha^{-1}\beta \rangle$

vii)$G_7 = D_4 \times C_2 = \langle \alpha, \beta, \gamma : \alpha^4 = \beta^2 = \gamma^2 = 1, \beta\alpha = \alpha^{-1}\beta, \gamma\alpha = \alpha\gamma, \gamma\beta = \beta\gamma \rangle$

viii) $G_8 = Q_4 \times C_2 = \langle \alpha, \beta, \gamma : \alpha^4 = \beta^4 = \gamma^2 = 1, \alpha^2 = \beta^2, \beta\alpha = \alpha^{-1}\beta, \gamma\alpha = \alpha\gamma, \gamma\beta = \beta\gamma \rangle$

ix) $G_9 = \langle \alpha, \beta, \gamma : \alpha^4 = \beta^2 = \gamma^2 = 1, \beta\alpha = \alpha\beta, \gamma\alpha = \alpha\gamma, \gamma\beta = \alpha^2\beta\gamma \rangle$

We apply the results in [1] and [2] to these groups except G_9, since it is not a semi direct product. It may be done by direct method.

Let F be a field. A ring A with unity is an algebra over F (F −algebra) if A is a vector space over F and the following compatibility condition holds $(sa).b = s(a.b) = a.(sb)$ for any $s \in F$. A is also called associative algebra (over F). The dimension of the algebra A is the dimension of A as a vector space over F.

Theorem 1[4]

Let A be a n −dimensional algebra over a field F. Then there is a one to one algebra homomorphism from A into $M_n(F)$, the algebra of n −matrices over F.

Let $G = \{g_1 = 1, g_2, \dots, g_n\}$ be a finite group of order n and F a field. Define $FG = \{a_1 g_1 + a_2 g_2 + \cdots + a_n g_n : a_i \in F\}$. FG is n −dimensional vector space over F with basis G. Multiplication of G can be extended linearly to FG. Thus, FG becomes an algebra over F of dimension n. FG is called group algebra. The following identifications should be realized:

i) $0_F g_G = 0_{FG} = 0$ for any $g \in G$

ii) $1_F g_G = g_{FG} = g$ for any $g \in G$. In particular $1_F g_G = 1_{FG} = 1$

$a_F 1_G = a_{FG}$ for any $a \in G$

A circulant matrix M on parameters $a_0, a_1, \dots, a_{n-1}$ is defined as follows:

$$M(a_0, a_1, \dots, a_{n-1}) = \begin{bmatrix} a_0 & a_{n-1} \cdots & a_1 \\ a_1 & a_0 \ \cdots & a_2 \\ \vdots & \vdots & \vdots \\ a_{n-1} & a_{n-1} & a_0 \end{bmatrix}$$

This matrix may be denoted in terms of its columns by $[col(a_0)|col(a_{n-1})| \dots |col(a_0)]$. M is said to be circulant block matrix if it is if the form $M(M_1, M_2, \dots, M_n)$. i.e, it is circulant blockwise on the blocks $M_1, M_2, \dots, M_n$.

Thus,

$$M = \begin{bmatrix} M_1 & M_n \cdots & M_2 \\ M_2 & M_1 \cdots & M_3 \\ \vdots & \vdots & \vdots \\ M_n & M_{n-1} & M_1 \end{bmatrix}$$

II) Main Results

Theorem 2) [1]

Let F be a field and $G = \langle \alpha : \alpha^n = 1 \rangle$ a cyclic group of order n. Then any element $a_1 1 + a_2 \alpha + \cdots + a_n \alpha^{n-1}$ of FG can be represented with respect to the order basis.

$\{1, \alpha, \dots, \alpha^{n-1}\}$ by the circulant matrix $M(a_1, a_2, \dots, a_n) = \begin{bmatrix} a_1 & a_n & \dots a_2 \\ a_2 & a_1 & \dots a_3 \\ \vdots & \vdots & \vdots\ \vdots \\ a_n & a_{n-1} \dots & a_1 \end{bmatrix}$.

Corollary 5) [1]

Let F be a field and $D_n = \langle \alpha, \beta : \alpha^n = \beta^2 = 1, \beta\alpha = \alpha^{n-1}\beta \rangle$ the dihedral group. Matrix representation of the general element $\sum_{i=0}^{n-1} a_i \alpha^i + \sum_{i=0}^{n-1} b_i \alpha^i \beta$ in FD_n is given by $M\big(M(a_0, a_1, \dots, a_{n-1}), M^{\beta}(b_0, b_1 \dots, b_{n-1})\big)$.

The general element of the group algebra FG is given by $w = w_0 + w_1 + \cdots + w_{m-1}$, where $w_i = (a_{0i} + a_{1i}\alpha + \cdots + a_{n-1i}\alpha^{n-1})\beta^i$ for $i = 0,1, \dots, m-1$. We take the following natural basis of the group algebra FG.

$B = \{1, \alpha, \dots, \alpha^{n-1}; \beta, \alpha\beta, \dots, \alpha^{n-1}\beta; \dots, \beta^{m-1}, \alpha\beta^{m-1}, \dots, \alpha^{n-1}\beta^{m-1}\}$. This can be written as follows: $B = \{1, \alpha, \dots, \alpha^{n-1}\}\beta^0 \cup \{1, \alpha, \dots, \alpha^{n-1}\}\beta \cup \dots \cup \{1, \alpha, \dots, \alpha^{n-1}\}\beta^{m-1}$. Briefly $B = B_0 \cup B_1 \cup \dots \cup B_{m-1}$, where $B_{j=}\{1, \alpha, \dots, \alpha^{n-1}\}\beta^j$. Let $T_B: FG \to M_n(F)$ be the linear transformation of our matrix representation relative to the basis B. Let $T_{B_j} = T_B|_{B_j}$. By theorem 2 we have the following

Lemma 6) [1]

$T_{B_0}(w_0) = M\big(a_{00}, a_{10}, \dots, a_{n-1,0}\big)$.

Lemma 7) [1]

$T_{B_j}(w_i)$ is obtained by columns interchange of $M\big(a_{0i}, a_{1i}, \dots, a_{n-1,i}\big)$ according to the elements $\alpha^{qt}, \alpha^{qt+r^i}, \dots, \alpha^{qt+(n-1)r^i}$.

Theorem 8 [1]

The matrix representation of $w = w_0 + w_1 + \cdots + w_{m-1}$ in FG relative to the basis $B = B_0 \cup B_1 \cup \dots \cup B_{m-1}$ is given by

$$T_B(w) = \begin{bmatrix} T_{B_0}(w_0) & T_{B_1}(w_{m-1}) \dots & T_{B_{m-1}}(w_1) \\ T_{B_0}(w_1) & T_{B_1}(w_0) \ \dots & T_{B_{m-1}}(w_2) \\ \vdots & \vdots \ \ \dots & \vdots \\ T_{B_0}(w_{m-1}) & T_{B_1}(w_{m-2}) \cdots & T_{B_{m-1}}(w_0) \end{bmatrix}$$

Note that if the order of the basis elements is changed, we obtain a different matrix of representation. The new matrix is obtained by suitable interchanging of the columns of the matrix $M(a_0, a_1, \dots, a_{n-1})$.

For more complicated finite groups we use the circulant block matrices to do the required representations.

Now, let G be an internal semidirect product of H and a cyclic group $K = \langle \alpha \rangle$ by $\emptyset$. Then the matrix representation $[w]$ of the general element w in FG is given as follows:

$G = H \rtimes_{\emptyset} K, \emptyset : K \to Aut(H)$ is a homomorphism, $\emptyset(\gamma)(h) = \gamma h \gamma^{-1}$. Suppose that $H = \{h_1, h_2, \ldots, h_n\}, K = C_m\langle\gamma\rangle = \{1, \gamma, \ldots, \gamma^{m-1}\}$ then the general element w in FG is $w = a_1 h_1 1 + a_2 h_2 1 + \cdots + a_n h_n 1 + a_{n+1} h_1 \gamma + a_{n+2} h_2 \gamma + \cdots + a_{2n} h_n \gamma + a_{2n+1} h_1 \gamma^2 + \cdots + a_{3n} h_n \gamma^2 + \cdots + a_{mn} h_n \gamma^{m-1}$.

Now we can write w as:

$w = w_1 + w_2 + \cdots + w_m$

Where

$$w_1 = a_1 h_1 1 + a_2 h_2 1 + \cdots + a_n h_n 1$$

$$w_2 = a_{n+1} h_1 \gamma + a_{n+2} h_2 \gamma + \cdots + a_{2n} h_n \gamma$$

$$\vdots$$

$$w_m = a_{(m-1)(n+1)} h_1 \gamma^{m-1} + \cdots + a_{mn} h_n \gamma^{m-1}$$

The matrix representation $[w]$ of w is $[w] = M\left([w_1], [w_2]^{\gamma}, \ldots, [w_m]^{\gamma^{m-1}}\right)$, where $\gamma^i : H \to H$ is the automorphism $\gamma^i = \emptyset(\gamma)(h) = \gamma^i h \gamma^{-i}$ and $[w_i] = \left[col(\gamma^i(h_1))\,\middle|\,col\left(\gamma^i(h_2)\right)\middle|\ldots|col\left(\gamma^i(h_n)\right)\right]$.

Theorem 9) [2]

The matrix representation $[w]$ of the general element w in FG is

$$[w] = \begin{bmatrix} [w_1] & [w_m]^{\gamma^{m-1}} & \ldots & [w_2]^{\gamma} \\ [w_2]^{\gamma} & [w_1] & \ldots & [w_m]^{\gamma^2} \\ \vdots & \vdots & \vdots & \vdots \\ [w_m]^{\gamma^{m-1}} & [w_m]^{\gamma^{m-2}} & [w_1] & \end{bmatrix}$$

III) Applications

i)$G_7 = (C_2\langle\alpha\rangle \times C_2\langle\beta\rangle) \rtimes C_4\langle\gamma\rangle = \langle \alpha, \beta : \alpha^4 = \beta^4 = 1, \beta\alpha = \alpha^{-1}\beta^{-1} \rangle$

$= \{1, \alpha, \alpha^2, \alpha^3, \beta, \alpha\beta, \alpha^2\beta, \alpha^3\beta, \beta^2, \alpha\beta^2, \alpha^2\beta^2, \alpha^3\beta^2, \beta^3, \alpha\beta^3, \alpha^2\beta^3, \alpha^3\beta^3\}$

$$w = a_1 1 + a_2\alpha + a_3\alpha^2 + a_4\alpha^3 + a_5\beta + a_6\alpha\beta + a_7\alpha^2\beta + a_8\alpha^3\beta + a_9\beta^2 + a_{10}\alpha\beta^2 + a_{11}\alpha^2\beta^2 + a_{12}\alpha^3\beta^2 + a_{13}\beta^3 + a_{14}\alpha\beta^3 + a_{15}\alpha^2\beta^3 + a_{16}\alpha^3\beta^3.$$

$G = (C_2\langle\alpha\rangle \times C_2\langle\beta\rangle) \rtimes C_4\langle\gamma\rangle; \varphi : C_4\langle\gamma\rangle \to \mathrm{Aut}(C_2\langle\alpha\rangle \times C_2\langle\beta\rangle)$

$w_0 = a_1 1 + a_2\alpha + a_3\alpha^2 + a_4\alpha^3,$

$w_1 = a_5\beta + a_6\alpha\beta + a_7\alpha^2\beta + a_8\alpha^3\beta,$

$w_2 = a_9\beta^2 + a_{10}\alpha\beta^2 + a_{11}\alpha^2\beta^2 + a_{12}\alpha^3\beta^2,$

$w_3 = a_{13}\beta^3 + a_{14}\alpha\beta^3 + a_{15}\alpha^2\beta^3 + a_{16}\alpha^3\beta^3.$

$$[w] = \begin{bmatrix} [w_0] & [w_3]^{\beta^3} & [w_2]^{\beta^2} & [w_1]^{\beta} \\ [w_1]^{\beta} & [w_0] & [w_3]^{\beta^3} & [w_2]^{\beta^2} \\ [w_2]^{\beta^2} & [w_1]^{\beta} & [w_0] & [w_3]^{\beta^3} \\ [w_3]^{\beta^3} & [w_2]^{\beta^2} & [w_1]^{\beta} & [w_0] \end{bmatrix}$$

$$[w_0] = \begin{bmatrix} a_1 a_4 a_3 a_2 \\ a_2 a_1 a_4 a_3 \\ a_3 a_2 a_1 a_4 \\ a_4 a_3 a_2 a_1 \end{bmatrix}$$

$G = (C_2\langle\alpha\rangle \times C_2\langle\beta\rangle) \rtimes_\varphi C_4\langle\gamma\rangle.$

$\varphi: C_4\langle\gamma\rangle \to \mathrm{Aut}(C_2\langle\alpha\rangle \times C_2\langle\beta\rangle)$is a homomorphism.

$\varphi(\beta)(1) = \beta 1 \beta^{-1} = 1$

$\varphi(\beta)(\alpha) = \beta\alpha\beta^{-1} = \alpha^3\beta^2$

$\varphi(\beta)(\alpha^2) = \beta\alpha^2\beta^{-1} = \alpha^2\beta^2$

$\varphi(\beta)(\alpha^3) = \beta\alpha^3\beta^{-1} = \alpha\beta^2$

$[w_1] = [\mathrm{col}(1)|\mathrm{col}(\alpha)|\mathrm{col}(\alpha^2)|\,\mathrm{col}(\alpha^3)],$

$[w_1]^\beta = [\mathrm{col}(1)|\mathrm{col}(\alpha^3\beta^2)|\mathrm{col}(\alpha^2\beta^2)|\,\mathrm{col}(\alpha\beta^2)].$

$\varphi(\beta^2)(1) = \beta^2 1 \beta^{-2} = 1$

$\varphi(\beta^2)(\alpha) = \beta^2\alpha\beta^{-2} = \alpha$

$\varphi(\beta^2)(\alpha^2) = \beta^2\alpha^2\beta^{-2} = \alpha^2$

$\varphi(\beta^2)(\alpha^3) = \beta^2\alpha^3\beta^{-2} = \alpha^3$

$[w_2] = [\mathrm{col}(1)|\mathrm{col}(\alpha)|\mathrm{col}(\alpha^2)|\,\mathrm{col}(\alpha^3)],$

$\varphi(\beta^3)(1) = \beta^3 1 \beta^{-3} = 1$

$\varphi(\beta^3)(\alpha) = \beta^3\alpha\beta^{-3} = \alpha^3\beta^2$

$\varphi(\beta^3)(\alpha^2) = \beta^3\alpha^2\beta^{-3} = \alpha^2\beta^2$

$\varphi(\beta^3)(\alpha^3) = \beta^3\alpha^3\beta^{-3} = \alpha\beta^2$

$[w_3] = [\mathrm{col}(1)|\mathrm{col}(\alpha)|\mathrm{col}(\alpha^2)|\,\mathrm{col}(\alpha^3)],$

$[w_3]^{\beta^3} = [col(1)|col(\alpha^3\beta^2)|col(\alpha^2\beta^2)|col(\alpha\beta^2)]$.

$$[w_3]^{\beta^3} = \begin{bmatrix} a_{13}a_6a_7a_8 \\ a_{14}a_7a_8a_5 \\ a_{15}a_8a_5a_6 \\ a_{16}a_5a_6a_7 \end{bmatrix}$$

$$[w] = \begin{bmatrix} [w_0] & [w_3]^{\beta^3} & [w_2]^{\beta^2} & [w_1]^{\beta} \\ [w_1]^{\beta} & [w_0] & [w_3]^{\beta^3} & [w_2]^{\beta^2} \\ [w_2]^{\beta^2} & [w_1]^{\beta} & [w_0] & [w_3]^{\beta^3} \\ [w_3]^{\beta^3} & [w_2]^{\beta^2} & [w_1]^{\beta} & [w_0] \end{bmatrix}$$

$$\left[\begin{array}{cccccccccccccccccccc}
a_1 & a_4 & a_3 & a_2 & \vdots & a_{13} & a_6 & a_7 & a_8 & \vdots & a_9 & a_{12} & a_{11} & a_{10} & \vdots & a_5 & a_{14} & a_{15} & a_{16} \\
a_2 & a_1 & a_4 & a_3 & \vdots & a_{14} & a_7 & a_8 & a_5 & \vdots & a_{10} & a_9 & a_{12} & a_{11} & \vdots & a_6 & a_{15} & a_{16} & a_{13} \\
a_3 & a_2 & a_1 & a_4 & \vdots & a_{15} & a_8 & a_5 & a_6 & \vdots & a_{11} & a_{10} & a_9 & a_{12} & \vdots & a_7 & a_{16} & a_{13} & a_{14} \\
a_4 & a_3 & a_2 & a_1 & \vdots & a_{16} & a_5 & a_6 & a_7 & \vdots & a_{12} & a_{11} & a_{10} & a_9 & \vdots & a_8 & a_{13} & a_{14} & a_{15} \\
\cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \cdots & \cdots & \cdots \\
a_5 & a_{14} & a_{15} & a_{16} & \vdots & a_1 & a_4 & a_3 & a_2 & \vdots & a_{13} & a_6 & a_7 & a_8 & \vdots & a_9 & a_{12} & a_{11} & a_{10} \\
a_6 & a_{15} & a_{16} & a_{13} & \vdots & a_2 & a_1 & a_4 & a_3 & \vdots & a_{14} & a_7 & a_8 & a_5 & \vdots & a_{10} & a_9 & a_{12} & a_{11} \\
a_7 & a_{16} & a_{13} & a_{14} & \vdots & a_3 & a_2 & a_1 & a_4 & \vdots & a_{15} & a_8 & a_5 & a_6 & \vdots & a_{11} & a_{10} & a_9 & a_{12} \\
a_8 & a_{13} & a_{14} & a_{15} & \vdots & a_4 & a_3 & a_2 & a_1 & \vdots & a_{16} & a_5 & a_6 & a_7 & \vdots & a_{12} & a_{11} & a_{10} & a_9 \\
\cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \cdots & \cdots & \cdots \\
a_9 & a_{12} & a_{11} & a_{10} & \vdots & a_5 & a_{14} & a_{15} & a_{16} & \vdots & a_1 & a_4 & a_3 & a_2 & \vdots & a_{13} & a_6 & a_7 & a_8 \\
a_{10} & a_9 & a_{12} & a_{11} & \vdots & a_6 & a_{15} & a_{16} & a_{13} & \vdots & a_2 & a_1 & a_4 & a_3 & \vdots & a_{14} & a_7 & a_8 & a_5 \\
a_{11} & a_{10} & a_9 & a_{12} & \vdots & a_7 & a_{16} & a_{13} & a_{14} & \vdots & a_3 & a_2 & a_1 & a_4 & \vdots & a_{15} & a_8 & a_5 & a_6 \\
a_{12} & a_{11} & a_{10} & a_9 & \vdots & a_8 & a_{13} & a_{14} & a_{15} & \vdots & a_4 & a_3 & a_2 & a_1 & \vdots & a_{16} & a_5 & a_6 & a_7 \\
\cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \cdots & \cdots & \cdots \\
a_{13} & a_6 & a_7 & a_8 & \vdots & a_9 & a_{12} & a_{11} & a_{10} & \vdots & a_5 & a_{14} & a_{15} & a_{16} & \vdots & a_1 & a_4 & a_3 & a_2 \\
a_{14} & a_7 & a_8 & a_5 & \vdots & a_{10} & a_9 & a_{12} & a_{11} & \vdots & a_6 & a_{15} & a_{16} & a_{13} & \vdots & a_2 & a_1 & a_4 & a_3 \\
a_{15} & a_8 & a_5 & a_6 & \vdots & a_{11} & a_{10} & a_9 & a_{12} & \vdots & a_7 & a_{16} & a_{13} & a_{14} & \vdots & a_3 & a_2 & a_1 & a_4 \\
a_{16} & a_5 & a_6 & a_7 & \vdots & a_{12} & a_{11} & a_{10} & a_9 & \vdots & a_8 & a_{13} & a_{14} & a_{15} & \vdots & a_4 & a_3 & a_2 & a_1
\end{array}\right]$$

ii) $G_6 = D_4\langle\alpha\rangle \times C_2\langle\beta\rangle = \langle \alpha, \beta, \gamma : \alpha^4 = \beta^2 = \gamma^2 = 1, \beta\alpha = \alpha^{-1}\beta, \gamma\alpha = \alpha\gamma . \gamma\beta = \beta\gamma \rangle$

$= \{1, \alpha, \alpha^2, \alpha^3, \beta, \alpha\beta, \alpha^2\beta, \alpha^3\beta, \gamma, \alpha\gamma, \alpha^2\gamma, \alpha^3\gamma, \beta\gamma, \alpha\beta\gamma, \alpha^2\beta\gamma, \alpha^3\beta\gamma\}$

$$w = a_1 1 + a_2\alpha + a_3\alpha^2 + a_4\alpha^3 + a_5\beta + a_6\alpha\beta + a_7\alpha^2\beta + a_8\alpha^3\beta + a_9\gamma + a_{10}\alpha\gamma + a_{11}\alpha^2\gamma + a_{12}\alpha^3\gamma + a_{13}\beta\gamma + a_{14}\alpha\beta\gamma + a_{15}\alpha^2\beta\gamma + a_{16}\alpha^3\beta\gamma.$$

$$T_{B_0}(w_0) = M(a_1, a_2, a_3, a_4) = \begin{bmatrix} a_1a_4a_3a_2 \\ a_2a_1a_4a_3 \\ a_3a_2a_1a_4 \\ a_4a_3a_2a_1 \end{bmatrix}$$

$$T_{B_0}(w_1) = M^{\beta}(a_5, a_6, a_7, a_8) = \begin{bmatrix} a_5a_6a_7a_8 \\ a_6a_7a_8a_5 \\ a_7a_8a_5a_6 \\ a_8a_5a_6a_7 \end{bmatrix}$$

$$T_{B_0}(w_2) = M(a_9, a_{10}, a_{11}, a_{12}) = \begin{bmatrix} a_9 & a_{12} & a_{11} & a_{10} \\ a_{10} & a_9 & a_{12} & a_{11} \\ a_{11} & a_{10} & a_9 & a_{12} \\ a_{12} & a_{11} & a_{10} & a_9 \end{bmatrix}$$

$$T_{B_0}(w_3) = M^{\beta}(a_{13}, a_{14}, a_{15}, a_{16}) = \begin{bmatrix} a_{13} & a_{14} & a_{15} & a_{16} \\ a_{14} & a_{15} & a_{16} & a_{13} \\ a_{15} & a_{16} & a_{13} & a_{14} \\ a_{16} & a_{13} & a_{14} & a_{15} \end{bmatrix}$$

$$T_{B_1}(w_0) = M(a_1, a_2, a_3, a_4) = \begin{bmatrix} a_1 & a_4 & a_3 & a_2 \\ a_2 & a_1 & a_4 & a_3 \\ a_3 & a_2 & a_1 & a_4 \\ a_4 & a_3 & a_2 & a_1 \end{bmatrix}$$

$$T_{B_1}(w_1) = M^{\beta}(a_5, a_6, a_7, a_8) = \begin{bmatrix} a_5 & a_6 & a_7 & a_8 \\ a_6 & a_7 & a_8 & a_5 \\ a_7 & a_8 & a_5 & a_6 \\ a_8 & a_5 & a_6 & a_7 \end{bmatrix}$$

$$T_{B_1}(w_2) = M(a_9, a_{10}, a_{11}, a_{12}) = \begin{bmatrix} a_9 & a_{12} & a_{11} & a_{10} \\ a_{10} & a_9 & a_{12} & a_{11} \\ a_{11} & a_{10} & a_9 & a_{12} \\ a_{12} & a_{11} & a_{10} & a_9 \end{bmatrix}$$

$$T_{B_1}(w_3) = M^{\beta}(a_{13}, a_{14}, a_{15}, a_{16}) = \begin{bmatrix} a_{13} & a_{14} & a_{15} & a_{16} \\ a_{14} & a_{15} & a_{16} & a_{13} \\ a_{15} & a_{16} & a_{13} & a_{14} \\ a_{16} & a_{13} & a_{14} & a_{15} \end{bmatrix}$$

$$[w] = M\left(M\left(M(a_1, a_2, a_3, a_4), M^{\beta}(a_5, a_6, a_7, a_8)\right), M\left(M(a_9, a_{10}, a_{11}, a_{12}), M^{\beta}(a_{13}, a_{14}, a_{15}, a_{16})\right)\right)$$

$$\left[\begin{array}{cccc:cccc:cccc:cccc} a_1 & a_4 & a_3 & a_2 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{12} & a_{11} & a_{10} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_2 & a_1 & a_4 & a_3 & a_6 & a_7 & a_8 & a_5 & a_{10} & a_9 & a_{12} & a_{11} & a_{14} & a_{15} & a_{16} & a_{13} \\ a_3 & a_2 & a_1 & a_4 & a_7 & a_8 & a_5 & a_6 & a_{11} & a_{10} & a_9 & a_{12} & a_{15} & a_{16} & a_{13} & a_{14} \\ a_4 & a_3 & a_2 & a_1 & a_8 & a_5 & a_6 & a_7 & a_{12} & a_{11} & a_{10} & a_9 & a_{16} & a_{13} & a_{14} & a_{15} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_5 & a_6 & a_7 & a_8 & a_1 & a_4 & a_3 & a_2 & a_{13} & a_{14} & a_{15} & a_{16} & a_9 & a_{12} & a_{11} & a_{10} \\ a_6 & a_7 & a_8 & a_5 & a_2 & a_1 & a_4 & a_3 & a_{14} & a_{15} & a_{16} & a_{13} & a_{10} & a_9 & a_{12} & a_{11} \\ a_7 & a_8 & a_5 & a_6 & a_3 & a_2 & a_1 & a_4 & a_{15} & a_{16} & a_{13} & a_{14} & a_{11} & a_{10} & a_9 & a_{12} \\ a_8 & a_5 & a_6 & a_7 & a_4 & a_3 & a_2 & a_1 & a_{16} & a_{13} & a_{14} & a_{15} & a_{12} & a_{11} & a_{10} & a_9 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_9 & a_{12} & a_{11} & a_{10} & a_{13} & a_{14} & a_{15} & a_{16} & a_1 & a_4 & a_3 & a_2 & a_5 & a_6 & a_7 & a_8 \\ a_{10} & a_9 & a_{12} & a_{11} & a_{14} & a_{15} & a_{16} & a_{13} & a_2 & a_1 & a_4 & a_3 & a_6 & a_7 & a_8 & a_5 \\ a_{11} & a_{10} & a_9 & a_{12} & a_{15} & a_{16} & a_{13} & a_{14} & a_3 & a_2 & a_1 & a_4 & a_7 & a_8 & a_5 & a_6 \\ a_{12} & a_{11} & a_{10} & a_9 & a_{16} & a_{13} & a_{14} & a_{15} & a_4 & a_3 & a_2 & a_1 & a_8 & a_5 & a_6 & a_7 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{13} & a_{14} & a_{15} & a_{16} & a_9 & a_{12} & a_{11} & a_{10} & a_5 & a_6 & a_7 & a_8 & a_1 & a_4 & a_3 & a_2 \\ a_{14} & a_{15} & a_{16} & a_{13} & a_{10} & a_9 & a_{12} & a_{11} & a_6 & a_7 & a_8 & a_5 & a_2 & a_1 & a_4 & a_3 \\ a_{15} & a_{16} & a_{13} & a_{14} & a_{11} & a_{10} & a_9 & a_{12} & a_7 & a_8 & a_5 & a_6 & a_3 & a_2 & a_1 & a_4 \\ a_{16} & a_{13} & a_{14} & a_{15} & a_{12} & a_{11} & a_{10} & a_9 & a_8 & a_5 & a_6 & a_7 & a_4 & a_3 & a_2 & a_1 \end{array}\right]$$

iii) $G_8 = \langle \alpha, \beta, \gamma : \alpha^4 = \beta^2 = \gamma^2 = 1, \beta\alpha = \alpha\beta, \gamma\alpha = \alpha\gamma . \gamma\beta = \alpha^2\beta\gamma \rangle$

The general element is

$$w = a_1 1 + a_2\alpha + a_3\alpha^2 + a_4\alpha^3 + a_5\beta + a_6\alpha\beta + a_7\alpha^2\beta + a_8\alpha^3\beta + a_9\gamma + a_{10}\alpha\gamma$$
$$+ a_{11}\alpha^2\gamma + a_{12}\alpha^3\gamma + a_{13}\beta\gamma + a_{14}\alpha\beta\gamma + a_{15}\alpha^2\beta\gamma + a_{16}\alpha^3\beta\gamma.$$

$$\left[\begin{array}{cccc:cccc:cccc:cccc}
a_1 & a_4 & a_3 & a_2 & a_5 & a_8 & a_7 & a_6 & a_9 & a_{12} & a_{11} & a_{10} & a_{15} & a_{14} & a_{13} & a_{16} \\
a_2 & a_1 & a_4 & a_3 & a_6 & a_5 & a_8 & a_7 & a_{10} & a_9 & a_{12} & a_{11} & a_{16} & a_{15} & a_{14} & a_{13} \\
a_3 & a_2 & a_1 & a_4 & a_7 & a_6 & a_5 & a_8 & a_{11} & a_{10} & a_9 & a_{12} & a_{13} & a_{16} & a_{15} & a_{14} \\
a_4 & a_3 & a_2 & a_1 & a_8 & a_7 & a_6 & a_5 & a_{12} & a_{11} & a_{10} & a_9 & a_{14} & a_{13} & a_{16} & a_{15} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
a_5 & a_8 & a_7 & a_6 & a_1 & a_4 & a_3 & a_2 & a_{13} & a_{16} & a_{15} & a_{14} & a_{11} & a_{10} & a_9 & a_{12} \\
a_6 & a_5 & a_8 & a_7 & a_2 & a_1 & a_4 & a_3 & a_{14} & a_{13} & a_{16} & a_{15} & a_{12} & a_{11} & a_{10} & a_9 \\
a_7 & a_6 & a_5 & a_8 & a_3 & a_2 & a_1 & a_4 & a_{15} & a_{14} & a_{13} & a_{16} & a_9 & a_{12} & a_{11} & a_{10} \\
a_8 & a_7 & a_6 & a_5 & a_4 & a_3 & a_2 & a_1 & a_{16} & a_{15} & a_{14} & a_{13} & a_{10} & a_9 & a_{12} & a_{11} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
a_9 & a_{12} & a_{11} & a_{10} & a_{15} & a_{14} & a_{13} & a_{16} & a_1 & a_4 & a_3 & a_2 & a_5 & a_8 & a_7 & a_6 \\
a_{10} & a_9 & a_{12} & a_{11} & a_{16} & a_{15} & a_{14} & a_{13} & a_2 & a_1 & a_4 & a_3 & a_6 & a_5 & a_8 & a_7 \\
a_{11} & a_{10} & a_9 & a_{12} & a_{13} & a_{16} & a_{15} & a_{14} & a_3 & a_2 & a_1 & a_4 & a_7 & a_6 & a_5 & a_8 \\
a_{12} & a_{11} & a_{10} & a_9 & a_{14} & a_{13} & a_{16} & a_{15} & a_4 & a_3 & a_2 & a_1 & a_8 & a_7 & a_6 & a_5 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
a_{13} & a_{16} & a_{15} & a_{14} & a_{11} & a_{10} & a_9 & a_{12} & a_5 & a_8 & a_7 & a_6 & a_1 & a_4 & a_3 & a_2 \\
a_{14} & a_{13} & a_{16} & a_{15} & a_{12} & a_{11} & a_{10} & a_9 & a_6 & a_5 & a_8 & a_7 & a_2 & a_1 & a_4 & a_3 \\
a_{15} & a_{14} & a_{13} & a_{16} & a_9 & a_{12} & a_{11} & a_{10} & a_7 & a_6 & a_5 & a_8 & a_3 & a_2 & a_1 & a_4 \\
a_{16} & a_{15} & a_{14} & a_{13} & a_{10} & a_9 & a_{12} & a_{11} & a_8 & a_7 & a_6 & a_5 & a_4 & a_3 & a_2 & a_1
\end{array}\right]$$

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References

[1] Kahtan H. Alzubaidy, Nabila M. Bennour, Matrix Representation of Group Algebras of General Metacyclic Groups, International Journal of Mathematics and its Applications, 2021,9(3), 33-36.

[2] Nabila M. Bennour and Kahtan H. Alzubaidy, Representation of Group Algebras of Non-Abelian Groups of order p^3, for a Prime $p \geq 3$, European Journal of Mathematics and Statistics, 2024; 5(2), 1-5.

[3] David Clausen, Classifying All Groups of Order 16, University of Puget Sound

Spring 2012, http://www.gnu.org/copyleft/fdl.html.

[4] Nathan Jacobson, Basic Algebra I, 2nd edition, W.H. Freeman New York 1996.

[5] David S. Dummit, Richard M. Foote, Abstract Algebra, 3rd edition. Wiley, 2003.