

Robust Methods of Assessing Fidelity Bank Share Price Movements in Nigeria Stock Market

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ABSTRACT

Markov chain is a viable tool for studying the share price formation; since each finite state communicates for proper management decisions in Fidelity bank. Therefore, this paper studied stochastic analysis of Markov chain and PCA in the closing share price data of Fidelity (2016-2022) in Nigeria Stock Exchange. The share prices were transformed into 3-steps transition probability matrix solution to cover this number of years. The future share prices changes were known. The criteria of obtaining four share prices which formed 2x2 matrices were given and analytical solution of principal component were considered for future stock price changes. From the solution matrix of stochastic analysis, showed that Fidelity bank, PLC has the best probability of price increasing in the near future: 10%, best probability of reducing in future by 23% and best probability of no-change in the near future by 22% which is a tool for proper decision making in the day-to-day management of the bank; which shows it is profit making organization and are hopeful for future investment plans both short or long term respectively.

KEYWORDS:

Markov Chain, PCA, Share price, Transition matrix and Stochastic Analysis.

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INTRODUCTION

In finance generally, Stock means the capital raised by a company or corporation through the issue and subscription of shares. It is a security that represents the ownership of a fraction of a corporation. In units, stocks are called shares. The stock market is made up of buyers and sellers of stocks, which represents ownership claims on businesses. It denotes to a number of exchanges and other venues in which shares of publicly held companies are bought and sold. The legal platform upon which the financial activities are done is referred to as stock exchange

The stock market is one of the most vital components of a free-market economy. It is among the best options to various companies for expansion or set up a new business venture. Investments can be done in stock, bond, mutual funds etc. So, an increase in price of stocks in the stock market indicates an increase in investments. However, the nature of stock prices has been unstable, seasonal, timedependent and highly volatile and therefore unpredictable. This is mostly due to uncertainties that arise from natural disasters, global trends, socio-political policies which may have unprecedented impact on the demand and supply of stocks. Thus, because of this, investors now have to go beyond studying the company's history, performance and development prospects of such fundamentals, but also be familiar with the variety of technical analysis in order to win a huge return on investment and become a successful investor. Stock trend analysis plays an important role in practical stock trading. Fundamental factors affect the stock price depending on a company's earnings and profitability from producing and selling goods and services. These includes: The level of the earnings base (represented by measures such as earnings per share, cash flow per share, dividends per share), the expected growth in the earnings base, the discount rate, which is itself a function of inflation and the perceived risk of the stock. Technical factors relate to a stock's price history in the market pertaining to chart patterns, momentum, and behavioral factors of traders and investors. They are the mix of external conditions that affect the supply of and demand for a company's stock. Some of these indirectly affect fundamentals. For example, economic growth indirectly contributes to earnings growth, Nkemnole and Okafor (2020).

Stochastic analysis is one of the tools which can be used to analyze stock trends. It begins with conditional probability under probability theory as it involves a random process. Hence it is based on chance. Various stochastic models can be applied to analyze stock market price changes. We have the Markov chain approach, the Wiener process also known as the Brownian motion, the Monte Carlo process, etc. When the market price starts from zero, and the increments are independent and stationery on a continuous path, then the stochastic process is said to have a Brownian motion. The increments here are usually distributed depending on their size. When the stock price predictions are based only on the present state and not on the previous state, then the process is said to have a Markov property, hence it is a Markov process. The Monte Carlo process generates samples of random variables which it analyses using a performance function to extract probabilistic information, thus, it is a random Markov process. The applications of stochastic models to stock market price all aims to predict the random behavior and the most accurate path for optimal profitability.

Research Problem

The forecast of possible states of the market is more complicated due to the intrinsic stochastic behavior of the share price movements. Share price are affected by many highly related factors such as economic, political, psychological and company-specific variables. Thus, to unify these factors it becomes very challenging to forecast share price movements. The greatest of it is when the stochastic analysis of the problem is combined with Principal Component Analysis (PCA). So, the ability to suitably understand the share price dynamics of Fidelity bank is not an easy one to grasp as to relates

at is factorily. Hence, this paper examined share price movement of Fidelity, PLC.; to realistically assess the future price changes and variations.

Objective of the Study

To apply stochastic analysis of Markov chain and PCA to assess share price changes of Fidelity bank PLC.

Significance of the Study:

Stochastic analyses of Markov chain are used to show when a share price has moved in one finite state to the other. The model presented in this study will give Fidelity bank clear directions when making decisions concerning share prices, as it helps them understand the dynamics and patterns of the share price movement that would be followed for optimum trading and profit. Secondly it will account the level of proportion of first Principal Component of future price changes. This paper compliments the work of Osu et al. (2019), by combining Principal Component Analysis (PCA) to study the share price variations of Fidelity bank which notifies the bank management and other traders, economist and even government to take vital decisions based on the levels of their investments.

LITERATURE REVIEW

The key to building accurate stochastic models remain to suit the intended purpose and not uniqueness as more than one model can represent a specific event but each model has a specific purpose. Amadi et al. (2022) applied Markov chain model to examine stock price formation in finite series. A 5-step transition matrix for independent stocks where transition matrix replicated the use of 3-states transition probability matrix. The stochastic analysis reveals from the study that stock price changes are memory-less and this satisfies the Markov chain property.

Mettle et al. (2014) developed a methodology for stochastic analysis of share prices. They showed that the method of using Markov chains as a stochastic analysis method in equity price studies improves equity portfolio decisions with strong statistical foundations. From their study, all states obtained have limiting distributions. Doroshkevych et al. (2020) conducted a study on the use of Markov chains for modeling of strategic planning innovative activity of the Enterprise. They investigated the process of strategic planning as an activity to determine the directions of Enterprise innovative activity (strategy) which would tally consistently with the available resource provision, innovative potential of the Enterprise and also satisfy the market needs using Markov chain. Haiying et al. (2021) carried out research on marketing prediction model based on Markov prediction. They presented the mathematical model of the product marketing prediction by establishing the probability matrix of product state transition. Oseni and Femi (2013) investigated the use of Markov analysis in Marketing of Telecommunication product; MTN, Airtel and Globacom as a case study. They analysed the proportion of the subscribers/users of each network using Markov's model.

Chang and Zhang (2016) conducted a study on the effects of Channel Experience and Direct marketing on Customer Retention in Multichannel settings. They use a multi-segment, multivariate hidden Markov's modeling framework to model three customer behaviors; purchase amount, purchase incidence and channel choice. Lakshmi and Jyothi (2020) applied the Markov process for predicting of stock market performance. The study proved Markov process to be relevant when stock market prices are analyzed for future predictions. Agwuegbo et al. (2010) analyzed the daily behavior of stock market prices through a random walk model which satisfies the Markovian property when the

nearest neighbor property of the model decreases. Zhang et al. (2016) carried out an empirical study of market share and marketing strategy based on Markov chain model. They first reviewed the basic theory of Markov chain model; they also cited two examples to analyze the market share and marketing strategy. Results from this would help enterprises achieve the objective of the maximum expected profit. Amadi and Vivian(2022)applied stochastic analysis of Markov chain in the closing stock price formation of three selected companies. The criteria of obtaining future stock price changes of the year were obtained in time varying investments. Lots of scholars has written extensively on this dynamic area such as Ochieng (2019), Chun-Wei C.(2012) and Aparna and Sarat (2012), etc.

However, reviews show that mathematical finance has copious applications as seen in the literature. This present project is aimed at complementing previous efforts by assessing stock price movement of Fidelity bank as this will add values in this dynamic area of study.

Mathematical Preliminaries

For proper understanding and for the purpose of this paper on Markov chain we start from defining stochastic process. It can also be seen as a statistical event that evolves time in accordance to probabilistic laws. Mathematically, a stochastic process may be defined as a collection of random variables which are ordered in time and defines at a set of time points which may be continuous or discrete. In view of the fact that a stochastic process is a relation of random variables, its requirement is similar to that for random vectors.

Definition 1: A stochastic process X is said to be a Markov chain if Markov property is satisfied:

$$P(X_{n+1} = j / X_0, X_1, \dots, X_n) = P(X_{n+1} = j / X_n)$$
(1)

For all $n \ge 0$ and $i, j \in S$ (state space).

It is sufficient to know that the Markov property given (1) is equivalent to the following for each $j \in S$.

$$P(X_{n+1} = j / X_{n1}, X_{n2}, \cdots, X_{nk}) = P(X_{n+1} = j / X_{nk})$$
(2)

(for any $n_1 < n_2 < \cdots, n_k \le n$)

Assuming $X_n = i$ implies that the chain is in the *ith* state at the *nth* step.it can also be said that the chain' having the value i' or 'being in state 'i'. The idea behind the chain is described by its transition probabilities:

$$P(X_{n+1} = j / X_n = i) \tag{3}$$

They are dependent on i, j and n.

Definition 2: The chain X is said to be homogeneous if the following are stated below

$$P(X_{n+1} = j / X_n = i) = P(X_1 = j / X_0 = i)$$
(4)

For all n, i, j.

The transition matrix $\mathbf{P} = (P_{ij})$ is $n \times n$ matrix of transition probabilities.

$$P_{ij} = P\left(X_{n+1} = j / X_n = i\right) \tag{5}$$

Hence, the transition probabilities with homogenous Markov chain are always stationary at a point.

Theorem 3: Suppose P is a stochastic matrix which implies the following:

i) P has non-negative entries or $P_{ij} \ge 0$ (ii) $\sum_{j} P_{ij} = \sum_{j} P(X_{n+1} = j / X_n = i) = \sum_{j} P(X_1 = j / X_0 = i)$

which is stationarity or point of convergence.

Proof:(i) each associated entry in P is a transition probability P_{ij} and being probability $P_{ij} \ge 0$.

(ii)
$$\sum_{j} P_{ij} = \sum_{j} P(X_{n+1} = j / X_n = i) = \sum_{j} P(X_1 = j / X_0 = i)$$

Which is stationarity.

$$P(X_i \in S / X_0 = i) = 1$$

Theorem 4 :(Chapman-Kolmogorov Equations).

$$P_{ij(m+n)} = \sum_{r=i}^{n} P_{ir(m)} P_{rj(n)} \text{ Since } P_{m+n} = P_m P_n \text{ and so on } P_n = P^n \text{ the } nth \text{ power of } P$$

$$P_{ij(m+n)} = P(X_{m+n} = j / X_0 = i)$$
Proof:
$$\sum_{r} P(X_{m+n} = j, X_m = r / X_0 = i)$$

$$\sum_{r} P(X_{m+n} = j / X_m = i / X_0 = i) P(X_m = r / X_0 = i)$$

Using the following in probability rule:

$$P(A \cap B / C) = P(A / B \cap C)P(B / C) \text{ and setting}$$
$$A = \{X_{m+n} = j\}, B = \{X_m = r\}, \text{and } C = \{X_0 = i\}$$

Using Markov property yields

$$P_{ij(m+n)} = \sum_{r} P(X_{m+n} = j / X_m = r) P(X_m = r / X_0 = i)$$

$$\sum_{r} P_{rj(n)} P_{ir(m)}$$

$$\sum_{r} P_{1r(m)} P_{r1(n)}$$

Hence $P_{m+n} = P_m P_n$ and so $P_n = P^n$, the power of P.

Problem Formulation

let $S_i(t)$ (i = 1, 2, ..., N, t = 1, 2, ..., n) be the daily closing share price data of Fidelity Bank selected years at time t be defined as three state Markov processes in finite states. Let $N \times n$ data matrix associated with $S_i(t)$ be X_{it} . We consider N closing share prices over n months, time horizon. To obtain an estimate of the transition probability as follows

$$P_{ij} = P(X_{t} = j / X_{t-1} = i), \text{ for } j = 0, 1, 2, 3, \dots, N$$

$$P_{ij} = \begin{cases} P \text{ if } j = 1 + j \\ q = 1 - P \text{ if } j = i - j \\ 0 \text{ otherwise} \end{cases}$$

where k + 1 is the number of states;

$$n_{ij} = \sum_{i=1}^{n} P_{ij} \text{ for } i \ j=0,2,3 \\ \frac{n_{ij}}{n_i} \text{ for } i \ j=0,1,\cdots k$$
(6)

However, for k = 3 is an estimate of the transition matrix.

$$\hat{P}ij(\text{FIDELITY})_{2016-2022} = \begin{pmatrix} \hat{P}_{00} & \hat{P}_{01} & \hat{P}_{03} \\ \hat{P}_{10} & \hat{P}_{11} & \hat{P}_{12} \\ \hat{P}_{20} & \hat{P}_{21} & \hat{P}_{22} \end{pmatrix}$$
(7)

Setting i = 0, 1, 2 for k = 3

Developing Markov Chain Model for Stochastic Analysis of Fidelity Share Price

For proper accuracy of Markov chain model for future events; it needs to be developed for prediction of share price movement. The initial share prices need to be in three finite states as follows:

R: represents the probability of share price reducing in near future

I : represents the probability of share price increasing in near future

NO-change: represents the probability of share price not changing in near future

However, probability of transition matrix shows the proper explanation of Markov chain. Every element in the matrix communicates. In order to form three states of Markov process we need to have the following table below:

Table 1: Transition Probability Matrix

State	1	2	3	Total of Row

1	<i>P</i> ₁₁	<i>P</i> ₁₂	<i>P</i> ₁₃	T_1
2	<i>P</i> ₂₁	<i>P</i> ₂₂	<i>P</i> ₂₃	T_2
3	<i>P</i> ₃₁	<i>P</i> ₃₂	<i>P</i> ₃₃	<i>T</i> ₃

In each entry P_{ij} indicates the number of times a transition is made from one state *i* to state *j*. the transition matrix is computed by simply dividing every element in each row through the total of each row. Nevertheless, this project studies Fidelity share price data collected from Osu et al.(2019).

Principal component Analysis(PCA) of the Stock Variables

Definition 3.1: Suppose \underline{X} has a joint distribution which has a variance matrix \sum with eigen values $\lambda_1 \ge \lambda_2 \ge ... \ge \lambda_p \ge 0$. consider the random variables $y_1 ... y_p$ which are linear combination of the $X'_1 s$, i.e.

$$\begin{array}{c} y_{1} = \underline{l} \underline{X} = l_{11} X_{1} + \ldots + l_{p_{1}} \lambda_{p} \\ \vdots \\ y_{p} = \underline{l} \underline{X} = l_{1} p X_{1} + \ldots + l_{p_{1}} \lambda_{p} \end{array}$$

$$(8)$$

The y_i 's will be PC if they are uncorrelated and the variances of y_1, y_2 are as large as possible. Recall that if $y_i = l'_i X$. In order to look at the amount of information that is in y_1 . We can consider the proportion of the total population variance due to y_i

$$\frac{\lambda_i}{\lambda_1 + \lambda_2 + \dots + \lambda_p}, i = 1, \dots, p \tag{9}$$

Hopefully the proportion is large for example i = 1, 2, 3. Udom(2015).

Results and Discussion

The data for this Project is gotten from the work of Osu et al.(2023). To demonstrate the closing share market price performances of Fidelity Bank PLC in finite states. The share price covers from 2016-2022 retrievable from Nigeria Stock Exchange (NSE).

Table 2: Share price of Fideli	ty Bank, PLC from 2016-2022
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Share price movements	Reducing(R) its share Price	Increasing(I) its share price	No(N) change in price	Row totals
R	415	62	138	615

Ι	61	121	81	263
Ν	139	80	384	603

Transition probability matrix for Fidelity bank

$$FIDELITY_{BANK}(P) = \begin{pmatrix} 0.6748 & 0.1008 & 0.2244 \\ 0.2319 & 0.4601 & 0.3080 \\ 0.2305 & 0.1327 & 0.6368 \end{pmatrix}$$

However, in order to predict the future price of entire of investments of different transition probability matrix of the companies: we therefore have the following explained row-wise:

Fidelity (2016-2022): Has 67% of reducing its price; 10% chance of increasing its price in the near future; 22% chance of no change in price. Also, in the same circumstances 23% chance of reducing its price; 46% chance of increasing its price and 31% chance of no change in price.

Finally, 23% chance of reducing its price 13% chance of increasing its price and 64% Chance of no change in price. In all, the overall predicted prices for the above companies gives: 23% chance of reducing its price,13% chance of increasing its price and 64% chance of no change in price.

The above assessments of the three companies provides an eye opener of these stochastic analysis that will be enhancing their investment decisions, The entire entry stipulates price changes for short and long terms business plans.

Following the method of Amadi and Wobo(2021) on minimum Fidelity bank share price criteria of 2x2 elements:

$$F_{FIDELITY BANK} = \begin{pmatrix} 0.1008 & 0.2244 \\ 0.2305 & 0.1327 \end{pmatrix}$$

we formed the matrix from the estimates of probability transition matrix of Fidelity bank, which information of the share price movements will account for the total proportion variability in the share price

Table 3: Transition Probability Matrix of Fidelity Bank Share Market Prices with Means, Standard deviations, Kurtosis and Skewness

<i>a</i> ₁₁	<i>a</i> ₁₂	<i>a</i> ₁₃	Mean	Std	Kurtosis	Skewness
0.6748	0.1008	0.2244	0.3333	0.3021	0.1008	0.5763
0.2319	0.4601	0.3080	0.3333	0.1162	1.500	0.3815
0.2305	0.1327	0.6368	0.3333	0.2675	1.500	0.6021

In Table 3, the mean indicates the probability of share price changes on average of Access bank which shows 0.3333 throughout the period of investments, see column 4. The standard deviations in column 5 indicate levels of different price changes which is always affected by volatility. Then, kurtosis measures the tail of entire share prices. Finally, the share price skewness measures level of distortion in the data set which guides an investor on basis of decision making.

Trading days	<i>a</i> ₁₁ = 0.1008	$a_{12} = 0.2244$	<i>a</i> ₂₁ = 0.2305	<i>a</i> ₂₂ = 0.1327
2	0.2016	0.4488	0.461	0.2654
4	0.4032	0.8976	0.922	0.5308
6	0.6048	1.3464	1.383	0.7962
8	0.8064	1.7952	1.844	1.0616
10	1.008	2.244	2.305	1.327
12	1.2096	2.6928	2.766	1.5924
14	1.4112	3.1416	3.227	1.8578
16	1.6128	3.5904	3.688	2.1232
18	1.8144	4.0392	4.149	2.3886
20	2.016	4.488	4.61	2.654

Table 4: Variations of Fidelity Bank future share prices according to the trading days

In Figure4, increase in the number of trading days also increases future share prices of Fidelity bank. It also connotes that profit increases over time; as gives business cycles to be more flexible adapting to market demands; the benefit of this assessment is to avert severe depletion of capital investments which may endanger profit making throughout the trading period of the capital investments.

In all, the above assessments of the Fidelity bank provide an eye opener of these stochastic analysis that will enhance investment decisions, The entire entry stipulates price changes for short and long terms business plans.

Principal Component Analysis of Fidelity share price Movement Variations

$$F_{FIDELITY \ BANK} = \begin{pmatrix} 0.1008 & 0.2244 \\ 0.2305 & 0.1327 \end{pmatrix}, \ F_{FIDELITY \ BANK} - \lambda I = 0$$

Solving the above share price matrix gives:

$$\lambda_1 = -0.1112, \ \lambda_2 = 0.3447$$

Solving for $\lambda_1 = -0.1112$, we have the following systems of equation

$$0.212K_1 + 0.2244K_2 = 0 \tag{10}$$

$$0.2305K_1 - 0.212K_2 = 0 \tag{11}$$

From (10) $0.212K_1 = -0.2244K_2$, $K_2 = \frac{0.212}{0.2244} = -0.9447$ putting K_2 in (4.2) gives

$$0.2305K_1 - 0.2244(-0.9447) = 0, \ 0.2305K_1 = -0.2120, \ K_1 = \frac{-0.2120}{0.2305} = -0.9197$$

Any vector of the form say form;

$$K_1 = \begin{pmatrix} -0.9197 \\ -0.9447 \end{pmatrix} = \begin{pmatrix} -0.9197C \\ -0.9447C \end{pmatrix}; \text{ say is an eigenvector corresponding to } \lambda_1 = -0.1112$$

$$-0.2439K_1 + 0.2244K_2 = 0 \tag{12}$$

$$0.2305K_1 - 0.212K_2 = 0 \tag{13}$$

From(13) $0.212K_1 = -0.2244K_2$, $K_2 = \frac{0.2439}{0.212} = 1.0869$ putting K_2 in (13) gives;

$$0.2305K_1 - 0.212(1.0869) = 0, \ 0.2305K_1 - 0.2304228 = 0_1, \ K_1 = \frac{-0.2304228}{0.2305} = 0.9997$$

Any vector of the form say form;

 $K_2 = \begin{pmatrix} 0.9997\\ 1.0869 \end{pmatrix} = \begin{pmatrix} 0.9997C\\ 1.0869C \end{pmatrix}; \text{ say is an eigenvector corresponding to } \lambda_2 = 0.3447$

To obtain anormalized eigenvectors for share price of Fidelity bank:

$$K_{1}'K_{1} = 1, (-0.9197C - 0.9447C) \begin{pmatrix} -0.9197C \\ -0.9447C \end{pmatrix}, 0.84584809C^{2} + 0.89245809C^{2} = 1, 1.73830618C^{2} = 1$$

$$C^{2} = \frac{1}{1.73830618}, \ C = \frac{1}{\sqrt{1.73830618}}, \ e_{1} = \begin{pmatrix} -0.9197 \\ \overline{\sqrt{1.73830618}} \\ -0.9447 \\ \overline{\sqrt{1.73830618}} \end{pmatrix}, \begin{pmatrix} -0.6976 \\ -0.7166 \end{pmatrix}$$

 $K_{2}'K_{2} = 1, (-0.9997C \ 1.0869C) \begin{pmatrix} -0.9997C \\ 1.0869C \end{pmatrix}, 0.99940009C^{2} + 1.18135161C^{2} = 1, 2.1807517C^{2} = 1$

$$C^{2} = \frac{1}{2.1807517}, \ C = \frac{1}{\sqrt{2.1807517}}, \ e_{1} = \begin{pmatrix} 0.9997 \\ \overline{\sqrt{2.1807517}} \\ 1.0869 \\ \overline{\sqrt{2.1807517}} \end{pmatrix}, \begin{pmatrix} 0.6770 \\ 0.7360 \end{pmatrix}$$

 $Y_1 = e_1'K = -0.6976K_1 - 0.7166K_2$ $Y_2 = e_2'K = -0.6770K_1 + 0.7360K_2$

To calculate the principal component of Fidelity bank share price accounted for

$$\lambda_1 = -0.1112, \ \lambda_2 = 0.3447, \ \frac{\lambda_1}{\lambda_1 + \lambda_2} = 0.2439 = 24.39\%$$

Two eigen values represent the total amount of Fidelity share price variance that can be explained by the principal component. The $\lambda_1 = -0.1112$ represents the levels of losses made all through the trading days by Fidelity bank PLC. So $\lambda_2 = 0.3447$ is greater than zero which is a good sign of high

level of investment return in the side of the Fidelity bank whose aim and passion is to maximize profit.

However, the eigenvectors determine the direction of the share price in terms of changes in short-run and long-run respectively. The 24% shows the levels of returns Fidelity bank will make in future based on their share price movements.

Conclusion And Recommendations

Markov chain is a precise tool for studying the share price formation; since each finite state communicates for proper management decisions in Fidelity bank. Consequently, this reearch studied stochastic analysis of Markov chain and PCA in the closing share price data of Fidelity bank PLC. (2016-2022) via Nigeria Stock Exchange. The share prices were transformed into 3-steps transition probability matrix solution to cover this number of years. The future share prices changes were known. The criteria of obtaining four share prices which formed 2x2 matrix were given and analytical solution of principal component were considered for future stock price changes. From the solution matrix of stochastic analysis showed that **Fidelity bank, PLC** has the best probability of price increasing in the near future: 10%, best probability of reducing in future by 23% and best probability of no-change in the near future by 22% which is a tool for proper decision making in the day-to-day management of the bank.

However, this study investigates three state case with PCA, the stochastic differential equation problem is suggested as an interesting area of further study, the application of fundamental matrix solution to determine future price changes will be an interesting study, introducing delay concept in studying of share price movement will be a good area to explore

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