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Application of Laplace-Adomian Decomposition Method (LADM) to Solving Zika Virus Model with Vector Control

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ABSTRACT

The emergence and spread of vector-borne diseases pose significant public health challenges worldwide, especially in regions where its primary vector, the *Aedes aegypti mosquito*, thrives. Zika virus (ZIKV) infection represents a pressing concern due to its potential for severe neurological complications and adverse pregnancy outcomes. Effective control strategies are imperative to mitigate ZIKV transmission and reduce the burden of disease. This study explores the application of the Laplace-Adomian Decomposition Method (LADM) to solve a mathematical model describing the dynamics of Zika virus transmission with vector control interventions. The Laplace-Adomian Decomposition Method (LADM) is used to numerically solve a Zika virus model incorporating vector control through Wolbachia-infected mosquitoes. This method allows for efficient and accurate computation of solutions, enabling insights into the impact of Wolbachia on reducing mosquito populations and controlling Zika transmission. The total population is dividing into three subpopulations: humans, Aedes aegypti mosquitoes, Wolbachia-infected mosquitoes and each of these subpopulations are further divided into epidemiological compartments. Advantages of the method over other numerical methods are clearly stated. Our findings demonstrate the effectiveness of the method and Wolbachia as a vector control measure. It also provides valuable insights for policymakers and public health authorities.

KEYWORDS:

Zika virus, Laplace-Adomian Decomposition Method, Vector control, Infectious disease dynamics, Epidemiology.

1. Introduction

Zika virus, a member of the *Flaviviridae family* transmitted primarily by Aedes mosquitoes, has emerged as a significant global health threat in recent years. Initially identified in the Zika forest of Uganda in 1947, the virus garnered little attention until outbreaks in Micronesia in 2007 and later in the Americas in 2015 raised alarm due to its association with severe neurological complications and adverse pregnancy outcomes [1, 2, 6]. Zika virus infection during pregnancy has been linked to microcephaly and other congenital abnormalities in newborns, as well as neurological disorders such as Guillain-Barré syndrome in adults. The rapid spread of Zika virus, facilitated by globalization, urbanization, and climate change, has led to widespread transmission across continents, with outbreaks reported in over 80 countries. The virus's ability to exploit diverse mosquito species, including Aedes aegypti and Aedes albopictus, as vectors, coupled with its potential for sexual and vertical transmission, poses challenges for containment and control efforts [4, 5].

Efforts to combat Zika virus have focused on vector control, vaccination development, and public health measures such as surveillance and mosquito abatement. However, the complex interplay between the virus, vectors, hosts, and environmental factors underscores the need for multidisciplinary approaches, including mathematical modeling, to understand transmission dynamics and evaluate intervention strategies. Mathematical models offer valuable tools for simulating and predicting the spread of Zika virus, assessing the impact of control measures, and guiding public health responses [4,5,6]. These models integrate epidemiological, entomological, and environmental data to elucidate the drivers of transmission and identify optimal intervention strategies. This introduction provides a brief overview of the Zika virus, highlighting its emergence, transmission dynamics, associated health risks, and challenges for control. Subsequent sections will delve into the mathematical modeling of Zika virus transmission and explore the role of modeling in informing evidence-based interventions to mitigate the impact of the virus [3,5]. Wolbachia-infected mosquitoes have garnered attention as a potential tool for controlling the spread of Zika virus, among other mosquito-borne diseases. Wolbachia is a genus of bacteria that naturally infects a wide range of arthropods, including mosquitoes. When introduced into mosquito populations, Wolbachia can interfere with the replication of certain viruses, including Zika virus, dengue virus, and chikungunya virus, thereby reducing the mosquitoes' ability to transmit these pathogens to humans [7].One approach to using Wolbachia for Zika control involves the release of Wolbachia-infected mosquitoes into the wild. These infected mosquitoes mate with wild mosquitoes, passing on Wolbachia to their offspring. Over time, the proportion of Wolbachia-infected mosquitoes in the population increases, reducing the ability of mosquitoes to transmit Zika virus.Another approach is to release male mosquitoes infected with Wolbachia. These males mate with wild females, but the eggs laid by these females do not hatch, leading to a reduction in the overall mosquito population. This method can indirectly reduce Zika transmission by decreasing the number of competent vectors in the population [8].

The Laplace-Adomian Decomposition Method (LADM) is a powerful numerical technique used for solving differential equations, particularly nonlinear ones. It combines the Laplace transform with the Adomian decomposition method to provide accurate and efficient solutions to a wide range of differential equations [9]. The method was first introduced by George Adomian in the late 1970s as an extension of the Adomian Decomposition Method, which he developed for solving nonlinear ordinary and partial differential equations. The key idea behind LADM is to transform a given differential equation into a series of simpler equations, which are then solved iteratively. This process involves decomposing the solution into a series of components using the Adomian polynomials and integrating each component using the Laplace transform [9,10]. By systematically solving the resulting equations, the LADM produces an approximate solution that converges to the exact solution as the number of terms in the series increases.LADM has been successfully applied to various fields, including mathematical biology, fluid dynamics, heat transfer, and engineering. Its ability to handle nonlinearities and its numerical efficiency make it a valuable tool for researchers and practitioners seeking solutions to differential equations that are difficult or impossible to solve analytically [9, 10, 11].

Mathematical model is a vital tool in studying and analyzing the transmission dynamics of contagious disease within a population, thus several models have been formulated by various researchers in a attempt to make recommendations to health care personnel so as to control infectious disases. For example Atokolo et al.[12] examined the impact of parameter values (θ, φ, h, and γ) on reducing the basic reproduction number (R0) of COVID-19, suggesting that adjusting these parameters could lead to the eventual elimination of the disease from the population. Their numerical simulations indicated that proper adherence to control measures such as social distancing, hand hygiene, and coughing etiquette could contribute to the eradication of the disease over time. Additionally, increasing rates of quarantine and isolation for suspected and confirmed cases were found to be effective in reducing the spread of the pandemic. Other valuable models including [10, 11, 12, 13, 14, 15,16, 17]

2. Model Formulation

Suppose $N_H(t)$ is the total human population at time t, where this population is further subdivided into four compartments viz susceptible human population at time t, $S_H(t)$, exposed human population $E_H(t)$, infected human population $I_H(t)$ and recovered human population $R_H(t)$. The total Ae.aegyptic mosquito population $N_M(t)$ is subdivided into three groups namely, the susceptible Ae.aegypitc mosquito population at time t $S_M(t)$, exposed Ae.aegyptic mosquito population $E_M(t)$ and infected Ae.aegyptic mosquito population $I_M(t)$. Mosquito with wolbachia known as wolbachia mosquitoes are used to control Ae.aegyptic mosquitoes. The total wolbachia mosquito population at time t is $N_W(t)$. This population includes, the susceptible wolbachia mosquito population $S_W(t)$, exposed wolbachia mosquito population $E_W(t)$ and infected wolbachia mosquito population $I_W(t)$. Let $\pi_H(t)$ be the constant recruitment rate of susceptible human population and $\lambda_H(t)$ represents infection strength of human where $\mu_H(t)$ is the natural death rate of human population. exposed human progresses to infected population at the rate $\alpha_H(t)$ and infected human recovered at the rateθ_H. Again let π_M denotes constant recruitment rate of Ae.aegyptic mosquito population and λ_M represents infection transmission strength of Ae.aegyptic mosquito. It is noted that only females Ae.aegyptic mosquitoes bite, the male mosquitoes do not bite instead they feed on nectar from flowers. The exposed Ae.aegyptic mosquitoes become infected at the rate b_M and μ_M is the natural death rate of Ae.aegyptic mosquito. π_W is the constant recruitment rate of wolbachia mosquito and μ_W natural death rate of wolbachia mosquito Exposed wolbachia mosquito progresses to infected class at the rate a_W where $\lambda_W$the infection transmission strength of wolbachia mosquito.

2.1 Variables and Parameters Interpretation

Table 1. Variables and Parameters used.

Variables	**Interpretation**
$N_H(t)$	Total human population
$s_H(t)$	Susceptible human population
$E_H(t)$	Exposed human population
$I_H(t)$	Infected human population
$R(t)$	Compartment of individuals who recover from both diseases.
$R_H(t)$	Recovered human population
$S_M(t)$	Susceptible mosquito without wolbachia population
$N_M(t)$	Total population of mosquito without wolbacchia
$E_M(t)$	Population of exposed mosquito without wolbachia
$I_M(t)$	Population of infected mosquito without wolbachia
$S_w(t)$	Susceptible wolbachia mosquito population
$N_w(t)$	Total population of wolbachia mosquito
$E_w(t)$	Exposed wolbachia mosquito
$I_w(t)$	Infected wolbachia mosquito
Parameter	**Description**
$\pi_H(t)$	Recruitment rate of human population
$\pi_M \pi_W(t)$	Recruitment rate of non wolbachia and wolbachia mosquito respectively
$\mu_{H,}\mu_{M,}\mu_W(t)$	Natural death rate of human, non wolbachia and wolbachia population respectively
$\lambda_{H,}\lambda_{M,}\lambda_W(t)$	Infection ability of human, non wolbachia and wolbachia mosquito respectively
$\beta_1(t)$	Mosquito biting rate
$\alpha_1(t)$	Transmission probability per biting of susceptible human with infected mosquito
$\alpha_2(t)$	Transmission probability per biting of susceptible mosquito with infected human
$\alpha_3(t)$	Transmission probability per mating of infected wolbachia mosquito with susceptible now wolbachia mosquito
$\alpha_4(t)$	Transmission probability per mating of susceptible non wolbachia mosquito with infected wolbachia mosquito
$\beta_2(t)$	Sexual contact rate between susceptible human to an infected human
$\alpha_H(t)$	Progression rate from exposed human to infected human

$\theta_H(t)$	Recovery rate of infected human
δ_H	Disease induced death rate in human population
$b_M(t)$	Progression from exposed to infected wolbachia mosquito
$a_W(t)$	Progression from exposed wolbachia to infected wolbachia mosquito

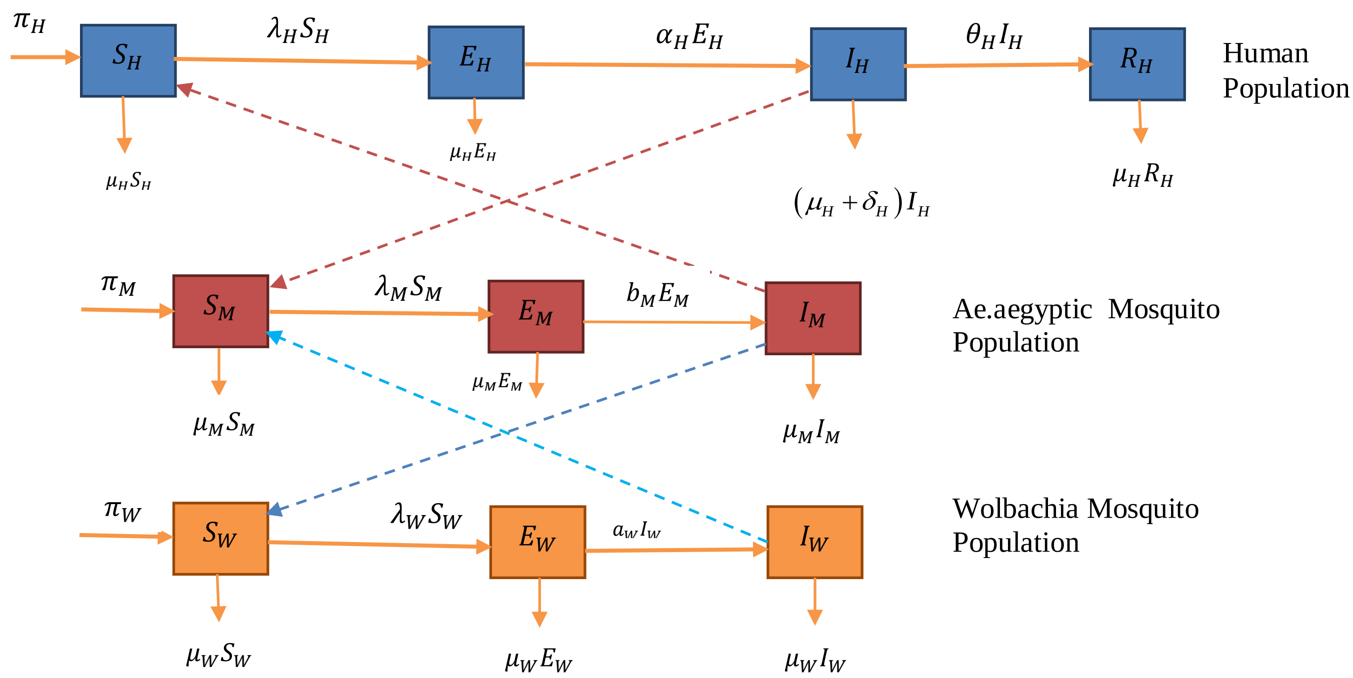


Fig: 1 Schematic diagram for the Model

The Model Equations

$$\frac{ds_H}{dt} = \pi_H - (\mu_H + \lambda_H)s_H$$

$$\frac{dE_H}{dt} = \lambda_H S_H - (\mu_H + \alpha_H)E_H$$

$$\frac{dI_H}{dt} = \alpha_H E_H - (\mu_H + \delta_H + \theta_H)I_H$$

$$\frac{dR_H}{dt} = \theta_H I_H - \mu_H R_H$$

$$\frac{dS_M}{dt} = \pi_M - (\mu_M + \lambda_M)s_M$$

(1)

$$\frac{dE_M}{dt} = \lambda_M S_M - (\mu_M + b_M)E_M$$

$$\frac{dI_M}{dt} = b_M E_M - \mu_M I_M$$

$$\frac{ds_W}{dt} = \pi_W - (\mu_W + \lambda_W)s_W$$

$$\frac{dE_W}{dt} = \lambda_W S_W - (\mu_W + a_W)E_W$$

$$\frac{dI_W}{dt} = a_W E_W - \mu_W I_W$$

Where,

$$\lambda_H = \frac{\beta_1\alpha_1 I_M + \beta_2\alpha_2 I_H}{N_H}, \lambda_M = \frac{\beta_1 C_1 I_H}{N_H}, \lambda_W = \frac{\alpha_3\beta_1 I_w + \alpha_4\beta_1 I_M}{N_H}$$

2.2 Fractional Order of the Zika virus Model

The Caputo derivative is measured as a differential operator in our model. We present in this segment some well-known definitions and effects that we shall be using throughout this research.

Definition 1 The Caputo fractional order derivative of a function (f) on the interval $[O,T]$ is defined by:

$$\left[{}^{C}D_0^{\beta} f(t)\right] = \frac{1}{\Gamma(n-\beta)}\int_0^t (t-s)^{n-\beta-1} f^{(n)}(s)ds, \tag{2}$$

Where $n = [\beta]+1$ and $[\beta]$ represents the integer part of β. In particular, for $0 < \beta < 1$, the Caputo derivative becomes:

$$\left[{}^{C}D_0^{\beta} f(t)\right] = \frac{1}{\Gamma(1-\beta)}\int_0^t \frac{f(s)}{(t-s)^{\beta}} ds, \tag{3}$$

Definition 2 Laplace transform of Caputo derivatives is defined as

$$\mathscr{L}[{}^{C}D^{\beta}q(t)] = S^{\beta}h(S) - \sum_{K=0}^{n} S^{\beta-i-1}y^{k}(0), \quad n-1 < \beta < n,\ n \in N, \tag{4}$$

For arbitrary $c_i \in R, i = 0,1,2,...n-1,\ n = [\beta]+1$ and $[\beta]$ represents the non-integer part of β.

Lemma 1. The following results hold for fractional differentiation equations

$$I^{\beta}[{}^{c}D^{\beta}h](t)=h(t)+\sum_{i=0}^{n-1}\frac{h^{(t)}(0)}{i!}t^{i}, \tag{5}$$

For arbitrary $\beta>0, i=0,1,2,...,n-1,$ where $n=[\beta]+1$ and $[\beta]$ represents the integer part of β Introducing fractional-order into the model, we now present a new model described by the following Introducing fractional order derivative into the model we present new mathematical model describe by set of fractional difference of order β for $0<\beta<1$

$$\left.\begin{aligned}
&D^{\beta}\left(S_{H}\right)=\pi_{H}-(\mu_{H}+\lambda_{H})s_{H},\\
&D^{\beta}\left(E_{H}\right)=\lambda_{H}S_{H}-(\mu_{H}+\alpha_{H})E_{H},\\
&D^{\beta}\left(I_{H}\right)=\alpha_{H}E_{H}-(\mu_{H}+\delta_{H}+\theta_{H})I_{H},\\
&D^{\beta}\left(R_{H}\right)=\theta_{H}I_{H}-\mu_{H}R_{H},\\
&D^{\beta}\left(S_{M}\right)=\pi_{M}-(\mu_{M}+\lambda_{M})s_{M},\\
&D^{\beta}\left(E_{M}\right)=\lambda_{M}S_{M}-(\mu_{M}+b_{M})E_{M},\\
&D^{\beta}\left(I_{M}\right)=b_{M}E_{M}-\mu_{M}I_{M},\\
&D^{\beta}\left(S_{W}\right)=\pi_{W}-(\mu_{W}+\lambda_{W})s_{W},\\
&D^{\beta}\left(E_{W}\right)=\lambda_{W}S_{W}-(\mu_{W}+a_{W})E_{W},\\
&D^{\beta}\left(I_{W}\right)=\alpha_{W}E_{W}-\mu_{W}I_{W}.
\end{aligned}\right\} \tag{6}$$

2.3 The Laplace-Adomian Decomposition Method (LADM) Implementation

We considered the general procedure of this method with the initial conditions. Applying Laplace transforms to both sides of the equation (1), and then we have:

$$\left.\begin{aligned}
&S^{\beta}\mathcal{L}(S_H)-S^{\beta-1}S_H(0)=\mathcal{L}\left[\pi_H-\left(\mu_H+\lambda_H\right)S_H\right]\\
&S^{\beta}\mathcal{L}(E_H)-S^{\beta-1}E_H(0)=\mathcal{L}\left[\lambda_H S_H-(\mu_H+\alpha_H)E_H\right]\\
&S^{\beta}\mathcal{L}(I_H)-S^{\beta-1}I_H(0)=\mathcal{L}\left[\alpha_H E_H-(\mu_H+\delta_H+\theta_H)I_H\right]\\
&S^{\beta}\mathcal{L}(R_H)-S^{\beta-1}R_H(0)=\mathcal{L}\left[\theta_H I_H-\mu_H R_H\right]\\
&S^{\beta}\mathcal{L}(S_M)-S^{\beta-1}S_M(0)=\mathcal{L}\left[\pi_M-(\mu_M+\lambda_M)s_M\right]\\
&S^{\beta}\mathcal{L}(E_M)-S^{\beta-1}E_M(0)=\mathcal{L}\left[\lambda_M S_M-(\mu_M+b_M)E_M\right]\\
&S^{\beta}\mathcal{L}(I_M)-S^{\beta-1}I_M(0)=\mathcal{L}\left[b_M E_M-\mu_M I_M\right]\\
&S^{\beta}\mathcal{L}(S_W)-S^{\beta-1}S_W(0)=\mathcal{L}\left[\pi_W-(\mu_W+\lambda_W)s_W\right]\\
&S^{B}L(E_W)-S^{\beta-1}E_W(0)=\mathcal{L}\left[\lambda_W S_W-(\mu_W+a_W)E_W\right]\\
&S^{B}\mathcal{L}(I_W)-S^{\beta-1}I_W(0)=\mathcal{L}\left[\alpha_W E_W-\mu_W I_W\right]
\end{aligned}\right\}$$

(7)

With initial conditions

$$S_H(0)=n_1,\ \ E_H(0)=n_2,\ I_H(0)=n_3,\ R_H(0)=n_4,\ S_M(0)=n_5,\ E_M(0)=n_6,\ I_M(0)=n_7,$$

$$S_W(0)=n_8,\ E_W(0)=n_9,\ I_W(0)=n_{10}$$

Dividing eqn. (7) by (S^{β}) we have:

$$\left.\begin{aligned}
\mathcal{L}(S_H) &= \frac{n_1}{S} + \frac{1}{S^{\beta}}\mathcal{L}\left[\pi_H - (\mu_H + \lambda_H)s_H\right] \\
\mathcal{L}(E_H) &= \frac{n_2}{S} + \frac{1}{S^{\beta}}\mathcal{L}\left[\lambda_H S_H - (\mu_H + \alpha_H)E_H\right] \\
\mathcal{L}(I_H) &= \frac{n_3}{S} + \frac{1}{S^{\beta}}\mathcal{L}\left[\alpha_H E_H - (\mu_H + \delta_H + \theta_H)I_H\right] \\
\mathcal{L}(R_H) &= \frac{n_4}{S} + \frac{1}{S^{\beta}}\mathcal{L}\left[\theta_H I_H - \mu_H R_H\right] \\
\mathcal{L}(S_M) &= \frac{n_5}{S} + \frac{1}{S^{\beta}}\mathcal{L}\left[\pi_M - (\mu_M + \lambda_M)s_M\right] \\
\mathcal{L}(E_M) &= \frac{n_6}{S} + \frac{1}{S^{\beta}}\mathcal{L}\left[\lambda_M S_M - (\mu_M + b_M)E_M\right] \\
\mathcal{L}(I_M) &= \frac{n_7}{S} + \frac{1}{S^{\beta}}\mathcal{L}\left[b_M E_M - \mu_M I_M\right] \\
\mathcal{L}(S_W) &= \frac{n_8}{S} + \frac{1}{S^{\beta}}\mathcal{L}\left[\pi_W - (\mu_W + \lambda_W)s_W\right] \\
\mathcal{L}(E_W) &= \frac{n_9}{S} + \frac{1}{S^{\beta}}\mathcal{L}\left[\lambda_W S_W - (\mu_W + a_W)E_W\right] \\
\mathcal{L}(I_W) &= \frac{n_{10}}{S} + \frac{1}{S^{\beta}}\mathcal{L}\left[\alpha_W E_W - \mu_W I_W\right]
\end{aligned}\right\}$$

(8)

Decomposing the non-linear term of equation (6) whereby we assume the solution of $S_H(t), E_H(t), I_H(t), R_H(t), S_M(t), E_M(t), I_M(t), S_W(t), E_W(t), I_W(t)$ are in the form of infinite series given by:

$$S_H(t) = \sum_{n=0}^{\infty} S_H(n), \quad E_H(t) = \sum_{n=0}^{\infty} E_H(n), \quad I_H(t) = \sum_{n=0}^{\infty} I_H(n), \quad R_H(t) = \sum_{n=0}^{\infty} R_H(n), \quad S_M(t) = \sum_{n=0}^{\infty} S_M(n),$$
$$E_M(t) = \sum_{n=0}^{\infty} E_M(n), \quad I_M(t) = \sum_{n=0}^{\infty} I_M(n), \quad S_W(t) = \sum_{n=0}^{\infty} S_W(n), \quad E_W(t) = \sum_{n=0}^{\infty} E_W(n), \quad I_W(t) = \sum_{n=0}^{\infty} I_W(n).$$

(9)

We have three (5) non-linear terms. The non-linear term in equation (6) are decomposed by Adomian polynomial as follows:

$$I_M(t)S_H(t)=\sum_{n=0}^{\infty}A(n),\quad I_H(t)S_H(t)=\sum_{n=0}^{\infty}B(n),\quad I_H(t)S_M(t)=\sum_{n=0}^{\infty}C(n),$$
$$I_W(t)S_W(t)=\sum_{n=0}^{\infty}D(n),\quad I_M(t)S_W(t)=\sum_{n=0}^{\infty}E(n)$$

(10)

Where $A(n),B(n),C(n),D(n),E(n)$ are Adomian polynomials given by

$$A(n)=\frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\left[\sum_{k=0}^{n}\lambda^k I_M(k)\sum_{k=0}^{n}\lambda^k S_H(k)\right]_{\lambda=0}$$

$$B(n)=\frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\left[\sum_{k=0}^{n}\lambda^k I_H(k)\sum_{k=0}^{n}\lambda^k S_H(k)\right]_{\lambda=0}$$

$$C(n)=\frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\left[\sum_{k=0}^{n}\lambda^k I_H(k)\sum_{k=0}^{n}\lambda^k S_M(k)\right]_{\lambda=0} \tag{11}$$

$$D(n)=\frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\left[\sum_{k=0}^{n}\lambda^k I_W(k)\sum_{k=0}^{n}\lambda^k S_W(k)\right]_{\lambda=0}$$

$$E(n)=\frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\left[\sum_{k=0}^{n}\lambda^k I_M(k)\sum_{k=0}^{n}\lambda^k S_W(k)\right]_{\lambda=0}$$

The polynomials are given by

$$A(0)=I_M(0)S_H(0),$$
$$A(1)=I_M(0)S_H(1)+I_M(1)S_H(0),$$
$$A(2)=I_M(0)S_H(2)+I_M(1)S_H(1)+I_M(2)S_H(0).$$

$$B(0)=I_H(0)S_H(0),$$
$$B(1)=I_H(0)S_H(1)+I_H(1)S_H(0),$$
$$B(2)=I_H(0)S_h(2)+I_H(1)S_H(1)+I_H(2)S_H(0).$$

$$C(0)=I_H(0)S_M(0),$$
$$C(1)=I_H(0)S_M(1)+I_H(1)S_M(0),$$
$$C(2)=I_H(0)S_H(2)+I_H(1)S_M(1)+I_H(2)S_M(0).$$

$$D(0) = I_W(0)S_W(0),$$
$$D(1) = I_W(0)S_W(1) + I_W(1)S_W(0),$$
$$D(2) = I_W(0)S_W(2) + I_W(1)S_W(1) + I_W(2)S_W(0).$$

(12)

$$E(0) = I_M(0)S_W(0),$$
$$E(1) = I_M(0)S_W(1) + I_M(1)S_W(0),$$
$$E(2) = I_M(0)S_W(2) + I_M(1)S_W(1) + I_M(2)S_W(0).$$

Substituting equation (9), (10) into equation (8) we obtained:

$$\left.\begin{aligned}
&\mathcal{L}\left\{\sum_{n=0}^{\infty} S_H(n)\right\}=\frac{n_1}{S}+\frac{1}{S^{\beta}}\mathcal{L}\left[\pi_H-\left(\frac{\beta_1\alpha_1\sum_{n=0}^{\infty}A(n)+\beta_2\alpha_2\sum_{n=0}^{\infty}B(n)}{N_H}\right)-\mu_H\sum_{n=0}^{\infty}S_H(n)\right]\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} E_H(n)\right\}=\frac{n_2}{S}+\frac{1}{S^{\beta}}\mathcal{L}\left[\left(\frac{\beta_1\alpha_1\sum_{n=0}^{\infty}A(n)+\beta_2\alpha_2\sum_{n=0}^{\infty}B(n)}{N_H}\right)-\left(\mu_H+\alpha_H\right)\sum_{n=0}^{\infty}E_H(n)\right]\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} I_H(n)\right\}=\frac{n_3}{S}+\frac{1}{S^{\beta}}\mathcal{L}\left[\alpha_H\sum_{n=0}^{\infty}E_H(n)-\left(\mu_H+\delta_H+\theta_H\right)\sum_{n=0}^{\infty}I_H(n)\right]\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} R_H(n)\right\}=\frac{n_4}{S}+\frac{1}{S^{\beta}}\mathcal{L}\left[\theta_H\sum_{n=0}^{\infty}I_H(n)-\mu_H\sum_{n=0}^{\infty}R_H(n)\right]\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} S_M(n)\right\}=\frac{n_5}{S}+\frac{1}{S^{\beta}}\mathcal{L}\left[\pi_M-\frac{\beta_1C_1\sum_{n=0}^{\infty}C(n)}{N_H}-\mu_M\sum_{n=0}^{\infty}S_M(n)\right]\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} E_M(n)\right\}=\frac{n_6}{S}+\frac{1}{S^{\beta}}\mathcal{L}\left[\frac{\beta_1C_1\sum_{n=0}^{\infty}C(n)}{N_H}-\left(\mu_M+b_M\right)\sum_{n=0}^{\infty}E_M(n)\right]\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} I_M(n)\right\}=\frac{n_7}{S}+\frac{1}{S^{\beta}}\mathcal{L}\left[b_M\sum_{n=0}^{\infty}E_M(n)-\mu_M\sum_{n=0}^{\infty}I_M(n)\right]\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} S_W(n)\right\}=\frac{n_8}{S}+\frac{1}{S^{\beta}}\mathcal{L}\left[\pi_W-\left(\frac{\beta_1\alpha_3\sum_{n=0}^{\infty}D(n)+\beta_1\alpha_4\sum_{n=0}^{\infty}E(n)}{N_H}\right)-\mu_W\sum_{n=0}^{\infty}S_W(n)\right]\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} E_W(n)\right\}=\frac{n_9}{S}+\frac{1}{S^{\beta}}\mathcal{L}\left[\left(\frac{\beta_1\alpha_3\sum_{n=0}^{\infty}D(n)+\beta_1\alpha_4\sum_{n=0}^{\infty}E(n)}{N_H}\right)-\left(\mu_W+\alpha_W\right)\sum_{n=0}^{\infty}E_W(n)\right]\\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} I_W(n)\right\}=\frac{n_{10}}{S}+\frac{1}{S^{\beta}}\mathcal{L}\left[\alpha_W\sum_{n=0}^{\infty}E_W(n)-\mu_W\sum_{n=0}^{\infty}I_W(n)\right]
\end{aligned}\right\} \tag{13}$$

Evaluating the Laplace transform of the 2nd terms in the RHS of (16), we obtain

$$\left.\begin{array}{l}
\mathcal{L}\left\{\sum_{n=0}^{\infty} S_H(n)\right\}=\frac{n_1}{S}+\left[\pi_H-\left(\frac{\beta_1\alpha_1\sum_{n=0}^{\infty}A(n)+\beta_2\alpha_2\sum_{n=0}^{\infty}B(n)}{N_H}\right)-\mu_H\sum_{n=0}^{\infty}S_H(n)\right]\frac{1}{S^{\beta+1}}\\
\mathcal{L}\left\{\sum_{n=0}^{\infty} E_H(n)\right\}=\frac{n_2}{S}+\left[\left(\frac{\beta_1\alpha_1\sum_{n=0}^{\infty}A(n)+\beta_2\alpha_2\sum_{n=0}^{\infty}B(n)}{N_H}\right)-\left(\mu_H+\alpha_H\right)\sum_{n=0}^{\infty}E_H(n)\right]\frac{1}{S^{\beta+1}}\\
\mathcal{L}\left\{\sum_{n=0}^{\infty} I_H(n)\right\}=\frac{n_3}{S}+\left[\alpha_H\sum_{n=0}^{\infty}E_H(n)-\left(\mu_H+\delta_H+\theta_H\right)\sum_{n=0}^{\infty}I_H(n)\right]\frac{1}{S^{\beta+1}}\\
\mathcal{L}\left\{\sum_{n=0}^{\infty} R_H(n)\right\}=\frac{n_4}{S}+\left[\theta_H\sum_{n=0}^{\infty}I_H(n)-\mu_H\sum_{n=0}^{\infty}R_H(n)\right]\frac{1}{S^{\beta+1}}\\
\mathcal{L}\left\{\sum_{n=0}^{\infty} S_M(n)\right\}=\frac{n_5}{S}+\left[\pi_M-\frac{\beta_1C_1\sum_{n=0}^{\infty}C(n)}{N_H}-\mu_M\sum_{n=0}^{\infty}S_M(n)\right]\frac{1}{S^{\beta+1}}\\
\mathcal{L}\left\{\sum_{n=0}^{\infty} E_M(n)\right\}=\frac{n_6}{S}+\left[\frac{\beta_1C_1\sum_{n=0}^{\infty}C(n)}{N_H}-\left(\mu_M+b_M\right)\sum_{n=0}^{\infty}E_M(n)\right]\frac{1}{S^{\beta+1}}\\
\mathcal{L}\left\{\sum_{n=0}^{\infty} I_M(n)\right\}=\frac{n_7}{S}+\left[b_M\sum_{n=0}^{\infty}E_M(n)-\mu_M\sum_{n=0}^{\infty}I_M(n)\right]\frac{1}{S^{\beta+1}}\\
\mathcal{L}\left\{\sum_{n=0}^{\infty} S_W(n)\right\}=\frac{n_8}{S}+\left[\pi_W-\left(\frac{\beta_1\alpha_3\sum_{n=0}^{\infty}D(n)+\beta_1\alpha_4\sum_{n=0}^{\infty}E(n)}{N_H}\right)-\mu_W\sum_{n=0}^{\infty}S_W(n)\right]\frac{1}{S^{\beta+1}}\\
\mathcal{L}\left\{\sum_{n=0}^{\infty} E_W(n)\right\}=\frac{n_9}{S}+\left[\left(\frac{\beta_1\alpha_3\sum_{n=0}^{\infty}D(n)+\beta_1\alpha_4\sum_{n=0}^{\infty}E(n)}{N_H}\right)-\left(\mu_W+\alpha_W\right)\sum_{n=0}^{\infty}E_W(n)\right]\frac{1}{S^{\beta+1}}\\
\mathcal{L}\left\{\sum_{n=0}^{\infty} I_W(n)\right\}=\frac{n_{10}}{S}+\left[\alpha_W\sum_{n=0}^{\infty}E_W(n)-\mu_W\sum_{n=0}^{\infty}I_W(n)\right]\frac{1}{S^{\beta+1}}
\end{array}\right\} \tag{14}$$

Taking the inverse Laplace transform of both sides of (14)

$$\left.\begin{array}{l}
\sum_{n=0}^{\infty} S_H(n) = n_1 + \left[\pi_H - \left(\dfrac{\beta_1\alpha_1 \sum_{n=0}^{\infty} A(n) + \beta_2\alpha_2 \sum_{n=0}^{\infty} B(n)}{N_H}\right) - \mu_H \sum_{n=0}^{\infty} S_H(n)\right]\dfrac{t^\beta}{\Gamma(\beta+1)} \\
\sum_{n=0}^{\infty} E_H(n) = n_2 + \left[\left(\dfrac{\beta_1\alpha_1 \sum_{n=0}^{\infty} A(n) + \beta_2\alpha_2 \sum_{n=0}^{\infty} B(n)}{N_H}\right) - \left(\mu_H + \alpha_H\right) \sum_{n=0}^{\infty} E_H(n)\right]\dfrac{t^\beta}{\Gamma(\beta+1)} \\
\sum_{n=0}^{\infty} I_H(n) = n_3 + \left[\alpha_H \sum_{n=0}^{\infty} E_H(n) - \left(\mu_H + \delta_H + \theta_H\right) \sum_{n=0}^{\infty} I_H(n)\right]\dfrac{t^\beta}{\Gamma(\beta+1)} \\
\sum_{n=0}^{\infty} R_H(n) = n_4 + \left[\theta_H \sum_{n=0}^{\infty} I_H(n) - \mu_H \sum_{n=0}^{\infty} R_H(n)\right]\dfrac{t^\beta}{\Gamma(\beta+1)} \\
\sum_{n=0}^{\infty} S_M(n) = n_5 + \left[\pi_M - \dfrac{\beta_1 C_1 \sum_{n=0}^{\infty} C(n)}{N_H} - \mu_M \sum_{n=0}^{\infty} S_M(n)\right]\dfrac{t^\beta}{\Gamma(\beta+1)} \\
\sum_{n=0}^{\infty} E_M(n) = n_6 + \left[\dfrac{\beta_1 C_1 \sum_{n=0}^{\infty} C(n)}{N_H} - \left(\mu_M + b_M\right) \sum_{n=0}^{\infty} E_M(n)\right]\dfrac{t^\beta}{\Gamma(\beta+1)} \\
\sum_{n=0}^{\infty} I_M(n) = n_7 + \left[b_M \sum_{n=0}^{\infty} E_M(n) - \mu_M \sum_{n=0}^{\infty} I_M(n)\right]\dfrac{t^\beta}{\Gamma(\beta+1)} \\
\sum_{n=0}^{\infty} S_W(n) = n_8 + \left[\pi_W - \left(\dfrac{\beta_1\alpha_3 \sum_{n=0}^{\infty} D(n) + \beta_1\alpha_4 \sum_{n=0}^{\infty} E(n)}{N_H}\right) - \mu_W \sum_{n=0}^{\infty} S_W(n)\right]\dfrac{t^\beta}{\Gamma(\beta+1)} \\
\sum_{n=0}^{\infty} E_W(n) = n_9 + \left[\left(\dfrac{\beta_1\alpha_3 \sum_{n=0}^{\infty} D(n) + \beta_1\alpha_4 \sum_{n=0}^{\infty} E(n)}{N_H}\right) - \left(\mu_W + \alpha_W\right) \sum_{n=0}^{\infty} E_W(n)\right]\dfrac{t^\beta}{\Gamma(\beta+1)} \\
\sum_{n=0}^{\infty} I_W(n) = n_{10} + \left[\alpha_W \sum_{n=0}^{\infty} E_W(n) - \mu_W \sum_{n=0}^{\infty} I_W(n)\right]\dfrac{t^\beta}{\Gamma(\beta+1)}
\end{array}\right\} \tag{15}$$

When $n=0$ we obtain,

$$S_H(0)=n_1,\ \ E_H(0)=n_2,\ I_H(0)=n_3,\ R_H(0)=n_4,\ S_M(0)=n_5,\ E_M(0)=n_6,\ I_M(0)=n_7,$$

$$S_W(0)=n_8,\ E_W(0)=n_9,\ I_W(0)=n_{10} \quad (16)$$

When $n=1$, we obtain,

$$\left.\begin{aligned}
S_H(1)&=\left[\pi_H-\left(\frac{\beta_1\alpha_1A(0)+\beta_2\alpha_2B(0)}{N_H}\right)-\mu_HS_H(0)\right]\frac{t^\beta}{\Gamma(\beta+1)}\\
E_H(1)&=\left[\left(\frac{\beta_1\alpha_1A(0)+\beta_2\alpha_2B(0)}{N_H}\right)-(\mu_H+\alpha_H)E_H(0)\right]\frac{t^\beta}{\Gamma(\beta+1)}\\
I_H(1)&=\left[\alpha_HE_H(0)-(\mu_H+\delta_H+\theta_H)I_H(0)\right]\frac{t^\beta}{\Gamma(\beta+1)}\\
R_H(1)&=\left[\theta_HI_H(0)-\mu_HR_H(0)\right]\frac{t^\beta}{\Gamma(\beta+1)}\\
S_M(1)&=\left[\pi_M-\frac{\beta_1C_1C(0)}{N_H}-\mu_MS_M(0)\right]\frac{t^\beta}{\Gamma(\beta+1)}\\
E_M(1)&=\left[\frac{\beta_1C_1C(0)}{N_H}-(\mu_M+b_M)E_M(0)\right]\frac{t^\beta}{\Gamma(\beta+1)}\\
I_M(1)&=\left[b_ME_M(0)-\mu_MI_M(0)\right]\frac{t^\beta}{\Gamma(\beta+1)}\\
S_W(1)&=\left[\pi_W-\left(\frac{\beta_1\alpha_3D(0)+\beta_1\alpha_4E(0)}{N_H}\right)-\mu_WS_W(0)\right]\frac{t^\beta}{\Gamma(\beta+1)}\\
E_W(1)&=\left[\left(\frac{\beta_1\alpha_3D(0)+\beta_1\alpha_4E(0)}{N_H}\right)-(\mu_W+\alpha_W)E_W(0)\right]\frac{t^\beta}{\Gamma(\beta+1)}\\
I_W(1)&=\left[\alpha_WE_W(0)-\mu_WI_W(0)\right]\frac{t^\beta}{\Gamma(\beta+1)}
\end{aligned}\right\} \quad (17)$$

When $n=2$, we obtain,

$$\left.\begin{aligned}
S_H(2) &= \left[\pi_H - \left(\frac{\beta_1\alpha_1 A(1) + \beta_2\alpha_2 B(1)}{N_H}\right) - \mu_H S_H(1)\right]\frac{t^\beta}{\Gamma(\beta+1)} \\
E_H(2) &= \left[\left(\frac{\beta_1\alpha_1 A(1) + \beta_2\alpha_2 B(1)}{N_H}\right) - (\mu_H + \alpha_H) E_H(1)\right]\frac{t^\beta}{\Gamma(\beta+1)} \\
I_H(2) &= \left[\alpha_H E_H(1) - (\mu_H + \delta_H + \theta_H) I_H(1)\right]\frac{t^\beta}{\Gamma(\beta+1)} \\
R_H(2) &= \left[\theta_H I_H(1) - \mu_H R_H(1)\right]\frac{t^\beta}{\Gamma(\beta+1)} \\
S_M(2) &= \left[\pi_M - \frac{\beta_1 C_1 C(1)}{N_H} - \mu_M S_M(1)\right]\frac{t^\beta}{\Gamma(\beta+1)} \\
E_M(2) &= \left[\frac{\beta_1 C_1 C(1)}{N_H} - (\mu_M + b_M) E_M(1)\right]\frac{t^\beta}{\Gamma(\beta+1)} \\
I_M(2) &= \left[b_M E_M(1) - \mu_M I_M(1)\right]\frac{t^\beta}{\Gamma(\beta+1)} \\
S_W(2) &= \left[\pi_W - \left(\frac{\beta_1\alpha_3 D(1) + \beta_1\alpha_4 E(1)}{N_H}\right) - \mu_W S_W(1)\right]\frac{t^\beta}{\Gamma(\beta+1)} \\
E_W(2) &= \left[\left(\frac{\beta_1\alpha_3 D(1) + \beta_1\alpha_4 E(1)}{N_H}\right) - (\mu_W + \alpha_W) E_W(1)\right]\frac{t^\beta}{\Gamma(\beta+1)} \\
I_W(2) &= \left[\alpha_W E_W(1) - \mu_W I_W(1)\right]\frac{t^\beta}{\Gamma(\beta+1)}
\end{aligned}\right\} \tag{18}$$

$$\vdots \quad = \quad \vdots$$

When $n = n+1$, we obtain,

$$\left.\begin{aligned}
S_H(n+1) &= \left[\pi_H - \left(\frac{\beta_1\alpha_1 A(n) + \beta_2\alpha_2 B(n)}{N_H}\right) - \mu_H S_H(n)\right]\frac{t^\beta}{\Gamma(\beta+1)} \\
E_H(n+1) &= \left[\left(\frac{\beta_1\alpha_1 A(n) + \beta_2\alpha_2 B(n)}{N_H}\right) - \left(\mu_H + \alpha_H\right)E_H(n)\right]\frac{t^\beta}{\Gamma(\beta+1)} \\
I_H(n+1) &= \left[\alpha_H E_H(n) - \left(\mu_H + \delta_H + \theta_H\right)I_H(n)\right]\frac{t^\beta}{\Gamma(\beta+1)} \\
R_H(n+1) &= \left[\theta_H I_H(n) - \mu_H R_H(n)\right]\frac{t^\beta}{\Gamma(\beta+1)} \\
S_M(n+1) &= \left[\pi_M - \frac{\beta_1 C_1 C(n)}{N_H} - \mu_M S_M(n)\right]\frac{t^\beta}{\Gamma(\beta+1)} \\
E_M(n+1) &= \left[\frac{\beta_1 C_1 C(n)}{N_H} - \left(\mu_M + b_M\right)E_M(n)\right]\frac{t^\beta}{\Gamma(\beta+1)} \\
I_M(n+1) &= \left[b_M E_M(n) - \mu_M I_M(n)\right]\frac{t^\beta}{\Gamma(\beta+1)} \\
S_W(n+1) &= \left[\pi_W - \left(\frac{\beta_1\alpha_3 D(n) + \beta_1\alpha_4 E(n)}{N_H}\right) - \mu_W S_W(n)\right]\frac{t^\beta}{\Gamma(\beta+1)} \\
E_W(n+1) &= \left[\left(\frac{\beta_1\alpha_3 D(n) + \beta_1\alpha_4 E(n)}{N_H}\right) - \left(\mu_W + \alpha_W\right)E_W(n)\right]\frac{t^\beta}{\Gamma(\beta+1)} \\
I_W(n+1) &= \left[\alpha_W E_W(n) - \mu_W I_W(n)\right]\frac{t^\beta}{\Gamma(\beta+1)}
\end{aligned}\right\} \tag{19}$$

The series solution of each compartment can be expressed as:

$$\begin{aligned}
S_H(t) &= S_H(0) + S_H(1) + S_H(2) + \ldots \\
E_H(t) &= E_H(0) + E_H(1) + E_H(2) + \ldots \\
I_H(t) &= I_H(0) + I_H(1) + I_H(2) + \ldots \\
R_H(t) &= R_H(0) + R_H(1) + R_H(2) + \ldots \\
S_M(t) &= S_M(0) + S_M(1) + S_M(2) + \ldots \\
E_M(t) &= E_M(0) + E_M(1) + E_M(2) + \ldots
\end{aligned} \tag{20}$$

$$I_M(t) = I_M(0) + I_M(1) + I_M(2) + \ldots$$

$$S_W(t) = S_W(0) + S_W(1) + S_W(2) + \ldots$$

$$E_W(t) = E_W(0) + E_W(1) + E_W(2) + \ldots$$

$$I_W(t) = I_W(0) + I_W(1) + I_W(2) + \ldots$$

3. Convergence Analysis for the Laplace-Adomian Decomposition Method (LADM).

The solution of (1) is expressed in the forms of infinite series which converged uniformly to its exact solution. To verify the convergence of the series (21), we employ the method used in [18]. For sufficient conditions of convergence of the LADM, we present the following theorem:

Theorem 1

Let X be a Banach space and $T: X \rightarrow X$ *be a constructive nonlinear operator such that for* $(x),(x)' \in X$, $\left\|T(x) - T(x')\right\|, 0 < k < 1$. *Then, T has a unique point x such that* $Tx = x$ *,where* $x = (S_H, E_H, I_H, R_H, S_M, E_M, I_M, S_W, E_W, I_W)$. *The series given () can be written by applying the Adominan decomposition method as follows:*

$$x_n = Tx_{n-1}, x_{n-1},$$

$$= \sum_{i=1}^{n-1} x_i, \; n = 1, 2, 3, \ldots$$

And we assume that $x_0 \in B_r(x)$, *where* $B_r(x) = \left\{x \in X : \left\|x' - x\right\| < r\right\}$; *then, we have as follows:*

(i) $x_n \in B_r(x)$

(ii) $\lim_{n \to \infty} x_n = x$

Proof

For condition (i), invoking mathematical induction,

For n=1, we have as follows:

$$\left\|x_0 - x\right\| = \left\|T(x_0) - T(x)\right\| \le \left\|x_0 - x\right\|.$$

If this is true for m-1, then

$$\left\|x_0 - x\right\| \le k^{m-1} \left\|x_0 - x\right\|.$$

This gives the following:

$$\|x_m - x\| = \|T(x_{m-1}) - T(x)\| \le k\|x_{m-1} - x\| \le k^n \|x_0 - x\|.$$

Therefore,

$$\|x_m - x\| \le k^n \|x_0 - x\| \le k^n r < r.$$

This directly implies that $x_n \in B_r(x)$.

Also, for (ii), we have that since $\|x_m - x\| \le k^n \|x_0 - x\|$ and $\lim_{n\to\infty} k^n = 0$, we can write $\lim_{n\to\infty} x_n = x$.

2.5 Numerical Solution of Laplace Adomian Decomposition Method (LADM)

In this section, we will see the numerical solution of the model. Using the initial conditions, the Laplace Adomian Decomposition Method (LADM) gives us an approximate solution in in terms of an infinite series presented as:

$$\left.\begin{aligned}
S_H(t) &= 200000 - 429.97\frac{t^{\beta}}{\Gamma(\beta+1)} - 824.16\frac{t^{2\beta}}{\Gamma(2\beta+1)} + \dots \\
E_H(t) &= 150000 - 25080.46\frac{t^{\beta}}{\Gamma(\beta+1)} + 5088.67\frac{t^{2\beta}}{\Gamma(2\beta+1)} + \dots \\
I_H(t) &= 13000 + 25298.11\frac{t^{\beta}}{\Gamma(\beta+1)} - 4656.55\frac{t^{2\beta}}{\Gamma(2\beta+1)} + \dots \\
R_H(t) &= 9000 + 168.73\frac{t^{\beta}}{\Gamma(\beta+1)} + 168.73\frac{t^{2\beta}}{\Gamma(2\beta+1)} + \dots \\
S_M(t) &= 10000 - 419.29\frac{t^{\beta}}{\Gamma(\beta+1)} - 75956098800\frac{t^{2\beta}}{\Gamma(2\beta+1)} + \dots \\
E_M(t) &= 8000 - 466.31\frac{t^{\beta}}{\Gamma(\beta+1)} - 75956098850\frac{t^{2\beta}}{\Gamma(2\beta+1)} + \dots \\
I_M(t) &= 9000 - 194.35\frac{t^{\beta}}{\Gamma(\beta+1)} - 1.88\frac{t^{2\beta}}{\Gamma(2\beta+1)} + \dots \\
S_W(t) &= 10000 - 12.49\frac{t^{\beta}}{\Gamma(\beta+1)} - 1.91\frac{t^{2\beta}}{\Gamma(2\beta+1)} + \dots \\
E_W(t) &= 8000 - -1427.93\frac{t^{\beta}}{\Gamma(\beta+1)} + 259.01\frac{t^{2\beta}}{\Gamma(2\beta+1)} + \dots \\
I_W(t) &= 9000 + 1439.78\frac{t^{\beta}}{\Gamma(\beta+1)} + -257.06\frac{t^{2\beta}}{\Gamma(2\beta+1)} + \dots
\end{aligned}\right\}$$

(21)

For $\beta = 1$, the series solution of our model becomes,

$$\left.\begin{aligned}
S_H(t) &= 200000 - 429.97t - 410.58t^2 + \ldots \\
E_H(t) &= 150000 - 25080.46t + 2544.34t^2 + \ldots \\
I_H(t) &= 13000 + 25298.11t - 2328.28t^2 + \ldots \\
R_H(t) &= 9000 + 168.73t + 84.37t^2 + \ldots \\
S_M(t) &= 10000 - 419.29t - 37978049400t^2 + \ldots \\
E_M(t) &= 8000 - 466.31t - 37978049425t^2 + \ldots \\
I_M(t) &= 9000 - 194.35t - 0.94t^2 + \ldots \\
S_W(t) &= 10000 - 12.49t - 0.96t^2 + \ldots \\
E_W(t) &= 8000 - 1427.93t + 129.51t^2 + \ldots \\
I_W(t) &= 9000 + 1439.78t - 128.53t^2 + \ldots
\end{aligned}\right\} \tag{22}$$

Table 5: Parameters Table of Values

PARAMETERS	VALUE	SOURCE
π_H	0.0647	[19]
π_M	0.0471	Estimated
π_W	0.03645	Estimated
μ_H	0.00003	[16]
μ_M	0.04	Estimated
μ_W	0.000025	[16]
β_1	0.45	[15]
β_2	0.37	Estimated
α_1	0.000006	[18]
α_2	0.16	Estimated
α_3	0.11	[18]
α_4	0.0000057	[16]
α_H	0.17	Estimated
θ_H	0.013	[13]
b_M	0.02070591	Estimated
a_W	0.0193	Estimated
δ_H	0.0025	[14]
C_1	0.12	Estimated

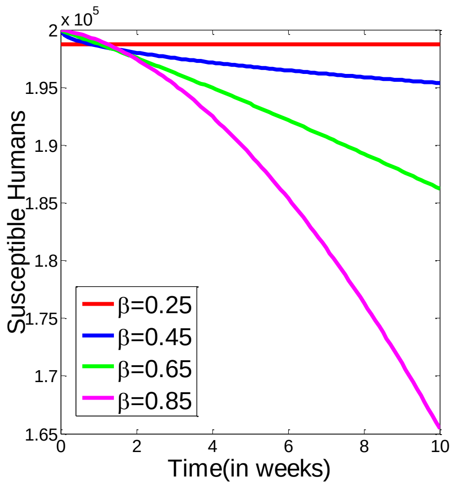


Fig 2a. Effect of varying β on susceptible Human population

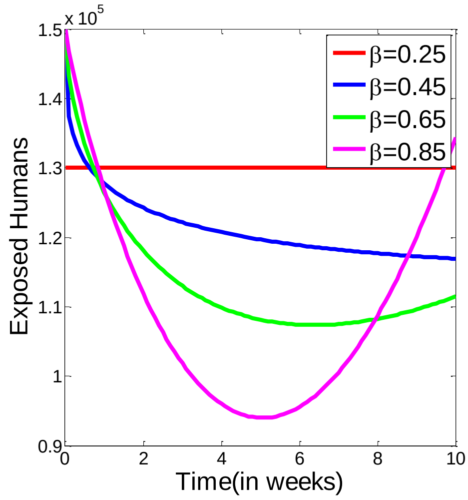


Fig 2b. Effect of varying β on Exposed Zika virus.

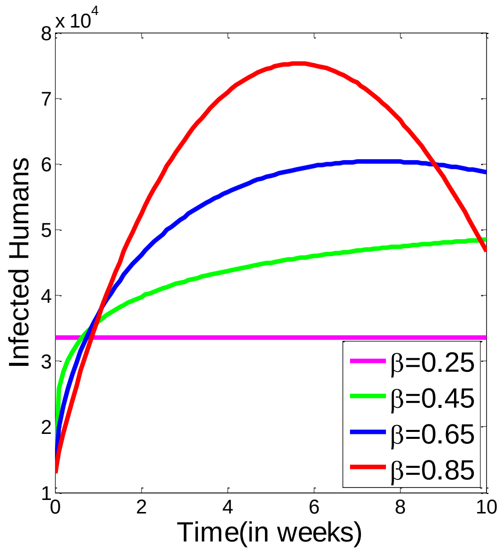


Fig 2c. Effect of varying β on Infected Humans population

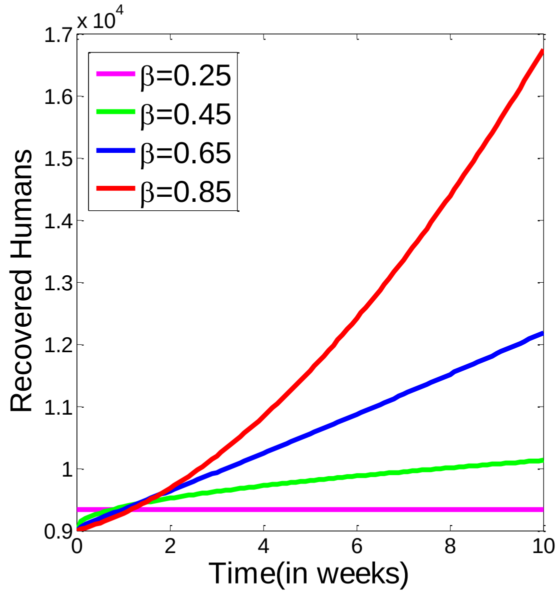


Fig 2d. Effect of varying β on Recovered Humans

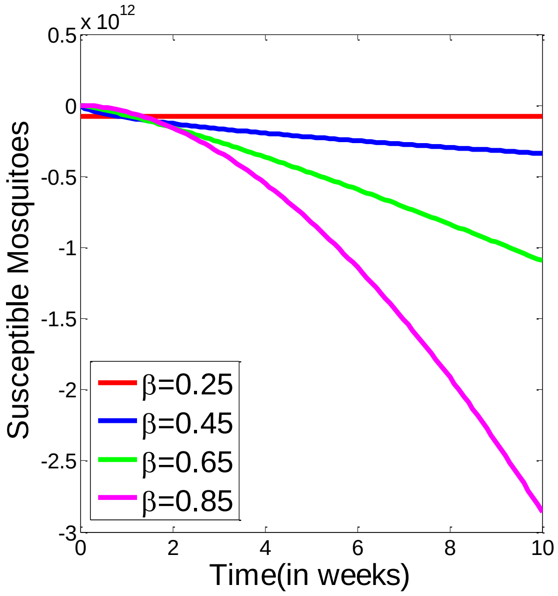


Fig 2e. Effect of varying β on Susceptible Mosquitoes

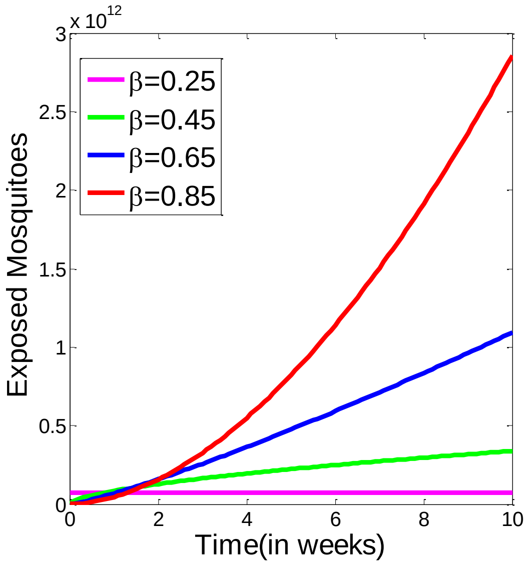


Fig 2f. Effect of varying β on Exposed Mosquitoes

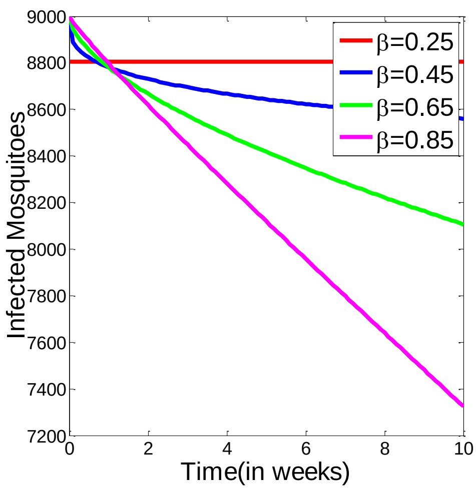


Fig 2g. Effect of varying β on Infected Mosquitoes

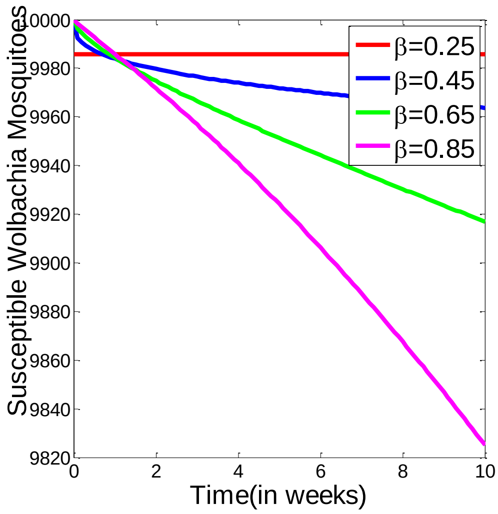
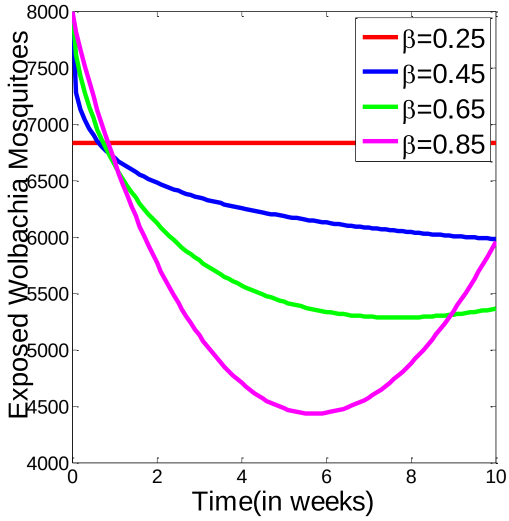


Fig 2h. Effect of varying β on Susceptible Wolbachia Mosquitoes



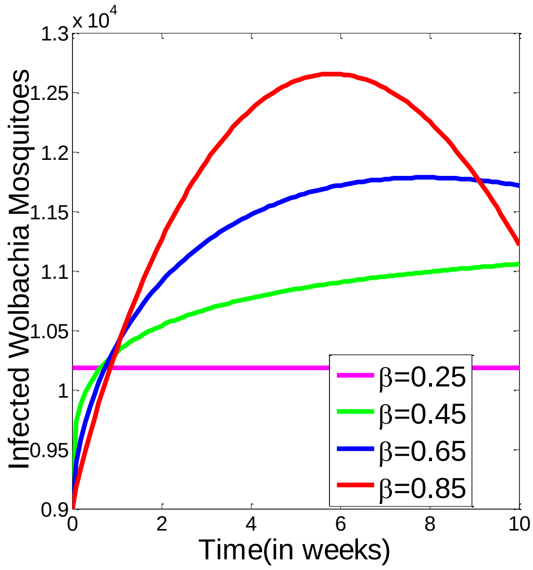


Fig 2i. Effect of varying β on Infected Wolbachia Mosquitoes

Fig 2j. Effect of varying β on Infected Wolbachia Mosquitoes

Conclusion

In figure 2a, the number of susceptible individuals decreases initially and later rises due to increase in the number of exposed humans in figure 2b. From figure 2c, the number of infected humans decreases with time leading to high recovery rate in figure 2d which indicates strong disease control within the population. It is observed that the number of susceptible mosquitoes decreases (see figure 2e) due to increase in the number of wolbachia-infected mosquitoes in figure 2h. It implies that with increasing time, the total mosquitoes population become wolbachia infected which means that the vector control measure is effective. Due the influx of mosquitoes from the susceptible population to the exposed population, the number of exposed mosquitoes increases in figure 2f leading to a decrease in the number of infected mosquitoes in figure 2g which implies disease control. A progressive increase is observed in number of wolbachia-infected population is observed in figure 2j which means transition of mosquitoes to wolbachia-infected mosquitoes is well implemented.

In conclusion, the application of the Laplace-Adomian Decomposition Method (LADM) to solve the Zika virus model incorporating Wolbachia-infected mosquitoes as a vector control strategy presents significant contributions to both mathematical modeling in epidemiology and practical disease control measures. Through numerical simulations and analysis, key findings have emerged, shedding light on the efficacy of Wolbachia-infected mosquitoes in reducing Zika virus transmission rates. The use of LADM has proven advantageous in accurately capturing the dynamics of the system, allowing for a comprehensive understanding of the interplay between different variables and their impact on disease spread.One of the primary findings of this research is the effectiveness of Wolbachia-infected mosquitoes in suppressing Zika virus transmission. By introducing Wolbachia-infected mosquitoes into the population, the model demonstrates a notable reduction in the number of infected individuals, thus highlighting the potential of this biological control method in mitigating disease burden.

Furthermore, the utilization of LADM offers several advantages in this context. Unlike traditional numerical methods, LADM provides analytical solutions, facilitating a more intuitive interpretation of the results and enabling rapid sensitivity analysis. Moreover, its flexibility allows for the incorporation of complex nonlinearities inherent in epidemiological models, ensuring the accuracy of the predictions. The significance of this research extends beyond academic interest, with implications for government and public health workers. By elucidating the effectiveness of Wolbachia-infected mosquitoes as a vector control strategy, policymakers can make informed decisions regarding the implementation of such interventions in Zika virus-endemic regions. Additionally, the insights gained from this study can inform resource allocation and intervention planning, ultimately aiding in the reduction of disease transmission and the protection of public health.

In summary, the application of LADM to analyze the Zika virus model with Wolbachia-infected mosquitoes as a vector control strategy represents a valuable contribution to both theoretical epidemiology and practical disease control efforts. By elucidating key findings through numerical simulations and analysis, this research underscores the importance of utilizing innovative approaches in combating emerging infectious diseases and underscores the potential of biological control methods in mitigating disease burden.

Conflict of interest

The authors declare that they have no competing interest

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Availability of data.

The data used in this study are referenced and presented in table 2 above.

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