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NUMERICAL VERIFICATION OF THE STRONG GOLDBACH CONJECTURE UP TO 9×10^{18}

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ABSTRACT

The Goldbach Conjecture states that every even integer ≥ 4 can be written as a sum of two prime numbers. It is known to be true for all even numbers up to 4 × 10^{18}[1]. Using the new formulation of a set of even numbers as $(P_1+P_2)+(P_2-P_1)^n$ [9], and the fact that, an even number of this formulation can be partitioned into all pairs of odd numbers [10], we present a computational algorithm that confirms the Strong Goldbach Conjecture holds true for all even numbers not larger than 9 × 10^{18}.

KEYWORDS

Goldbach Conjecture, Goldbach partition, Even numbers, Odd numbers, Prime numbers, Natural numbers.

1. Introduction

The strong or binary Goldbach Conjecture asserts that all positive even integers can be expressed as the sum of two primes. Any two primes (p, q)such that$p + q = 2n$ for n a positive integer are called a Goldbach partition[2]. Goldbach's original conjecture was formulated in such a way that, every even integer greater than 2 can be expressed as the sum of two prime numbers [3]. For example, 4 = 2 + 2, 6 = 3 + 3, 8 = 3 + 5, 10 = 3 + 7 = 5 + 5, and so on. It remains one of the most tantalizing and intriguing open problems in mathematics. The conjecture has inspired a great deal of research in Number Theory and has also led to the development of new techniques and methods in the field [4].

The Strong Goldbach Conjecture has been verified for large even values as summarized in Table 1.

bound	reference
1×10^{4}	Desboves 1885
1×10^{5}	Pipping 1938
1×10^{8}	Stein and Stein 1965
2×10^{10}	Granville et al. 1989
4×10^{11}	Sinisalo 1993
1×10^{14}	Deshouillers et al. 1998
4×10^{14}	Richstein 1999, 2001
2×10^{16}	Oliveira e Silva (Mar. 24, 2003)
6×10^{16}	Oliveira e Silva (Oct. 3, 2003)
2×10^{17}	Oliveira e Silva (Feb. 5, 2005)
3×10^{17}	Oliveira e Silva (Dec. 30, 2005)
12×10^{17}	Oliveira e Silva (Jul. 14, 2008)
4×10^{18}	Oliveira e Silva (Apr. 2012)
9×10^{18}	This paper

Table 1: Verification of the Strong Goldbach Conjecture [5].

The latest result on the Strong conjecture shows that it holds up through 4×10^{18} [1] but remains unproven for almost 250 years despite considerable effort by many mathematicians throughout history [6].

2. Partitioning any even number into all pairs of odd numbers

Using the new formulation of a set of even numbers as $(P_1+P_2)+(P_2-P_1)^n$ [9], it has been shown that it is always possible to partition any even number into all pairs of odd numbers using the following algorithm:

Let P be the set of all prime numbers, $\mathbb{N}$ be the set of all natural numbers and O the set of all odd numbers.

Step 1 : Let P_1 and $P_2 \in P$, then $(P_1 + P_2) + (P_2 - P_1)^n$ is even, $\forall n \in \mathbb{N}$, and $p_2 > p_1$[9].

Step 2: Let d be even and belong to the half-open interval $[1, [\frac{1}{2}((p_1 + p_2) + (p_2 - p_1)^n]]$.

Step 3: Let z_i *and* $y_i \in 1 \le O \le \frac{1}{2}((P_1 + P_2) + (P_2 - P_1)^n)$ for $i \in O$ and belong to the half-open interval $[1, [\frac{1}{2}((p_1 + p_2) + (p_2 - p_1)^n]]$.

With p_1, p_2, d and z_i, we partition $(p_1 + p_2) + (p_2 - p_1)^n$ as follows:

Partition 1 : $((P_1 + P_2) + (P_2 - P_1)^n) - (d + z_1) = y_1$

Partition 2 : $((p_1 + p_2) + (p_2 - p_1)^n) - (d + z_3) = y_3$

Partition 3: $((p_1 + p_2) + (p_2 - p_1)^n) - (d + z_5) = y_5$

.
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.

Partition i: $((p_1 + p_2) + (p_2 - p_1)^n) - (d + z_{((\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)-1)}) = y_{((\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)-1)}$

The set of pairs $(d + z_1, y_1), (d + z_3, y_3), (d + z_5, y_5), \ldots,$
$(d + z_{((\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)-1)}, y_{((\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)-1)})$ of odd numbers are all partitions of the even number $(p_1 + p_2) + (p_2 - p_1)^n$ [10].Since prime numbers greater than 2 are subsets of odd numbers, from these set of pairs of odd numbers, the possibility is that, there exist at least one pair of prime.

Example 1

Step 1 : Let $p_1 = 13$, $p_2 = 23$ and $n = 1$, then
$(p_1 + p_2) + (p_2 - p_1)^1 = (13 + 23) + (23 - 13)^1 = 36 + 10 = 46$ is even.

Step 2 : we take , $d = p_2 - p_1 = 23 - 13 = 10 > 0$

Step 3 : Odd numbers in the range $1 \le O \le \frac{1}{2}(46) \rightarrow (1,3,5,7,9,11,13,15,17,19,21,23)$

Then, using $d = 10$ and the set of odd numbers in step 3, we partition 46 as follows:

I. $46 - (10 + 1) = 35$
II. $46 - (10 + 3) = 33$
III. $46 - (10 + 5) = 31$
VII. $46 - (10 + 13) = 23$
VIII. $46 - (10 + 15) = 21$
IX. $46 - (10 + 17) = 19$

IV. $46-(10+7)=29$ X. $46-(10+19)=17$
V. $46-(10+9)=27$ XI. $46-(10+21)=15$
VI. $46-(10+11)=25$ XII. $46-(10+23)=13$

The partitions of 46 are therefore: $\big((10+1),35\big),\big((10+3),33\big),\big((10+5),31\big),\big((10+7),29\big),\big((10+9),27\big),\big((10+11),25\big),\big((10+13),23\big),\big((10+15),21\big),\big((10+17),19\big),\big((10+19),17\big),\big((10+21),15\big),((10+23),13)\Rightarrow$
$(11,35),(13,33),(15,31),(17,29),(19,27),(21,25),(23,23),(25,21),(27,19),(29,17),(31,15),$

$(33,13)$. Therefore, the algorithm has partitioned 46 into 12 pairs of odd numbers. From these 12 pairs of odd numbers, the pairs $(17,29)$ *and* $(23,23)$ are prime. Which shows that 46 can be written as a sum of two distinct pairs of prime.

In general, the same algorithm can be used to partition 46 into12 pairs of odd numbers for $n=1$ and any multiples of $d$belonging to the half-open interval[1, $[\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n]]$[10].

Remark 1

The multiples of $d$belonging to the half-open interval[1, $[\frac{1}{2}(46)]$ are $\{2,4,6,8,10,12,14,16,18,20,22\}$.

Example 2

Step 1: For $(p_1+p_2)+(p_2-p_1)^n=46$ and $n=1$

Step 2 : $d=22$

Step 3 : Odd numbers in the range $1\le O\le \frac{1}{2}(46)\rightarrow(1,3,5,7,9,11,13,15,17,19,21,23)$

Then we partition 46 as follows:

I. $46-(22+1)=23$ VII. $46-(22+13)=11$
II. $46-(22+3)=21$ VIII. $46-(22+15)=9$
III. $46-(22+5)=19$ IX. $46-(22+17)=7$
IV. $46-(22+7)=17$ X. $46-(22+19)=5$
V. $46-(22+9)=15$ XI. $46-(22+21)=3$
VI. $46-(22+11)=13$ XII. $46-(22+23)=1$

The partitions of 46 are therefore: $\big((22+1),23\big),\big((22+3),21\big),\big((22+5),19\big),\big((22+7),17\big),\big((22+9),15\big),\big((22+11),13\big),\big((22+13),11\big),\big((22+15),9\big),\big((22+17),7\big),\big((22+19),5\big),\big((22+21),3\big),((22+23),1)\Rightarrow$
$(23,23),(25,21),(27,19),(29,17),(31,15),(33,13),(35,11),(37,9),(39,7),(41,5),(43,3)$

and $(45,1)$. Again, the algorithm has partitioned 46 into 12 pairs of odd numbers. From these 12 pairs of odd numbers, the pairs $(29,17),(23,23)$ *and* $(43,3)$ are prime. Which shows further that 46 can be written as a sum of three distinct pairs of prime.

The results show that since any even number can be partitioned into all pairs of odd numbers, in these pairs of odd numbers, there is a possibility of obtaining at least one pair of prime fulfilling the Strong Golbach Conjecture. As the even number grows bigger, the odd partitions increases significantly and it comes also difficult to obtain the prime partitions. In the past, various researchers have utilized several different ways in trying to solve the conjecture, ranging from computational, statistical and probabilistic approaches to analytic number theory [7].

Although computer results cannot prove the conjecture, mathematicians have used computational approach and extended the verification of Goldbach Conjectures to larger numbers. This study presents in section 3a simple Java program that implements the algorithm developed for partitioning any even number into all pairs of odd numbers. Ordinarily it requires a powerful computer to partition huge numbers since Operations involving very large numbers can be computationally expensive. Performing arithmetic operations on extremely large numbers requires more processing power and time. Algorithms designed for standard-sized numbers may not scale efficiently to handle very large inputs [8].The Java program was coded in a computer with the following specifications:

2.1 Hardware Specifications and System Information

- Processor: Intel(R) Core(TM) i7-10510U CPU @ 1.80GHz   2.30 GHz
- Installed RAM:16.0 GB (15.8 GB usable)
- System Type: 64-bit operating system, x64-based processor

3. Computational Environment

The program was coded in an Integrated Development Environment called Apache Netbeans IDE 15. Java Runtime Environment (Java 8 Update 371 (64-bit)) was pre-installed to provide an environment for the program to execute. Additionally, Java Development Kit (Java(TM) SE Development Kit 19 (64-bit) was also installed to provide resources needed for the program to execute. The ultimate result when the program runs is that it partitions an even number as an input into all pairs of odd numbers. The program executes in different sequential stages as follows:

Step 1:

The program allows the user to enter an even number say $2n$in a Java Text Field that allows up to 40 characters.

Code extract 1: JTextFieldenterEven = new JTextField (40);

The even number $2n$ is captured as a String and converted into Long data type.

Code extract 2: String evenTotal = enterEven.getText();

Code extract 3: evenNumber = Long.parseLong(evenTotal);

Step 2:

The even number $2n$ is then divided by 2 and an even random number is picked from the open half internal [1, n]. The even random number is then displayed inside a Java Text Area that is inside a Java Scroll Pane.

Code extract 4: halfEvenNo = (evenNumber)/2;

Code extract 5: randomEvenNo = randomEven.nextLong(halfEvenNo);

Code extract 6: result.append("The random Even No is:" + randomEvenNo + "\n");

Step 3:

The even random number is incremented by adding odd numbers starting from 1 and the result is Subtracted each time from $2n$ that was initially entered to get the second partition as an odd number.

Remark 2: Note that the below terms primeOne and primeTwo are just variables and do not necessarily imply they are prime numbers.

Code extract 7: primeOne = randomEvenNo + i;

Code extract 8: primeTwo = evenNumber - primeOne;

Step 4

The primeOne and primeTwo becomes the first partitions of the even number $2n$and are then displayed side by side separated by a comma in a Java Text Area that's inside a Java Scroll Pane.

Code extract 8: JTextArea result= new JTextArea(8, 40);

Code extract 9: sPane.setViewportView(result);

Code extract 10: {result.append(primeOne + "," + primeTwo + "\n");

The process continues until the even number 2n is partitioned into all pairs of odd numbers. The following figures show the results obtained when the program executes:

For $2n = \mathbf{1,000,000,000,000,000,000}$, some partitions of this even number includes:

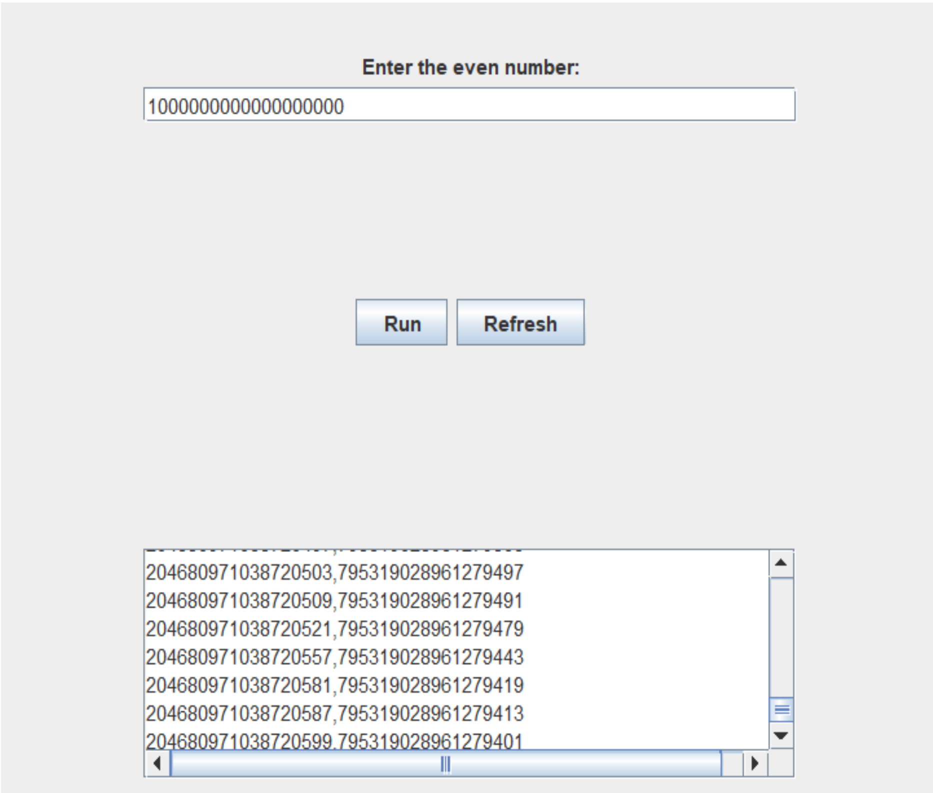

Figure 1 *shows Partitions of 1,000,000,000,000,000,000 of all pairs of odd numbers. The figure displays a subset of the set of all partitions of odd numbers.*

For $2n = \mathbf{9,000,000,000,000,000,000}$, some partitions of this even number includes:

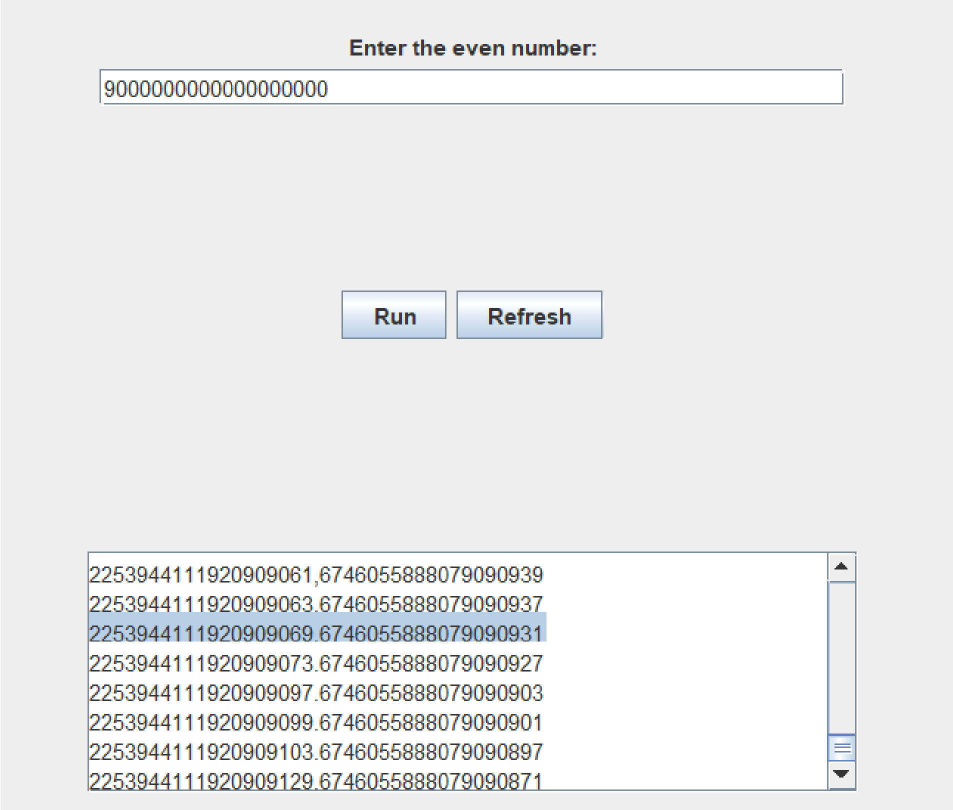

Figure 2 *shows Partitions of 9,000,000,000,000,000,000 for all pairs of odd numbers. The figure displays a subset of the set of all partitions of odd numbers. The highlighted pair is prime.*

The following Table 2 summarizes the partitioning of selected even numbers into all pairs of odd numbers up to 9×10^{18} while showing the Goldbach partition for the even number.

Table 2 *Summarizes the partitioning of selected even numbers into all pairs of odd numbers and finding the distinct Goldbach partitions*

S/no	Even number: $(p_1 + p_2) + (p_2 - p_1)^n$	Pairs of odd numbers whose sum is $(p_1 + p_2) + (p_2 - p_1)^n$	Distinct Goldbach partitions whose sum is $(p_1 + p_2) + (p_2 - p_1)^n$
1	10	(3,7), (5,5), (7,3)	… (5,5), (7,3)…
2	24	(3,21), (5,19), (7,17), (9,15), (11,13), (13,11)	… (5,19), (7,17), (11,13)..
3	46	(11,35), (13,33), (15,31), (17,29), (19,27) , (21,25), (23,23), (25,21), (27,19), (29,17), (31,15), (33,13)	… (17,29),(23,23)…
4	500	…,(271,229), (487,13), (463,37), (397,103)	…, (271,229), (487,13),

		(373,127), …	(463,37), (397,103) (373,127),…
5	10,000	…, (7559,2441), (7919,2081), (7793,2207), (7727,2273),…	… (7559,2441), (7919,2081), , (7793,2207), (7727,2273), …
6	10^6	…, (999809,191), (999827,173),(999863,137) (999893,107), (999899,101),(999917,83),…	…, (999917,83), (999863,137), (999809,191), …
7	9,989,748	…, (9989537,211), (9989549,199), (9989569,179), (9989597,151),(9989599,149) (9989621,127), …	…, (9989689,59), (9989677,71), (9989569,179), …
8	10^8	…, (99999821,179), (99999827,173), (99999833,167), (99999941,59), (9999994753), …	**…,** (17422763,82577237), (99999941,59),(99999827,173), (99999821,179), **…**
9	10^9	…, (857330003,142669997), (857330021,142669979), (857330051,142669949), (857330069,142669931) (857330093,142669907), …	…,(857330021,142669979), …
10	10^{18}	…, (204680971038720509,795319028961279491) (204680971038720521,795319028961279479) (204680971038720557,795319028961279443) (204680971038720581,795319028961279419), …	…, (204680971038720557,795319028961279443) , …
11	9×10^{18}	… (2253944111920909063,6746055888079090937) (2253944111920909069,6746055888079090931), (2253944111920909073,6746055888079090927), (2253944111920909097,6746055888079090903), (2253944111920909099,6746055888079090901), (2253944111920909103,6746055888079090897) …	… (2253944111920909069,6746055888079090931) …

The results therefore show that the even number 9×10^{18} can be written as a sum of two prime numbers 2253944111920909069 and 6746055888079090931. To confirm that indeed the two numbers are actually prime, we used the Prime Numbers Generator and Checker[11] available online, giving us the following two figures:

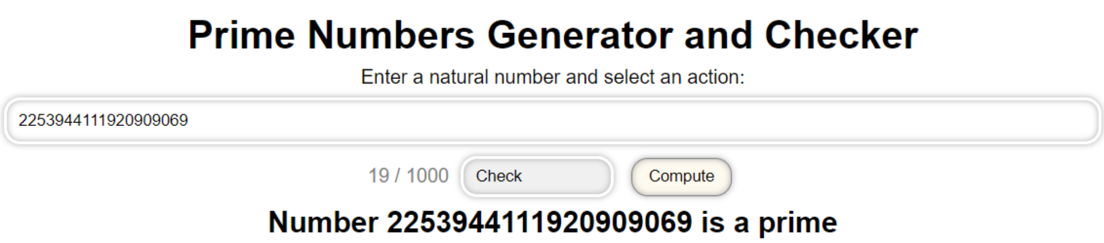

Figure 3 *shows the number 2253944111920909069 is prime*

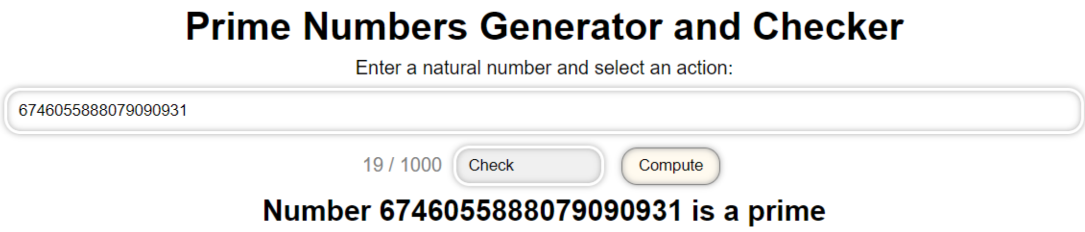

***Figure 4** shows the number 6746055888079090931 is prime*

This therefore implies that the equations $9 \times 10^{18} = 2253944111920909069 + 6746055888079090931$ agrees with the Strong Goldbach Conjecture statement that any even number greater than 2 can be written as a sum of two primes.

Conclusion

The Strong Goldbach Conjecture, which states that every even integer greater than 2 can be expressed as the sum of two prime numbers, has been extensively verified for all integers up to 4×10^{18}[12]. However, despite the significant computational effort and the absence of counterexamples mathematicians are nowhere near to providing a general proof [3]. This study has extended the verification of the Strong Goldbach Conjecture for all even numbers up to 9×10^{18}which is rather an advancement from the latest results by Oliveira et al. in 2014 showing that the Strong Goldbach conjecture holds true for all even numbers not larger than 4×10^{18}.

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