

A NARRATIVE INQUIRY EXPLORING THE ROLE OF WRITING IN MATHEMATICS TO FOSTER UNDERSTANDING

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Abstract

In recent years, there has been a growing trend in mathematics education research circles, advocating for more opportunities to allow students to be able to communicate their mathematical reasoning process with their peers. This paper takes readers through a narrative inquiry into the role of writing in mathematics, and how creating opportunities for students to write-down their mathematical ideas and the procedures they undergo in solving a problem 'in their own words' can increase motivation and interest in the study of mathematics and at the same time help students with developing a deeper understanding of concepts in mathematics, and the skills associated with 'learning how to learn' in a safe and collaborative learning environment.

Keywords

Mathematics communication, writing in mathematics, conceptual understanding.

INTRODUCTION

What constitutes an effective approach to learning and teaching in mathematics? Is there more than one way to teach mathematics? Debates about these and other related questions about effective mathematics teaching continue on even to this day. Some teachers and researchers advocate for direct instruction while others push for a more dialogic approach to instruction in the classroom (Hattie, et.al. 2017). Both models propose similar goals (namely the development of conceptual understanding in mathematics), however they differ in the ways in which learning opportunities are organized within the context of a lesson. *Direct instruction* is where the teacher decides the learning intentions and success criteria; makes them transparent to the students; demonstrates them by modeling; check for understanding and tie it all together with a closure. *Dialogic instruction* places more emphasis on ensuring students are provided with ample opportunities to collaboratively engage in their own discourse focusing on mathematical concepts;

derive their own problem-solving techniques; justify their mathematical reasoning; and evaluate the accuracy of their solutions. Neither model advocates that students simply memorize formulas and procedures, as the National Council of Teachers of Mathematics (2014) states, "emphasis should be placed on developing procedural fluency built on a foundation of conceptual understanding in mathematics", however despite all that is said and published regarding this topic, and despite all the discussions that continue to take place amongst teachers and researchers regarding this topic, for many students, mathematics continues to be a daunting and sometimes terrifying subject. Why is that? Boaler (2016) suggests that it is because people hold beliefs about mathematics that they do not hold about other subjects. "If you ask most students what they think their role is in math classrooms they will tell you it is to get questions right. Students rarely think that they are in math classrooms to appreciate the beauty of mathematics, to ask deep questions, to explore the rich set of connections that make up the subject, or even to learn about the applicability of the subject. They think they are in math class to perform." (Boaler, 2016, p.21) These students are often of the opinion that mathematics does not require discussion, debate or exploration and that it offers little in the way of meaningful connection to their daily lives (Marks-Krpan, 2013).

This study explores how a structured approach that promotes the development of conceptual understanding in mathematics in a more explicit way can help in developing procedural fluency in mathematics, regardless of how the learning engagements are facilitated in the classroom (i.e. via direct instruction or dialogic instruction). Marks-Krpan (2013) suggests, that one of the most important aspects of mathematics teaching is to teach students how to better 'communicate' their own ideas in mathematics with one another. "Communication is a critical component of learning new concepts because through articulating their understanding of a concept, students often crystallize that understanding." (Marks-Krpan, 2013, p. viii) This paper takes readers through a narrative inquiry into the role of writing in mathematics, and how creating opportunities for students to write-down their mathematical ideas and the procedures they undergo in solving a problem 'in their own words' can increase motivation and interest in the study of mathematics and at the same time help in developing a deeper understanding of concepts in mathematics, and the skills associated with 'learning how to learn' in a safe and collaborative learning environment.

THE ROLE OF WRITING IN MATHEMATICS

The benefits of engaging students in exploring their thinking through writing are recognized in many subject areas. The National Council of Teachers of Mathematics (2000) notes that writing needs to be an essential part of a mathematics program as it enables students to explore their own mathematical ideas and build on those of others. Unlike mathematical discourse, student writing can also provide documentation of thinking that they can later revisit, to see how their understanding of a specific concept has changed and share these ideas with their peers during collaborative group engagements. When students first arrive at school, they often lack personal experiences discussing, reading, and writing about mathematics. Students require opportunities to discuss their ideas from an early age before expressing them in writing. Mathematical discourse plays an important role in developing the foundation for writing skills in mathematics as it allows students to consolidate their thinking before they begin to write (Ediger, 2006).

Some teachers assume that if students know how to write in language arts, they will know how to write in mathematics. Mathematical writing, however, involves learning processes that may be unfamiliar to students (Marks-Krpan, 2013). Students are required to analyze, make conjectures, use specific mathematical vocabulary, and explore their mathematical reasoning. In addition, students need to be comfortable working with special symbols and numbers to represent abstract mathematical concepts and represent their thinking process. As with writing in language arts, students will require explicit instruction and a lot of opportunities to practice how to effectively communicate their mathematical ideas in writing.

RESEARCH METHODOLOGY

Narrative inquiry is based on the premise that we understand or make sense of our experiences through narrative (Bruner, 1990) and is a research approach that focuses on the use of stories to generate data. The idea of narrative inquiry is that stories are collected as a means of understanding experience as lived and told, through both research and literature. In narrative inquiry the researcher does not observe the participant objectively; instead it is important that the researcher takes a subjective position in connecting relationally with the participants' social, cultural and environmental influences on their experience. Narrative inquiry is a way of understanding and inquiring into experience through "collaboration between researcher and participants, over time, in a place or series of places, and in social interaction with milieus" (Clandinin & Connelly, 2000, p. 20). This inquiry process, analysis, and interpretation begins with field texts (initial data sources), which are later used to formulate research texts which are narrative accounts of the interpretations of the different experiences. These narrative accounts are then used to inquire into resonant threads or patterns that could be discerned from them. Exploration within this three-dimensional narrative inquiry space (time, space, social) allows the researcher to study the complexity of the relational composition of the teacher and students' experiences in and out of the mathematics classroom.

MATHEMATICS AND WRITING: A NARRATIVE INQUIRY

As a teacher and a reflective practitioner, I have often wondered about the role of literacy (reading and writing) in mathematics in helping to develop mathematical literacy (competence or knowledge in the subject). If it is to ensure students can read and interpret word problems better, should this not be the responsibility of the language teachers to support students through this development? Working in an authorized International Baccalaureate (IB) World School, there is an expectation that 'all teachers are language teachers' however during my first few years of teaching in the IB, it was not quite clear to me if 'language' was referring to the language of instruction in a mathematics classroom, or about mathematics as a language in general. It was not until my exposure to writer's workshop (Thompson Writing Program, 2013) and its emphasis on building an environment of sharing and collaboration that the dots began to connect, and the gears began to turn inside my head allowing me to see that 'language teaching' in mathematics actually refers to both teaching to support development in the language of instruction and mathematics as a language. Language is what helps us formulate concepts in our minds, and these concepts are the building blocks of understanding. It was then that I began to understand why it is so important for students to get in the habit of writing down their thoughts freely 'in their own words', to help in deepening understanding in mathematics.

Language learning and teaching in some respects is another way of describing constructivist learning and teaching in the sense that language is both the tool and the medium used to promote the active and ongoing construction of new understandings. Constructivism as a learning theory assumes that "knowledge" of any kind does not exist independent of the knower. Knowledge is rather something that is constructed individually or socially to make sense of the world around us during the learning process. This is carried out through experiencing things and reflecting on those experiences. In the constructivist worldview, learning involves the active creation of knowledge, and therefore it is important that these constructivist learners are provided with ample opportunities to continually ask questions, explore new possibilities and findings, and assess what they know. A constructivist classroom, "provides students with opportunities to develop deep understandings of the material, internalize it, understand the nature of knowledge development and develop complex cognitive maps that connect together bodies of knowledge and understanding" (Richardson, 2003, p.1628) however since its inception there has been active debate over its effectiveness as a learning theory in various educational circles around the world, as well as on whether or not it is possible to develop a prescribed teaching methodology for it.

Supporters of what is known as 'direct instruction' or the 'transmission model' of teaching, where the teaching and learning happens when teachers are seen as the bearers of all knowledge telling the students what they need to know, appear to be skeptical of constructivist pedagogy stating that they are "critical of approaches that involve students being left by teachers to find things out for themselves" (Taber, 2011, p.257) however I believe this is a very big misconception of what it means to teach using the constructivist approaches in a classroom. My understanding of constructivist approaches is that students are provided with opportunities to unpack what they know and share that knowledge with others and in the process "construct" new knowledge and compare that with the accepted knowledge published in various subject-area textbooks. It is however not a free for all as some people claim it to be. Students are not simply left to their own devices, because then I fail to see how it could even be considered as teaching. There needs to be a certain level of directed guidance by the teacher as facilitator, to lead the students towards the desired path. The desired path being, the awareness of and the ability to critically analyze how knowledge is gained and accepted by oneself and by society in the different subject disciplines. Thompson suggests that "constructivism is not a theory of learning but a model of knowing, and constructivism may be used to build a theory of learning." (Thompson, 2000, p.414), and I believe this is where the heart of constructivist pedagogy lies and why 'writing' is so important in teaching mathematics. As students communicate their mathematical thoughts through writing, they make implicit learning explicit, allowing educators to see how they understand and make connections between mathematical concepts.

Returning back to the idea of 'all teachers are language teachers', the transformation observed in students during collaborative engagements after making it a requirement for students to keep a journal of their learning process as a reflection tool and as a source for prompts during group discussions, was an empowering experience. It has allowed me to see firsthand how the simple act of documenting thoughts onto paper can help students to learn more conceptually in mathematics and interact more objectively and critically with the knowledge they gain.

Carter (2009) in her paper on connecting mathematics and writing workshop describes how bringing the two curricular areas together, namely writing and mathematics resulted in improvements in both areas. "Adding writing in reflective journals to my daily routine in math class extended my students' thinking about the strategies they use to problem solve in math class." (Carter, 2009, p.610) Her incorporation of journal writing to keep logs of their mathematical exploration, put students in a position where they had to explain their strategy or thinking to the rest of the class just as a teacher might explain a lesson. She found also that allowing students to justify their explanations using examples based on their own experiences provided a non-threatening climate in the room where everyone's ideas and ways of thinking are valued and respected. Wilcox and Monroe (2011) provide further examples in their paper on ways in which writing can be integrated and support the learning of mathematics. I particularly liked the "Think-Write-Share" strategy that "involves everyone in the class by allowing think time and expecting students to write before sharing their answer to a teacher question." (Wilcox & Monroe, 2011, p.522) I thought this was a great idea, and one that not only allows students to share their own understandings and preconceptions in a non-threatening way, but it also puts students accountable for their own mathematical understanding. After a number of student answers were shared and discussed, the students were asked to write their answers again, by revising their original answer in their notebook to reflect on their increased understanding. I like this, and this is a great way of supporting higher-order cognitive thinking. Strategies like these will undoubtedly support students in internalizing what they are being taught, with all the opportunities they will be provided with to think in their own way into the actual math, and in the process, making it more meaningful to them through the active process of writing. It also supports a learner-centered classroom where students are given choices and a more active role in the classroom allowing them to experiment, analyze, and discover solutions on their own, all the while benefiting from the diversity of learners and their individual input.

I see writing now as an essential and very important tool in supporting this process. Writing can help in organizing one's thoughts, and not only that but also to keep careful record of the changes in thinking over time, and an explicit and accessible medium for students to share their ideas and understandings with their peers. What is key to all of this I think is the environment of collaboration that writing can open up for students to allow for time to share ideas and ways of thinking which can further fuel the inquiry within each individual. Peterson (2008) also stresses the importance of providing students with opportunities to engage in what she calls discovery writing, which is based on the view that "the process of writing helps learners think more deeply about ideas and information that they encounter when reading, listening, viewing, and moving about their worlds, which in turn leads to a fuller understanding of the information, than were they not to write about them". (Peterson, 2008, p.2) This may very well be what is missing in a 'traditional' mathematics teaching curriculum. Students' views of mathematics are often limited to numbers and procedural steps to remember (Aspinwall & Aspinwall, 2003). Writing about mathematics encourages students to investigate concepts from a variety of perspectives, broadening their understanding of what mathematics entails (Marks-Krpan, 2013). Writing also affords students time to reflect, reshape, and rework what they know. As they organize and revise their writing, students can learn more about their own mathematical thinking and identify possible gaps in their mathematical knowledge (McIntosh & Draper, 2001; Sousa, 2008). As one Year 7 student shared, "When I write about math, I really have to think about how I understand mathematics. I get to know myself in a different way."

Jaworski (1994) sees a necessity not only for mathematics teachers to facilitate constructivist learning environments for their students (to enable opportunities for meaningful collaboration in their classroom) but to also contribute as active members of the school's learning community as lifelong learners as well. Fosnot (1993) also sees constructivist approaches producing fruitful outcomes to help in shifting the focus of the collaborative engagements to meaning making and knowledge construction, with a view of knowledge as temporary, nonobjective, internally constructed, developmental, and socially and culturally mediated within a learning community. While constructivist theories of learning in mathematics may not explicitly address the need to bring writing into the classroom specifically, it does however reinforce the notion of learning as a social process, and the importance of facilitating learning opportunities for students to work collaboratively together and communicate their own ideas on a particular mathematical concept or problem. As Wathall (2016) suggests, "it is important for learners to not only be given opportunities to explain and describe their ideas and stages in learning but also to be given opportunities to listen and discuss descriptions and explanations." (Wathall, 2016, p. 38) This suggests that writing within a collaborative social context (rather than in isolation), may be what is needed to provide space for students to engage in meaningful inquiry and contribute to the collective construction of new understandings in a mathematics classroom.

CONCLUSION

Knowledge and understanding in mathematics, like many of the other disciplines cannot be transferred without the use of language. The constructivist theory of learning is based on the idea that learning is an active process of "creating" rather than simply acquiring knowledge, which is a model that I believe we all, as teachers, should consider when designing and re-designing lessons to fit the needs and the demands of the student, to provide a more conducive environment for more meaningful learning to happen in the classroom. The integration of writing exercises as a means to support understanding and discovery in mathematics should not be overlooked. If the goal of mathematics education is to develop students with the ability to think conceptually, transfer understandings across contexts and situations, then I believe teachers must engage their students both intellectually and emotionally, and most importantly to enjoy learning mathematics as they engage in their own inquiries, revealing the mystery and beauty of mathematics as a discipline and as a pursuit of knowledge one axiom at a time.

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