GPH-International Journal of Educational Research
(Online ISSN 3050-9599)
Volume 9 | Issue 10 | 2026
*Published by*: Global Publication House
*DOI:* https://doi.org/10.5281/zenodo.24395800
*Journal homepage:* https://gphjournal.org/index.php/er/index

# Design, fabrication and evaluation of an apparatus for investigating the conditions for maximum range of a projectile in teaching projectile motion (Physics 10)

***Phan Hồng Giang[a], Lưu Quang Hải *[b], Nguyễn Hương Quỳnh[b], Trương Thúy Hằng[a]***
[a] Physics teacher and research supervisor, Lương Ngọc Quyến High School, Thái Nguyên, Vietnam
[b] Student, Lương Ngọc Quyến High School, Thái Nguyên, Vietnam.
(Lưu Quang Hải: Class 10A11; Nguyễn Hương Quỳnh: Class 11A5)
* Corresponding author: hluu9255@gmail.com

## Abstract

The 2018 general education physics curriculum in Vietnam requires Grade 10 students to carry out a project to find the launch conditions that give the maximum range of an object thrown from a given height. The launching methods commonly used in classrooms do not keep the launch speed and launch angle stable between trials, and the national list of minimum teaching equipment for upper-secondary schools contains no apparatus dedicated to this topic. This study describes the design, fabrication and evaluation of a launcher that allows the launch angle, launch speed and launch height to be set independently. Six design criteria were defined, five launching options were compared, and a prototype driven by the limbs of a recreational crossbow was built from locally available materials at a cost of about 300,000 VND. Initial tests showed scattered landing points caused by a varying contact point between the string and the ball; a pusher pad with two wire hooks on which the string slides were added to keep the line of action on the tube axis. After this modification, the deviation between the set and the independently measured launch angle did not exceed 0.6°, the coefficient of variation of the range was 0.81% for horizontal launches and between 0.14% and 0.34% at 30°, 45° and 60°, and the measured ranges exceeded the values of the drag-free model by 1.5% to 1.9%. For the launch height of 1.10 m, the model gives an optimal angle of 43.1°; with an angle step of 15°, the data do not separate this value from 45°. Angle setting was identified as the main contributor to range uncertainty away from the optimal angle. Possible uses in project-based teaching and directions for further development are discussed.

## Keywords:

*Projectile motion; maximum range; low-cost apparatus; school physics laboratory; control of variables; 2018 general education curriculum.*

## 1. Introduction

The 2018 general education curriculum for physics in Vietnam is organised around the development of competencies. For the Grade 10 topic of projectile motion, it states a learning outcome that requires students to carry out a project or study to find the conditions under which an object thrown in air from a certain height reaches the greatest height or the greatest range

© 2026 Author(s). Published by Global Publication House (GPH).
*This is an Open Access article distributed under the terms of the CC BY 4.0 License.*

(Ministry of Education and Training [MOET], 2018). This outcome describes a research activity rather than the recall of a formula. To achieve it, students need equipment that allows them to change one quantity at a time, keep the other quantities constant, and obtain quantitative data that are reliable enough to support a conclusion. The ability to design and interpret such controlled comparisons, often described as the control-of-variables strategy, is a component of scientific reasoning that develops through opportunities to plan and evaluate experiments (Chen & Klahr, 1999), and laboratory work is one of the main settings in which these opportunities occur in school science (Hofstein & Lunetta, 2004).

The physics of the task makes this requirement concrete. The range of a projectile launched from a point at height $h$ above the landing plane depends jointly on the launch angle $\alpha$, the launch speed $v_0$ and the height $h$. When the launch and landing points lie on the same level, the range is greatest at 45°, and this is the result that students usually remember. When the launch point lies above the landing plane, the optimal angle is smaller than 45° and depends on the ratio $2gh/v_0^2$ (Halliday et al., 2014; Lichtenberg & Wills, 1978). Research on students' understanding of kinematics reports that students who appear to understand a concept in one representation or situation often have difficulty transferring it to related ones (Lichtenberger et al., 2017). Applying the 45° result outside the conditions under which it holds can be seen as one instance of this difficulty. An apparatus that makes the dependence of the range on $\alpha$, $v_0$ and $h$ measurable gives students evidence with which to examine the conditions of validity of this result.

Classroom practice, however, faces practical constraints. The methods commonly used to produce projectile motion in lessons, namely throwing by hand, shooting with a hand-held rubber band and releasing a ball along an inclined chute, do not keep the launch speed stable between trials and do not allow the launch angle to be set with a small error. Data collected in this way cannot separate the effects of the individual variables. The list of minimum teaching equipment for upper-secondary schools issued with Circular No. 39/2021/TT-BGDĐT contains no apparatus dedicated to projectile motion (MOET, 2021), and an informal survey by the authors at five upper-secondary schools in the province did not find such an apparatus in use. Commercial projectile launchers from overseas suppliers meet the technical requirements, but their price is beyond the equipment budget of most Vietnamese upper-secondary schools. The same circular allows teachers to use self-made teaching equipment in addition to the minimum list (MOET, 2021), which provides a basis for developing low-cost apparatus locally.

Recent Vietnamese studies have reported self-built apparatus for upper-secondary physics, for example a spectrometer that uses a diffraction grating and an ESP32-CAM module for teaching atomic line spectra (Nguyễn et al., 2026) and microcontroller and sensor kits connected to smartphones for investigating harmonic oscillation and circular motion (Quách & Trần, 2026). These studies indicate that apparatus built from locally available components can provide quantitative data for curriculum topics that are not covered by the minimum equipment list. In the sources reviewed for the present study, we did not find a self-made apparatus for the maximum-range learning outcome that allows the three launch variables to be set independently and that has been evaluated against the projectile model.

This paper reports the design, fabrication and evaluation of such an apparatus, developed as a student science research project at an upper-secondary school. The study addresses three research questions. RQ1: Does the apparatus meet the design criteria set for the stability of the launch speed, the accuracy of the launch angle, the adjustability of the launch height, the agreement with the projectile model, safety, and fabrication with school tools? RQ2: To what extent do the measured ranges agree with the drag-free projectile model, and which source of error contributes most to the uncertainty of the range? RQ3: With its present resolution, can the apparatus distinguish the optimal launch angle from 45° experimentally?

## 2. Methods

### *2.1. Theoretical model and design criteria*

Consider an object launched with speed $v_0$ at an angle $\alpha$ above the horizontal from a point at height $h$ above the ground. Air resistance is neglected and the gravitational acceleration $g$ is taken as constant, so the motion is the combination of uniform motion along the horizontal and uniformly accelerated motion along the vertical (Halliday et al., 2014; Lương, 2020). The origin O of the coordinate system is placed on the ground directly below the launch point, the $x$-axis is horizontal in the direction of launch and the $y$-axis points vertically upward (Figure 1). The initial conditions are $x_0 = 0$, $y_0 = h$, $v_{0x} = v_0\cos\alpha$ and $v_{0y} = v_0\sin\alpha$; the acceleration components are $a_x = 0$ and $a_y = -g$. The equations of motion are

$$x = v_0 \cos\alpha \cdot t \tag{1}$$

$$y = h + v_0 \sin\alpha \cdot t - \frac{gt^2}{2} \tag{2}$$

Eliminating $t$ and setting $y = 0$ gives the range

$$L = \frac{v_0 \cos\alpha}{g}\left[v_0 \sin\alpha + \sqrt{{v_0}^2 \sin^2\alpha + 2gh}\right] \tag{3}$$

Maximising Eq. (3) with respect to $\alpha$ gives the maximum range and the optimal angle $\alpha*$:

$$L_{max} = \frac{v_0}{g}\sqrt{{v_0}^2 + 2gh} \tag{4}$$

$$\tan\alpha^* = \frac{v_0}{\sqrt{{v_0}^2 + 2gh}} \tag{5}$$

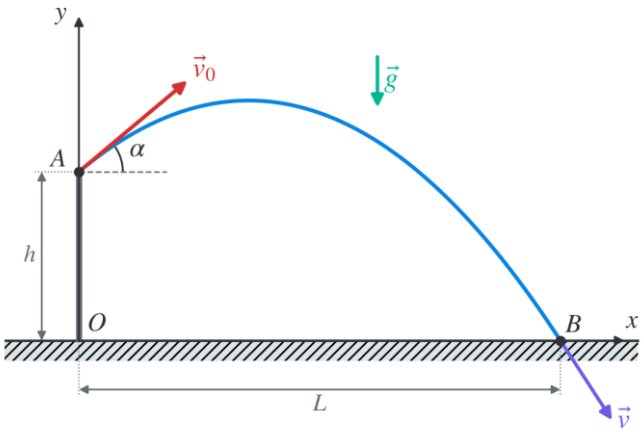



**Figure 1.** Motion of an object launched at angle $\alpha$ with speed $v_0$ from point A at height $h$; $L$ is the range measured from the foot O of the launch point to the landing point B.

For $h \to 0$, Eq. (5) gives $\alpha* \to 45°$. For $h > 0$, $\alpha*$ is smaller than 45°, and the difference increases with the dimensionless ratio $2gh/v_0^2$. For the values used in this study ($h$ = 1.10 m and $v_0 \approx 12.4$ m/s), this ratio is about 0.14 and $\alpha* \approx 43.1°$. The sensitivity of the range to the launch angle, $\partial L/\partial \alpha$, was computed numerically from Eq. (3) with these parameters. It is about 0.22 m per degree at 30° and 0.30 m per degree at 60°, and it approaches zero near $\alpha*$. This has two consequences for the design. First, locating $\alpha*$ experimentally requires a launch speed that changes little between trials and a fine angle step near the maximum. Second, away from $\alpha*$, an error in setting the angle can be a large contributor to the error in the range.

On this basis, six criteria were set for the apparatus. (C1) The launch speed is stable: its coefficient of variation is below 3%, checked by repeated horizontal launches. (C2) The launch angle is set and read with a deviation of no more than one scale division (1°), checked against a digital angle gauge. (C3) The launch height can be set at no fewer than four defined positions and is held firmly during firing. (C4) The measured range deviates from Eq. (3) by less than 10%. (C5) No part is ejected during firing and the moving parts are stopped mechanically, checked in supervised tests. (C6) All parts can be made with hand tools, a bench drill and a welding machine, checked against the record of materials and fabrication steps.

Five options for producing the launch speed were compared against these criteria. Throwing by hand and shooting with a hand-held rubber band are simple and need no construction, but the launch speed varies between trials and the angle is difficult to fix. A ball rolling down an inclined chute and leaving through a circular arc and an adjustable chute has a launch speed determined by the release height and an adjustable angle, but energy losses at the transitions and friction reduce and vary the speed, and the arc and joints require precise machining. A guide tube with a compressed spring gives a stable and adjustable launch speed with simple assembly, but the attainable speed is limited by the compression of the spring and the spring may lose stiffness with use. A pair of crossbow limbs driving the ball from the front gives a stable launch speed, an adjustable draw, an angle that is easy to set and hold, and a low cost through reuse of a recreational crossbow; its drawbacks are a more complex structure, the need for precise machining and the need for a safety stop. The crossbow-limb option was selected, and the spring option was retained as a possible alternative.

### *2.2. Design and fabrication of the apparatus*

The apparatus consists of five functional assemblies (Figure 2). The support assembly comprises a base and a vertical aluminium column (Ø32 × 2 mm, 1000 mm long) carrying a row of 15 locating holes of Ø8 mm at a pitch of 50 mm. The height-lock assembly comprises a sliding sleeve, a spring-loaded locating pin and a wing screw with a plastic pad, which fix the launch height at the positions defined by the holes. The pivot assembly allows the guide tube to rotate in a vertical plane to change the launch angle. The guide tube (outer/inner diameter 42/37 mm, 420 mm long) has two symmetric open slots in its side walls for the string, a viewing slot on its top that is narrower than the ball, and three catch marks inside that correspond to three draw levels. The launch assembly consists of two half-limbs fitted into a bow block through whose centre the guide tube passes (Figure 3). The projectiles are steel balls of 20 mm diameter.

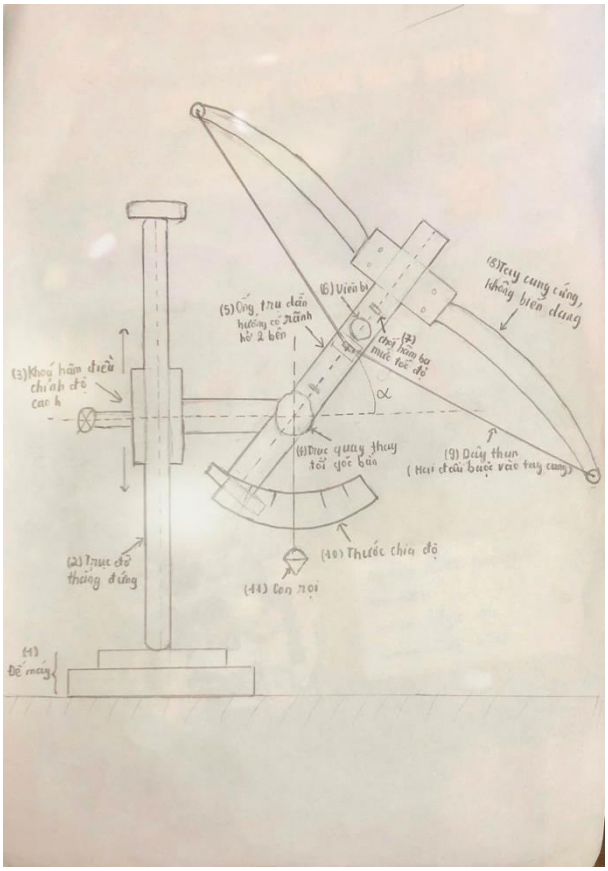



**Figure 2.** Schematic of the apparatus (original drawing; labels in Vietnamese). (1) base; (2) vertical support column; (3) height-adjustment lock; (4) pivot for changing the launch angle; (5) guide tube with two side slots; (6) steel ball; (7) catch with three draw levels; (8) rigid bow limbs; (9) string, both ends attached to the limb tips; (10) protractor; (11) plumb bob.

The launch angle is set and read with a protractor fixed rigidly to the guide tube and a plumb bob hung at the pivot. The protractor rotates with the tube while the plumb line remains vertical, so the graduation crossed by the plumb line gives the angle between the tube axis and the horizontal. Because this reading uses the direction of gravity as its reference, it is not affected by a floor that is not exactly level.

The holes in the column were drilled along one generatrix with a drilling jig to keep the pitch constant. The two side slots and the viewing slot were cut in the guide tube. The bow was separated into two half-limbs, which were fitted into two symmetric sockets in the bow block and held by a clamping cover and bolts. Before assembly, the two half-limbs were checked with a spring balance at the same draw length to ensure similar stiffness. A rubber stop pad in front of the bow block, a steel pin that prevents rotation of the bow block and a safety stop ring on the column were included for safe operation (Figure 3). All parts were made with a bench drill and hand tools from materials available in Vietnam: aluminium tubes for the column, sleeve and guide tube, a recreational crossbow for the limbs and string, locating pins, wing screws and bolts, a protractor, a plumb bob, a measuring tape, five steel balls and consumables such as paint, glue and carbon paper. The total cost of materials was about 300,000 VND. Figure 4 shows the finished apparatus.

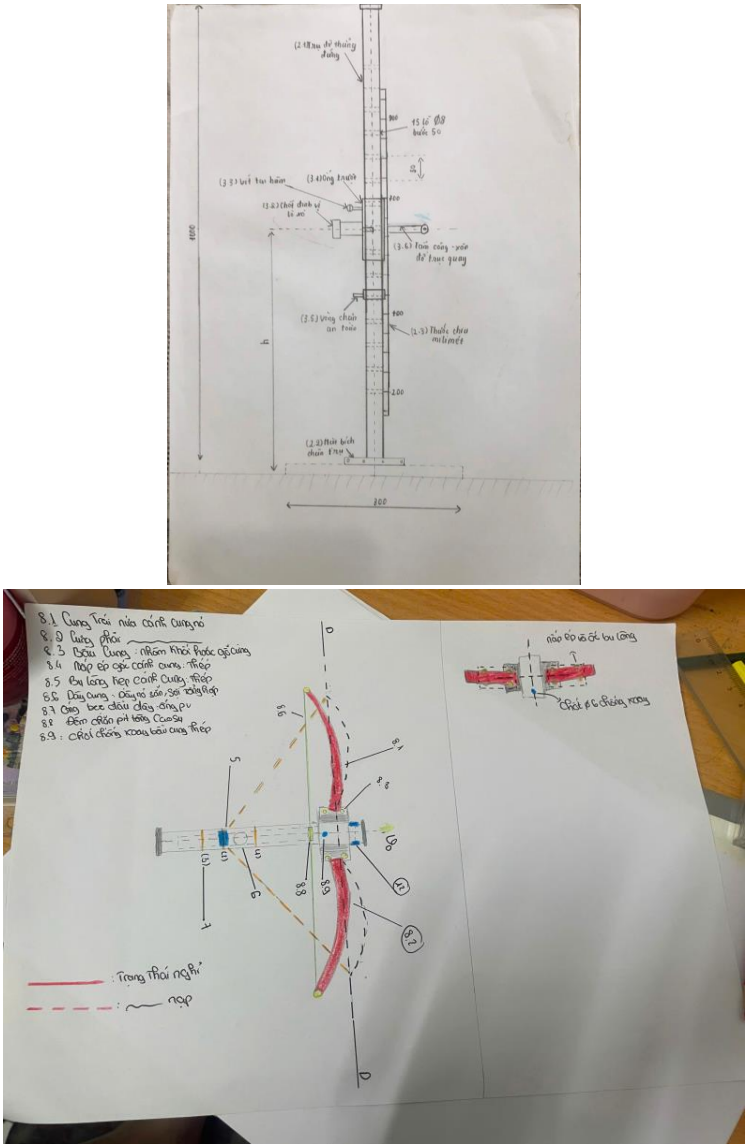


**Figure 3.** Detailed drawings (original; labels in Vietnamese, dimensions in mm). (a) Support column and height lock: 2.1 vertical column; 2.2 base flange; 2.3 millimetre scale; 3.1 sliding sleeve; 3.2 spring-loaded locating pin; 3.3 wing locking screw; 3.5 safety stop ring; 3.6 cantilever plate carrying the pivot; 15 holes Ø8 mm at 50 mm pitch. (b) Bow assembly: 8.1 and 8.2 left and right half-limbs; 8.3 bow block (aluminium block with milled limb sockets); 8.4 steel clamping cover; 8.5 steel clamping bolts; 8.6 string (synthetic fibre); 8.7 PU sleeve at the string ends; 8.8 rubber stop pad; 8.9 steel anti-rotation pin of the bow block. Solid line: rest position; dashed line: drawn position.

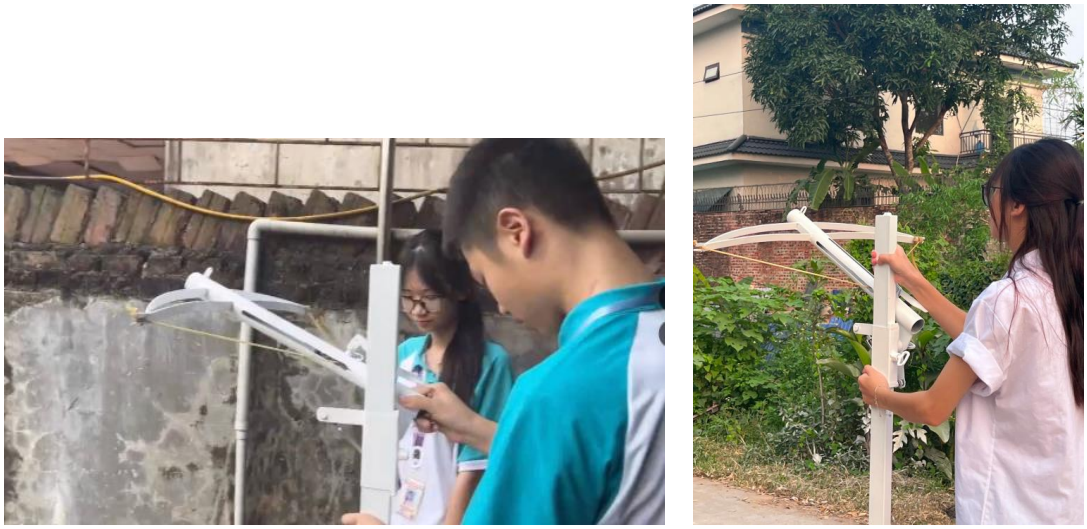


**Figure 4.** The apparatus after fabrication, during setting of the launch angle (left) and during loading (right).

### *2.3. Measurement procedure and data analysis*

The launch height $h$ was measured with a measuring tape from the tube axis to the floor; all measurements reported here were made at $h$ = 1.10 m and at the same catch level. The launch speed was determined indirectly from a horizontal launch: with $\alpha = 0$, the horizontal distance $L_0$ from the vertical projection of the muzzle to the landing point was measured and the launch speed was obtained from

$$v_0 = L_0\sqrt{\frac{g}{2h}} \tag{6}$$

The launch angle was set with the protractor and plumb bob and then checked with a digital angle gauge. To assess the accuracy of angle setting, each of the values 0°, 30°, 45° and 60° was set three times, and the tube was turned to another position between settings. The range was measured with a measuring tape from the vertical projection of the muzzle on the floor to the landing point, which was marked on carbon paper laid over the landing zone. Each condition was repeated five times. For each condition, the mean $M$, the standard deviation SD and the coefficient of variation were calculated, and the relative deviation of the mean from the model value $L_{\text{th}}$ given by Eq. (3) was computed:

$$CV = \frac{SD}{M} \times 100\% \tag{7}$$

$$\delta = \frac{M - L_{th}}{L_{th}} \times 100\% \tag{8}$$

The value $g$ = 9.81 m/s$^2$ was used throughout. The contribution of the angle-setting error to the range was estimated as

$$\Delta L_\alpha = \left|\frac{\partial L}{\partial \alpha}\right| \cdot \Delta\alpha \tag{9}$$

in which the partial derivative is taken in absolute value from Eq. (3) and $\Delta\alpha$ is the largest deviation found in the angle check. Slow-motion video recorded with a smartphone was used to observe the launch process during the initial tests, in line with the use of video recording to examine fast mechanical events in school physics (Brown & Cox, 2009).

## 3. Results

### *3.1. Initial testing and refinement of the launch mechanism*

In the first tests, the catch level, launch angle and launch height were kept constant, yet the ranges of successive launches differed noticeably and the landing points were scattered both along and across the launch direction. Observation through the viewing slot and slow-motion video showed that the string acted directly on the ball and that the point of contact changed from one launch to the next. Because the force was not applied on the tube axis, part of the energy went into rotation of the ball and a velocity component out of the launch plane appeared, so both the magnitude and the direction of the muzzle velocity varied. These observations are qualitative. When the trigger is released, the string pulls the hooks, the pad slides along the tube and pushes the ball. The modification acts in three ways. First, the force of the string is transmitted to the ball through a plane perpendicular to the tube axis, so the line of action of the force on the ball lies on the axis. Second, because the string slides freely in the two hooks, the tensions in the two branches of the string equalise, and the resultant force on the pad lies on the axis even when the string is slightly off-centre at loading. Third, the hooks protrude into the two side slots and prevent the pad from rotating about the axis. The front face of the pad has a spherical recess 2 mm deep that centres the ball; the depth was kept small so that the ball separates easily from the pad when the pad is stopped by the rubber stop pad. All measurements reported in Sections 3.2 and 3.3 were made after this modification.

### *3.2. Accuracy of angle setting and repeatability of the launch speed*

For set angles of 0°, 30°, 45° and 60°, the readings of the digital angle gauge over three settings were 0°–0.5°, 30.0°–30.5°, 45.0°–45.6° and 60.0°–60.5°, respectively. The largest deviation between the set and the measured angle was 0.6°, which corresponds to about half of the 1° division of the protractor. Criterion C2 is therefore met.

The five horizontal launches gave ranges from 5.82 m to 5.93 m, with $M = 5.87$ m, SD = 0.047 m and CV = 0.81% (Table 1). Equation (6) then gives $v_0 = 12.40$ m/s. Under the model, the range at $\alpha = 0$ is proportional to $v_0$, so 0.81% equals the coefficient of variation of the launch speed if other sources of scatter are negligible; this value is below the 3% threshold of criterion C1. Other sources, however, also act at $\alpha = 0$. The range of a horizontal launch is sensitive to the elevation of the barrel (about 0.27 m per degree, Table 1), so the observed SD would correspond to a variation of about 0.17° in barrel elevation between launches. The data at 45° give a complementary estimate: there the range depends little on the angle, whereas its sensitivity to the launch speed is high (a 1% change in $v_0$ changes the range by about 1.9%). The coefficient of variation of 0.14% at 45° therefore indicates that the launch speed varied between launches by about 0.1% or less. Taken together, the value of 0.81% can be treated as an upper bound for the variation of the launch speed; with five launches per condition, these estimates are approximate.

### *3.3. Range as a function of the launch angle and sources of error*

Table 1 and Figure 6 present the ranges measured at 0°, 30°, 45° and 60° together with the model values. At 30°, 45° and 60°, the coefficients of variation lay between 0.14% and 0.34%. The mean ranges exceeded the model values computed with $v_0 = 12.40$ m/s by 1.47% to 1.91%,

which is below the 10% threshold of criterion C4. The deviation has the same sign at the three angles and is larger than the scatter of the measurements (0.21 m to 0.29 m compared with SD values of 0.02 m to 0.05 m), so it is systematic. When a single launch speed is fitted to the data at 30°, 45° and 60°, the best-fitting value is 12.50 m/s; with this value the deviations at these angles lie within ±0.33%, but the model then predicts 5.92 m for the horizontal launch, 0.05 m (0.9%) more than the measured value. This difference corresponds to a downward offset of the barrel of about 0.18° during the horizontal launches.

**Table 1.** Ranges measured at different launch angles ($h$ = 1.10 m, same catch level, $n$ = 5 per angle) and comparison with Eq. (3)

| $\alpha$ (°) | Ranges of the five launches (m) | $M$ (m) | SD (m) | CV (%) | $L_{th}$ (m) | $\delta$ (%) | $\lvert\partial L/\partial\alpha\rvert$ (m/°) |
|---|---|---|---|---|---|---|---|
| 0 | 5.82, 5.93, 5.85, 5.91, 5.84 | 5.87 | 0.047 | 0.81 | 5.87[a] | – | 0.27 |
| 30 | 15.49, 15.61, 15.53, 15.60, 15.52 | 15.55 | 0.052 | 0.34 | 15.26 | +1.91 | 0.22 |
| 45 | 16.92, 16.98, 16.94, 16.97, 16.94 | 16.95 | 0.024 | 0.14 | 16.69 | +1.53 | 0.03 |
| 60 | 14.33, 14.43, 14.36, 14.41, 14.37 | 14.38 | 0.040 | 0.28 | 14.17 | +1.47 | 0.30 |

*Note.* $L_{th}$ was calculated from Eq. (3) with $v_0$ = 12.40 m/s, $h$ = 1.10 m and $g$ = 9.81 m/s$^2$. $\lvert\partial L/\partial\alpha\rvert$ is the sensitivity of the range to the launch angle calculated from Eq. (3). [a] The horizontal launches were used to determine $v_0$, so this value is not an independent comparison.

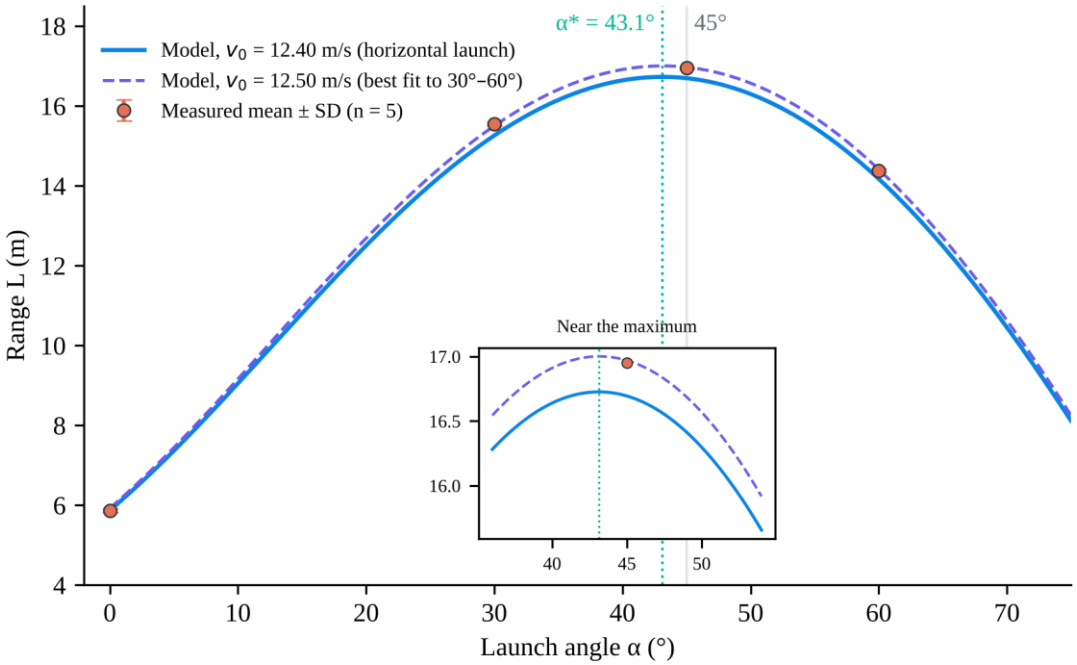



**Figure 6.** Measured ranges (individual launches in grey, means ± SD in orange) and the range given by Eq. (3) for the launch speed determined from the horizontal launches (solid line) and for the launch speed that best fits the data at 30°–60° (dashed line). The dotted line marks the optimal angle $\alpha* = 43.1°$ given by Eq. (5).

Among the four angles, the largest range was measured at 45° (16.95 m). Equation (5) gives $\alpha* = 43.1°$ for the present $h$ and $v_0$, and the difference between the ranges at $\alpha*$ and at 45° given by Eq. (3) is 0.03 m, which is of the same order as the SD at 45° (0.024 m). With an angle step of 15°, the measured data therefore cannot distinguish $\alpha*$ from 45°; such a distinction would require smaller angle steps near the maximum. What the data do show is that the range depends on the launch angle along a curve with a single maximum, in agreement with the shape predicted by Eq. (3).

Figure 7 compares, for each angle, the observed SD of the range, the change in range produced by an angle-setting deviation of 0.6° according to Eq. (9), and the systematic deviation from the model. The angle-setting contribution is 0.13 m at 30° and 0.18 m at 60°, larger than the random scatter of 0.04 m to 0.05 m at these angles, whereas at 45° it falls to 0.02 m because the range is insensitive to the angle near the maximum. Among the error sources evaluated, angle setting is therefore the largest contributor to the uncertainty of a single range measurement away from $\alpha*$, and its small effect near the maximum is consistent with the smallest SD being observed at 45°. Regarding the remaining criteria, the column provides 15 defined height positions (C3), the rubber stop pad, the anti-rotation pin and the safety stop ring address C5, and the fabrication with a bench drill and hand tools described in Section 2.2 addresses C6.

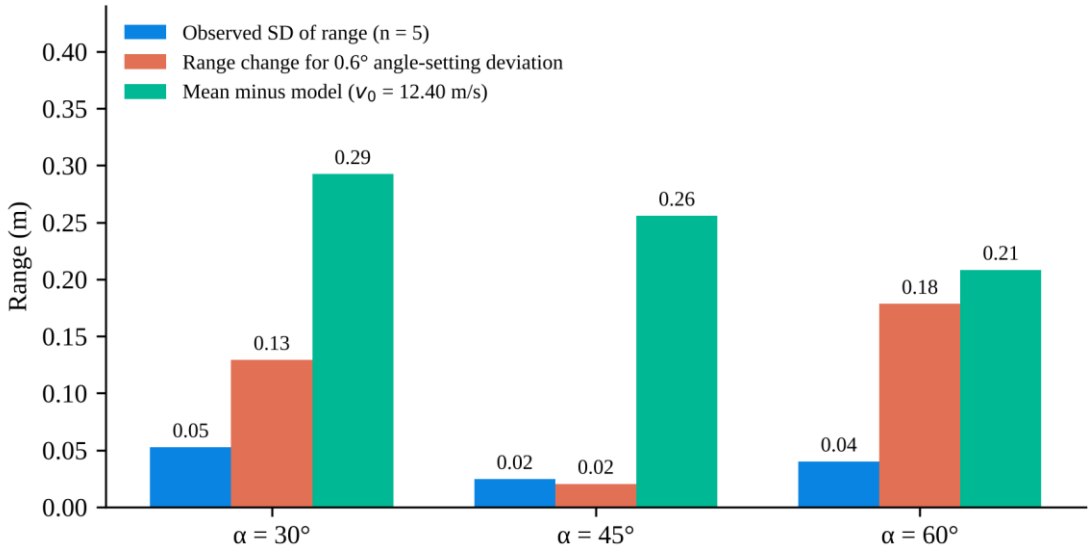



**Figure 7.** Observed standard deviation of the range, change in range produced by a 0.6° deviation in angle setting (Eq. 9), and deviation of the mean range from the model with $v_0$ = 12.40 m/s, at 30°, 45° and 60°.

## 4. Discussion

With respect to RQ1, the apparatus met criteria C1, C2 and C4 in quantitative tests: the launch speed varied by less than 1% between launches, the launch angle was set within 0.6° of an independent measurement, and the measured ranges deviated from the projectile model by less than 2%. Criteria C3, C5 and C6 were addressed through design features and the fabrication process. The ability of the height lock to hold its position during firing at all 15 positions and

the outcome of supervised safety tests were not documented quantitatively in this study and should be reported when the apparatus is evaluated further.

The pusher pad appears to be the element that made repeatable launches possible. The initial problem was not the stiffness or the draw length of the limbs but the geometry of force transmission: a string acting directly on a sphere applies its force at a point that depends on how the ball and string are placed at loading. The pad converts this arrangement into a piston-like drive in which the line of action is set by the geometry of the tube, the free sliding of the string in the hooks equalises the two string tensions, and the hooks running in the side slots prevent rotation. After the modification, the coefficient of variation of the range was below 1% at all four angles. Because the scatter before the modification was recorded only qualitatively, the size of the improvement cannot be stated numerically.

With respect to RQ2, the measured ranges were 1.5% to 1.9% larger than the drag-free model when the launch speed obtained from the horizontal launches was used. Air resistance, which the model neglects, would reduce rather than increase the range, so it does not explain the sign of this deviation. Two explanations are consistent with the data. The first concerns the launch point. The height $h$ was measured at the tube axis, and at $\alpha = 0$ this coincides with the height of the muzzle; when the tube is inclined about the pivot, the muzzle rises by approximately $\ell \sin \alpha$, where $\ell$ is the distance from the pivot to the muzzle. Reproducing the observed deviations with Eq. (3) would require $\ell \approx 0.41$ m to 0.45 m at the three angles, which is of the same order as the 420 mm length of the guide tube; $\ell$ was not recorded, so this explanation cannot be confirmed. The second concerns the determination of $v_0$. If the barrel pointed slightly downward during the horizontal launches, by about 0.18°, the launch speed would be underestimated, and a launch speed of 12.50 m/s would describe the data at 30°–60° within 0.33%. Such an offset lies within the range of the angle check at 0°, where one of the three readings was 0.5°. Measuring the muzzle height at each angle and measuring $v_0$ directly, for example with a pair of light gates at the muzzle, would allow the two explanations to be separated. The error analysis also indicates that angle setting is the main contributor to the uncertainty of a single measurement away from the optimal angle, which identifies a finer angle scale or an electronic angle sensor as the next improvement.

With respect to RQ3, the apparatus in its present configuration does not distinguish the optimal angle of 43.1° from 45°, because the expected difference in range (0.03 m) is comparable to the scatter of the measurements and the angle step was 15°. Two routes follow from Eq. (5). One is to measure with steps of 2° to 3° between about 38° and 50°. The other is to work under conditions in which the ratio $2gh/v_0^2$ is larger, so that $\alpha*$ lies further from 45°: according to Eq. (5), with $h = 1.10$ m, a launch speed of about 4 m/s would give $\alpha* \approx 33°$. Whether the lower draw levels of the apparatus give launch speeds in this range was not tested in this study. For teaching, the flat top of the range curve can itself be used as evidence: students can observe that ranges measured a few degrees on either side of the maximum differ by less than the scatter of the measurements, which bears on how a conclusion about an optimum should be stated.

The apparatus can be used for the Grade 10 projectile motion topic as a learning project in which students propose a hypothesis about the optimal angle, plan how to change one variable while holding the others constant, collect repeated measurements, plot the range against the

angle and compare their data with Eq. (3). This sequence corresponds to the research activity described in the curriculum (MOET, 2018) and gives students practice in the control-of-variables strategy (Chen & Klahr, 1999). In science club activities, the deviation between the measured ranges and the drag-free model could be used as a starting point for studying the influence of the launch geometry and of air resistance. Because all parts can be made with a bench drill and hand tools from materials available in Vietnam, a subject group of teachers or a group of students could reproduce the apparatus from the drawings, and Circular No. 39/2021/TT-BGDĐT provides a basis for using such self-made equipment (MOET, 2021). Compared with the electronic apparatus reported in recent Vietnamese studies (Nguyễn et al., 2026; Quách & Trần, 2026), the present design relies on a mechanical launch and a gravity-referenced angle reading; electronic sensing could be added later to measure the launch speed directly.

The study has several limitations. All reported measurements were made at a single launch height and a single draw level, with one type of projectile and five launches per condition. A series in which the launch height was varied was begun but is not reported here because it does not yet cover the full range of the column. The model neglects air resistance and treats the launch point as fixed. The launch speed was determined indirectly. The evaluation concerns the technical performance of the apparatus; no data on students' learning were collected, and the evaluation of the apparatus by teachers and experts has not yet been completed. Claims about the educational value of the apparatus therefore remain to be examined in classroom trials.

## 5. Conclusion

A launcher that allows the launch angle, launch speed and launch height to be set independently was designed and fabricated from locally available materials at a cost of about 300,000 VND. A pusher pad with two wire hooks on which the string slides was introduced to keep the line of action of the string on the tube axis. After this modification, the deviation of the set launch angle from an independent measurement did not exceed 0.6°, the coefficient of variation of the range was below 1%, and the measured ranges at 30°, 45° and 60° exceeded the values of the drag-free model by 1.5% to 1.9%. The data reproduce the single-maximum dependence of the range on the launch angle, but with a 15° step they do not separate the optimal angle of 43.1° predicted for the launch height of 1.10 m from 45°. Further work will replace the protractor with a finer scale or an angle sensor, measure the muzzle height at each angle, measure the launch speed directly with light gates, use angle steps of 2° to 3° near the maximum, complete the series with varying launch height, and evaluate the use of the apparatus in classroom projects.

## References

Brown, D., & Cox, A. J. (2009). Innovative uses of video analysis. *The Physics Teacher, 47*(3), 145–150. https://doi.org/10.1119/1.3081296

Chen, Z., & Klahr, D. (1999). All other things being equal: Acquisition and transfer of the control of variables strategy. *Child Development, 70*(5), 1098–1120. https://doi.org/10.1111/1467-8624.00081

Halliday, D., Resnick, R., & Walker, J. (2014). *Fundamentals of physics* (10th ed.). John Wiley & Sons.

Hofstein, A., & Lunetta, V. N. (2004). The laboratory in science education: Foundations for the twenty-first century. *Science Education, 88*(1), 28–54. https://doi.org/10.1002/sce.10106

Lichtenberg, D. B., & Wills, J. G. (1978). Maximizing the range of the shot put. *American Journal of Physics, 46*(5), 546–549. https://doi.org/10.1119/1.11258

Lichtenberger, A., Wagner, C., Hofer, S. I., Stern, E., & Vaterlaus, A. (2017). Validation and structural analysis of the kinematics concept test. *Physical Review Physics Education Research, 13*(1), Article 010115. https://doi.org/10.1103/PhysRevPhysEducRes.13.010115

Lương, D. B. (Ed.). (2020). *Vật lí đại cương, tập 1: Cơ học và nhiệt học* [General physics, Vol. 1: Mechanics and thermodynamics]. Vietnam Education Publishing House.

Ministry of Education and Training. (2018). *Chương trình giáo dục phổ thông môn Vật lí* [General education curriculum: Physics] (Issued with Circular No. 32/2018/TT-BGDĐT). https://thuvienphapluat.vn/van-ban/Giao-duc/Circular-32-2018-TT-BGDDT-promulgating-generaleducation-program-519827.aspx

Ministry of Education and Training. (2021). *Thông tư số 39/2021/TT-BGDĐT ban hành Danh mục thiết bị dạy học tối thiểu cấp Trung học phổ thông* [Circular No. 39/2021/TT-BGDĐT on the list of minimum teaching equipment for upper-secondary schools]. https://thuvienphapluat.vn/van-ban/Giao-duc/Thong-tu-39-2021-TT-BGDDT-thiet-bi-day-hoc-toi-thieu-cap-Trung-hoc-pho-thong-499697.aspx

Nguyễn, T. L., Trần, Đ. Q. D., & Đoàn, T. Y. L. (2026). Nghiên cứu, thiết kế và chế tạo bộ thí nghiệm máy đo phổ phục vụ dạy học nội dung "Quang phổ vạch của nguyên tử" thuộc chuyên đề "Vật lí lượng tử" lớp 12 [Design and fabrication of a spectrometer kit for teaching atomic line spectra in the Grade 12 quantum physics module]. *Tạp chí Khoa học Trường Đại học Sư phạm Thành phố Hồ Chí Minh, 23*(5), 1005–1017. https://doi.org/10.54607/hcmue.js.23.5.5332(2026)

Quách, U. L., & Trần, V. L. (2026). Ứng dụng vi điều khiển và cảm biến để chế tạo các bộ thí nghiệm khảo sát kết nối với thiết bị thông minh vận dụng trong dạy học Vật lí trung học phổ thông theo Chương trình 2018 [Using microcontrollers and sensors to build experiment kits connected to smart devices for teaching upper-secondary physics under the 2018 curriculum]. *Tạp chí Khoa học Trường Đại học Sư phạm Thành phố Hồ Chí Minh, 23*(SI1), 432–443. https://doi.org/10.54607/hcmue.js.23.SI1.5422(2026)