Universes symmetries for energies

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From the research for particles in physics it is clear that discrete symmetries guide their existence. One Higgs boson H1 is found on particle base, gravitational waves GRW are found. There is a particle-wave duality postulated in physics. Hence H1 needs a frequency and GRW a particle presentation. Studying the four basic forces of physics, mass and gravity GR has not a special symmetry. The standard model of physics adds to the weak interaction WI the SU(2) symmetry, to the strong interaction the SI SU(3) symmetry and to the electromagnetic interaction EMI a circle U(1).
In developing shapes and geometries for the particles of U(1), SU(2), SU(3) it is clear that the standard model relates on the number of generators for its symmetry. The purpose of this article is to look at suitable geometries in different dimensions and for noncommutative measures which can include also gravity and mass such that the standard model can be extended as a well-understood old theory. Some modifiactions are necessary since for instance neutral leptons are found having mass and some old claims have been that they have mass 0.

Section 1 Introduction

First we make some remarks for a universes existence. The model is taken from two colliding galaxies:

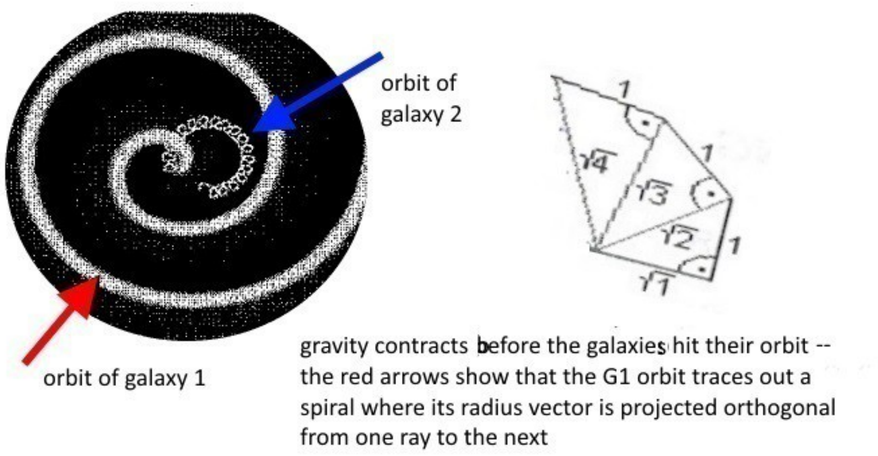
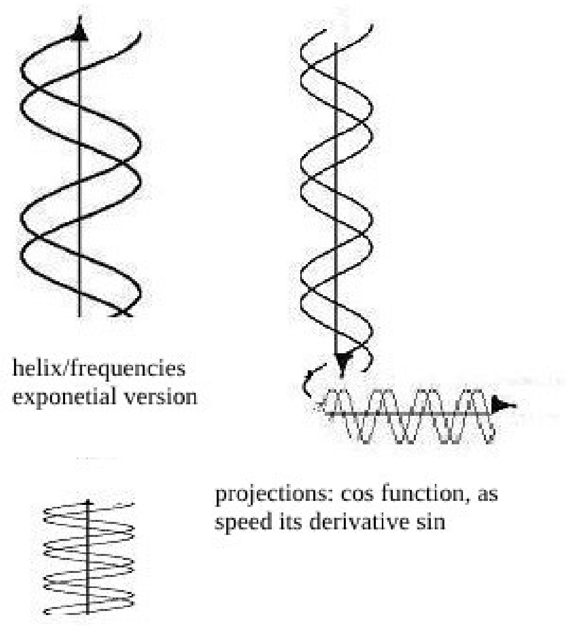
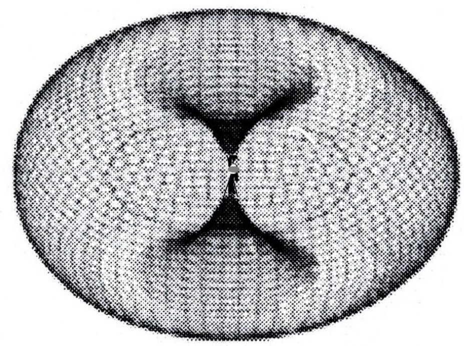


Figure 0 Collision of galaxies, spiralic contraction

There is no view for a H1 Higgs boson in physics which is a particle carrying some energy of the colliding huge galaxies for a universe. We take as geometry a frequency f presentation of mass m, using $mc^2 = hf$, c speed of light, h Planck constant. As for light waves the mass energy f is rolled up as helix on a finite cylinder which for H1 is closed to a Horn torus with a singular point in the middle.





For the singular point ∞ take from Lissajous hitting two orthogonal frequencies at $t = t0$ and from the phase angle on the vertical y-axis the radius $r(t0)= \sin^2\varphi$ which is then also the radius of the two cyclinders on which the frequencies expand in time; as time moves to $t = t1>t0$ let the radius $r(t) \rightarrow 0 = r(t1)$; then the two cylinders are closed at infinity by a Minkowski (double) cone with surface $r(t)^2=c^2(t - t1)^2$ and with tip ∞ (figure 4)

Figure 1 helix and Horn torus

There are 3 kinds of 2-dimensional surfaces, the first one below rotates a loop/circle about a

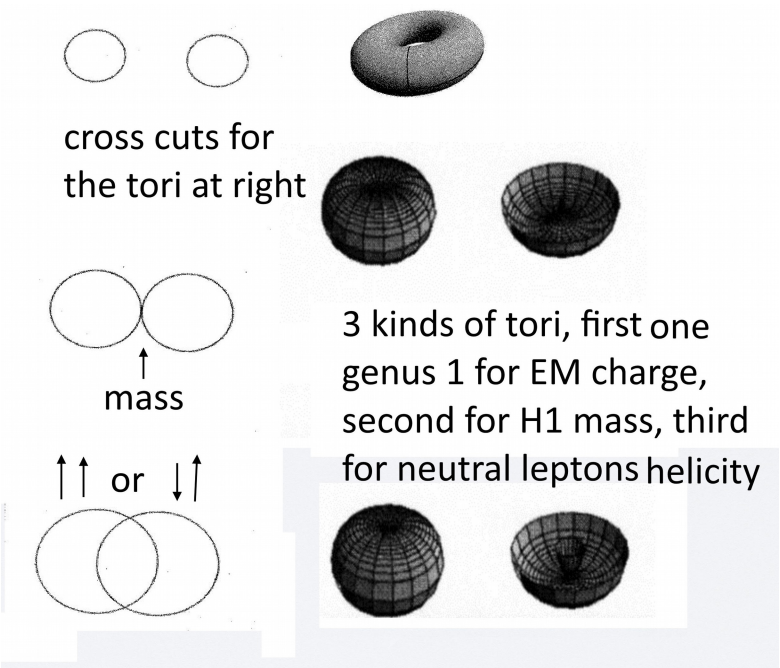


Figure 2 the inverse Hopf map torus

second loop C. If C is contracted, the rotated cross cut circles touch for a Horn torus, a further contraction shows the spindle geometry of neutral leptons with helicity. H1 is for heavy mass with spin 0. Since neutral leptons are taken for presenting part of the dark matter in the universe, H1 is also taken for dark matter. If it gets a push or pull by some interactions, we add as for neutral leptons a spin to H1 with two orientations for inertial mass as Higgs bosons H2, H3. The momentum vector of the helicity is for H2, H3 an external force acting on these Higgs bosons. Since H1 as dedected by CERN we postulate its fast decay. In the Hopf geometry the mass charge location is a leaning rotating circle which fills out the torus, blown up from a scalar weight at an initial point of a momentum vector for mass on the Hopf sphere S^2. In the Tool Bag this is replaced by a spring, representing graviton waves frequency. If we change dimensions, H1 is presented as 5-dimensional sphere S^5 in a complex space C^3. The blow up dimensions are knwon for SI with S^5 belonging to its geometry. The Horn torus belongs to the SU(2) WI geometry as S^3 decay products. This S^5 from SI is taken in its stereographic projection for a 5-dimensional vector field of Schmutzer [5]. It is a common field for electromagnetism EM and GR. With its projective projector the field decays into three 4-dimensional fields, one for EM and the second GR. Added is a third scalar field. We take for the EM, GR fields a quark particle presentation: they have an EM and a mass charge as poles on a 2-dimensional brezel surface of genus 2. Two potential energies POT are generated, one for EM and the second for GR. As charge they have a potential field where the force vector is integrated mathematically as second derivative. Potentials are first derivates and use the scaled inverse radius 1/r about the pole for their geometry, projected into the universes space.

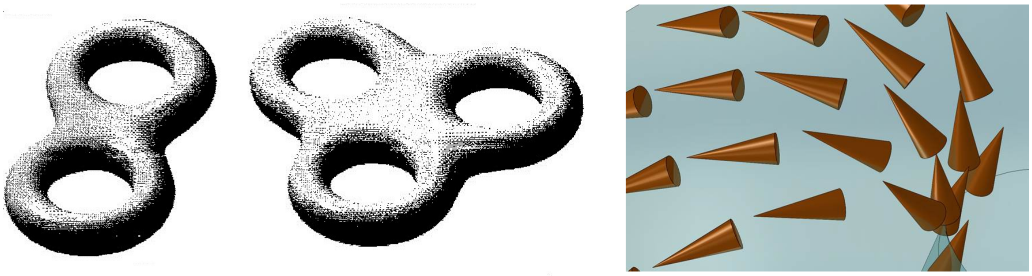


Figure 3 Heegard splittings of S^3 into 2 quark brezel of genus 2 with 2 poles and into 2 nucleon handle bodies of genus 3 where only the mass poles of 3 quarks are shown; S^3 can be splitted in general into 2 handle bodies of the same genus with their surfaces identified. As a replacement for a wave presentation the annihilation operator (it lowers the number of particles in a given state by one) and a generation operator (it increases the number of particles in a given state by one) are used in physics. They can be applied to the Heegard splittings, to Feynman diagrams for colliding particles and to other kinds of decay processes. At right: magnetic field quantum whirls.

Section 2 Symmetries and Geometries

We started a new universe with the Higgs bosons as new energy carriers and their decay as parts of a 3-dimensional projected S^3 into quarks or nucleons and fields. For a description of other energies and other particles it is necessary to introduce now geometrically the vectorial based presentation of SU(3) with the trivial fiber bundle S^3xS^5 and fiber S^1 (a loop/circle), an octonian 8-dimensional vectorspace H. The coordinates are named j = 0,1,2...,7 if no confusion occurs, otherwise ej. The coordinate 0 sets somewhere in H a vector e0, having an initial and an endpoint and a direction where its energy acts in H.
Dihedral groups Dn, n = 1,2,3,4,6 can be used for generating the octonians and energies. They are transformations of an n-edge in a (complex) plane. D1 = Z2 with matrices id, -id has an up-down rotation of an octonian e0 vector by π on a real u-axis. It can also be presented as a circle U(1) having as symmetry two clockwise cw and counterclockwise mpo orientations. As projective line it has a point ∞ for stereographic projecting the circle of D1 to a real number line. The e7 circle U(1) is cut at the angle $0 = 2\pi$ to a second parallel (not aligned) interval I to the vector e0 in a complex

plane and D2 = Z2xZ2 (id, rotation by π, two reflections at the u-,w-axes) applies to e0, I for the C,P, T operators of physics. If the two sides e0, I with endpoints pasted for bounding a disk in a plane are identified as lines, a Riemannian sphere S^2 is obtained having for instance the two points 0,∞ as north/south poles. Using the Heisenberg couplings of coordinates in the hedgehog for S^2 (figure 8) and deuteron, the remaining SI (C^3) coordinates ej, j = 1,...,6 of the octonians are generated. More details for a Cayley-Dickson constructions of coordinate dimensions are found in [1] and in section 3. Drawn as 6-edge (figure 7 is for color charges), the symmetry D6 = D3xZ2 uses for Z2 the conjugation operator C. A spin-like coordinate cross product triple GF sets 6 masses as weights for the quark series and C adds their duals for the 12 series. For leptons are two GF1, GF2 where GF1 sets the 3 masses for EM charges e,μ,τ and C sets their conjugates, GF2 sets the 3 masses as oscillation for neutral leptons νj and C adds their conjugates. For all Dj with j even one can paste the sides of this j-edge together in pairs and get surfaces for the inner location of particles, for instance the above torus as aba(-1)b(-1) or abca(-1)b(-1)c(-1) for the genus 3 surface of nucleons. The D3 symmetry as complex cross ratios and invariants under the MT symmetry of S^2 applies for six energy vectors, the nucleons quark triangle and the strong SI rotor (see [1]). D4 (4 rotations, 4 reflections) is for the SU(2) Pauli matrices symmetry and for the weak WI rotor. They are listed as octonian spacetime coordinates ej, j = 1,2,3 for space and e4 is added for time. The magnetic EM group applies for this (figure 4).

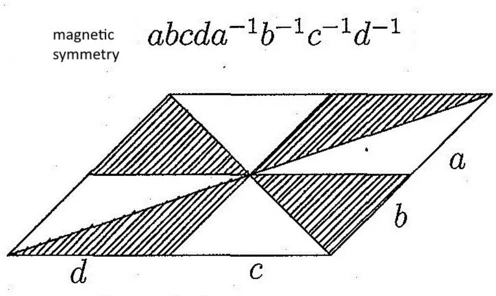
The D2 circle can be used for a 1-dimensional decay, decaying into a magnetic momentum vector and a quark-gluon handle (figure 9) with 2 quarks as verices and a gluon inerval confining the paired quarks. The above mentioned blow up S^2 of the D2, with 2 energy vector at the poles 0,∞ can present 2 color charge gluons of SI. The decay along an equator can be into 2 color charge caps. In the EM case this decay is for a magnetic field quantum whirl and a disc for a complex integration along the boundary of a function having an energy pole in its center. Use the residuation thery for instance. In figure 9, 10 the tetrahedron and ooctahedron triangulations add more poles on S^2.

If S^2 is Hopf blown up to S^3 also more poles can be added for its Heegard decomposition decays (handles, figure 3). If it decays into two solid 3-dimensional balls it can be for observable grids in space as demonstrated for nucleons or other baryons and mesons.

D3 is a nucleon triangles symmetry (figure 3 middle). It acts for generating barycentrical coordinates for GR. D4 generates the weak interaction SU(2) with 3 WI bosons W+, W-, Z0.

Its cross product as spin S = (Sx,Sy,Sz) generates in an deuteron atomic kernel through isospin exchanges between paired u-, d-quarks locally the Euclidean xyz-coordinates in the universes space (a WI rotor called wheel).

In use are also cylindrical coordinates and the light cone for EMI and Minkowski metric of the universes spacetime. Ψ-waves use exponential, periodic functions and e7 as U(1): roll a strip of the plane together to a cylinder and let the light frequency expand on it as helix. In projective geometry the cylinder Z is closed by one point ∞ not in Z. This gives another version H' of the Horn torus. It is alternatively obtained by closing the Minkowski light cone at infinity by a circle.



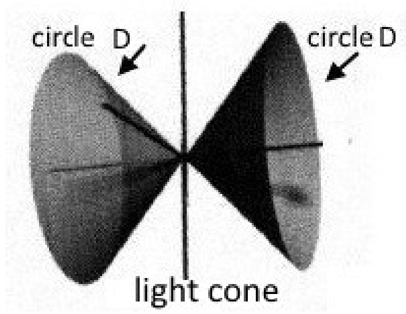


Figure 4 magnetic symmetry (a genus 4 surface) and light cone (Minkowski metric)

The difference between Euclidean WI generated spin coordnates and barycentrical SI conic generated coordinates can explain the discrete symmetry with the e0, I intervals of D2 or other changing geometries, for instance changing Bohr radii in atoms shells or discrete spin length values which start with spin ½ for fermions, spin 1 for bosons and spin 2 for gravitons. In the WI wheel the flat S^1 orbit counts winding numbers around a circle. In the complex numbers residuation theory they use the center O of S^1 where in this model the complex function to be integrated has a pole (figure 10 Lissajous, also the hedgehog). For SI the S, N marked conic whirls with tip P are counting also winding numbers and $O \neq P$ as center for its bounding S^1 circle is sitting on the cones axis. The pulsations of nucleons are attributed to this (figure 14 at right). They are generated by *rgb*-graviton whirls actions as neutral color charge of nucleons and heat. The use of exponential functions exp with discrete period 2π counts windings through integer numbers as $2\pi n$, n = 1,2,... where the cw or mpo orientation for a time expansion on S^1 can add negative integers. Time expanding exp waves or whirls have in the two WI, SI versions of figure 6 circular frequency $\omega = 2\pi f$. In cylindrical expansions of energy E = hf the exp expansion is drawn as (left or right handed spring) helix line on the tubes surface. The tube Z can have a line as linear rotation axis which coincides with its eigentime world line. Z can also be curved as in the Hopf geometry tubes for EM tori (figures 1,2). Let O be a center on the Z axis where the radius r of Z is measured for the S^1 cruss cut of the tube. Then the two D2 intervals e0, I are parallel to the axis at radii $r = r1 < r2$. If the energy E is changing such that no full winding of the helix line is obtained the absorbed or emitted surplus energy is changing discrete the radius numbers r1, r2. In an atoms shell this is for Bohr radii of electrons, for electrons spin the integer multiplication is used. For the emitted or absorbed light the spectral series of light are observed.

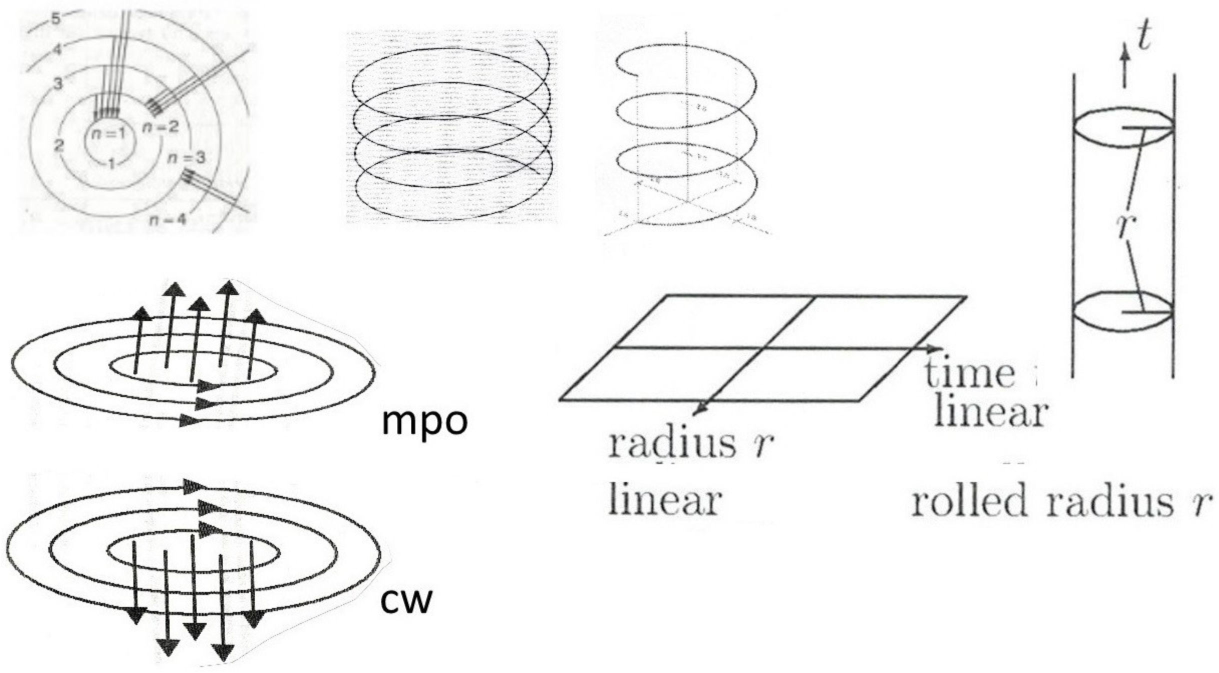


Figure 5 orientations and radius on surfaces or in space

For the helix lines on a cylinder one winding is also for the energies wave length λ and exp is observed as cosine projection in space. Radius changes the waves amplitude and wave length occurs in the equation speed $v = f \cdot \lambda = \omega / k$, $k = 2\pi / \lambda$ wave number. In figure 5 for orientations the spin GF Euclidean coordinate triple is with mpo on +x, +y, +z, in figure 10 left the orientation for an – EM charge is +x, +y, -z as it is also for angular momentum L = rxp. For the magnetic momentums changing orientation towards spin s the + EM charge has a mpo + sign in the gyromagnetic relation μ = γ·s. The gyromagnetic Landé factor is g = 2 for electrons eigen-angular momentum in $\gamma = g \cdot e_o/2m$. In general g = 2 for eigen-spin momentum and g = 1 for orbital (no spin involved) momentum, otherwise 1 ≤ g ≤ 2 holds. In a classical paradoxa of physics it is not taken into account that the eigenrotation of an electron is blown up from its Hopf sphere in figure 10 by a rotating transversal fiber S^1 which is not acting like en electrical current but traces out the torus surface where the electrical charge is then located, Another harmonic wave is listed for this below, using the torus equations. For neutral leptons the helicity couples the + spin either with momentum p = m(+v) or p = m(-v), m mass v speed vector, the eigen-spin momentum rotates with mpo spin for anti-neutrinos. For the EMI light cylinder the reflection of light at matter (figure 5) alllows to change with the broken world line to emit an energy part of its frequency, the inverse absorption of energy pasing by a huge matter system P is observed as double lensing for the light emitted from a star behind P. The added frequency energy can come from gravity. For the redshift of light is assumed that moving along its world line, the cylinder radius is changing using the discrete symmetry D2 (figure 5 right).For the observed neutrino oscillation it is assumed that its carrying mass GF carries all three possible masses at its vectors. They change stochastically in time like spin vectors in the Stern-Gerlach experiment the observable GF mass vector such that in the helicity of figure 6 the mass of the momentum p but not its direction changes. This GF has two of its vectors in undetermined direction not available for space observations. Also in the Stern-Gerlach experiment only one of the prepared spin coordinates is measured and changes after the performed experiment its direction in a stochastic up or down direction. The Copenhagen interpretation is used: systems generally have no definite properties before they are measured; quantum mechanics can only predict the probability distribution of a given measurement's possible results. The system can change states and give only one result as value after the measurement. - Added is by the author that the result can come from a multivalued function like the complex cross ratios, and vectors or matrices as values for instance can replace numerical scalars. The Stern-Gerlach experiment and the neutrino oscillation measures a vectors state as outcome of a measurement.

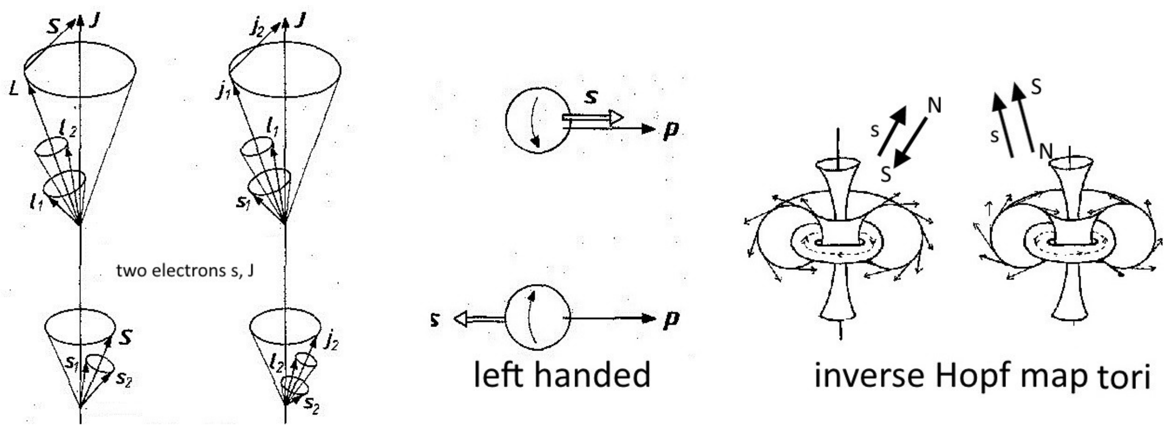


Figure 6 precision motion for electrons in atoms shells, coupling of spin and orbital rotational momentum, helicity of neutral leptons, S^3 Hopf figure for the figure 10 left Hopf sphere S^2; spin lets the leaning EM charged circles rotate on the torus, the eigenrotation is along the z-axis; the same spin action holds for figure 11 the heavy mass torus when spin is added for inertial mass

Concerning measurements and logic: useful are the seven 3-dimensional cross products of the octonians which are like spin triple vectors GF but carry other weights than spin length on its vectors. From the new symmetries introduced for the particle series only D1, D2 are commutative. It was suggested in 1936 by Birkhoff and von Neumann that Quantum Mechanics should use a finite dimensional real or complex projective geometry. Since projection operators onto subspaces

of H are not commuting, this adds that also the logical implications get restrictions. There is no subspace deduction or modus ponens theory, only weak replacements, for instance if the corresponding operators commute.
The H' is taken as local presentation for dark energy: inverted is mathematically speed v<c of systems in the universe to a dark energy speed v'>c with $v'v = c^2$. If this method of invertion through Moebius transformations MT 1/z of a Riemannian sphere is applied to H1 then its radius r is inverted at the dark matter radius r' at the Schwarzschild radius Rs for a mass with $r'r = (Rs)^2$.
For the postulated gravitons as particles there is open what they are. In the MINT-Wigris model they are presented as the neutral color charge of nucleons, as *rgb*-graviton whirls, the superposition of quarks 3 color charges red, green, blue. The Schwarzschild scaling factor of the general relativistic Schwarzschild metric is taken as an MT G of order 6 for a cyclic subgroup of D3xZ2. The G-compass of figure 7 shows how its discrete (with the sixth roots of unity of D6) turning needle generates the 6 color charges for SI and SU(3).

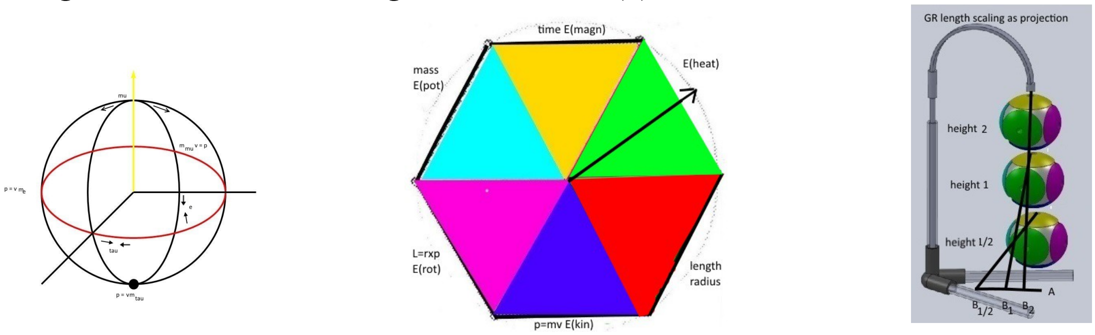


Figure 7 G-compass (middle), projective/stereographic map GR (right) length scaling by graviton waves and *rgb*-graviton whirls action; at left: mass GF for neutrinos 3 masses oscillation with a minimum my, at -y, a maximum my < mz at -z and using the real cross product, measuring the mz·my area at -x with the mass mx = 1 – mz – ((1-mz)/(1-mz)) = 1 -mz – my. For the SI rotor, p sits in up or down orientation on the nulceon triangles quark vertices, setting the same way for quarks a GF with the u,d,s masses, a second quark GF carries then the c,b,t masses. The Higgs fields diagonal matrix transformation generates with its 3 eigenvalues diag[M,1+M+m,m] the GF masses by associated von Neumann operators.

An energy vector interpretation of the sectors is that red carries EM, green carries heat, blue carries kinetic energy and momentum PM, yellow carries magnetism MG, turquoise carries mass GR, magenta carries angular momentum and rotation AM (figure 7).
In the first H1 bifurcation quoted above quarks arise with the POT energies EM and GR. EM bifurcates into MG and heat, GR bifurcates into PM and AM. After that, in a Feigenbaum bifurcation, 8 gluons are bifurcated for generating the SU(3) 8-dimensional space. Then heat chaos occurs in the early development of an universe. The linear octonian vector space is another presentation for SU(3).
The SU(3) GellMann projection matrices for 8 gluons are essentially 3-dimensional blown up Pauli matrix triples by adding a column and row with 0 coordinates. The first 3 for *rgb*-gravitons GF project the fiber S^3 of the SU(3) geometry S^3xS^5 down to the Hopf S^3 of SU(2). The S^5 geometry is projectively normed to a 2-dimensional complex inner spacetime CP^2 by a fiber S^1 for deuteron and nucleons. It is observable as a grid in the 4-dimensional spacetime of the physics universe. The GellMann blown up σ3 matrices are linearly dependent which are then listed as two λ3, λ8 matrices for 2 not 3 gluons of SU(3). λ1, λ4, λ6 are blown up σ1 matrices, λ2 λ5, λ7 blown up σ2 matrices.

Section 3 Mathematical Remarks

From the geometrical point of view, the affine Minowski space of physics and SU(2) has to be

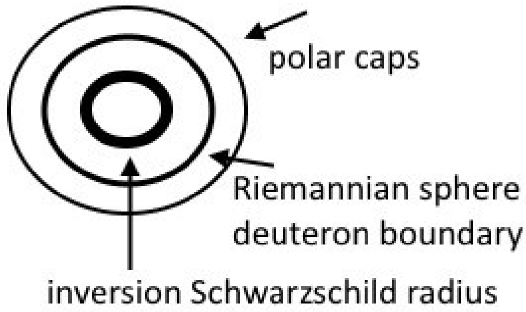
Cayley-Dickson extended to octonians as a vector space for energies. The dimensions occur in another presentation as the SU(3) geometry of SI. It projects down into the SU(2) symmetry and spacetime and adds a grid for atomic kernels and atoms locations. A dynamics inside such a grid is described in [1] and the handbook of the MINT-Wigris Tool Bag. For the differential and integration energy calculus the Gleason frames GF are recommended which are pairwise orthogonal octonian coordinate triples like the spin triples, carrying suitable weights. Spin has length as weights, neutral leptons have masses as 3 weights (for instance). The third important addition to quantum mechanics is that boundaries or shells as 2-dimensional Riemannian spheres S^2 are introduced by the CP^2 space for deuteron: there are three concentric spheres as Bohr shells, one with its Schwarzschild Rs radius, the next larger shell for its boundary and the third outer one as polar caps where it has in vectorial energy form its energy exchange with the universes outer environment (figure 8). S^2 has the MT as symmetry group which allows to add GR to the standard model of physics. Concerning the projective and projection geometry in this model we refer to other publications of the author (see the references below and articles of 2018). The coordinates use different number systems in 2x2-matrix form. The real numbers x arise as a stereographic projection of an U(1) loop to a line. The coordinates are added to a 2x2-matrix as x·id, the complex D2 plane extend this by an $\pi/2$ rotation matrix y·A with a11 = a22 = 0 and a12 = 1 = -a21 for u = x + iy, y real. coordinates. Twice the coordinate extensions are now in pairs of the already constructed numbers. D4 uses two complex numbers for C^2 in the matrix form a11 = z1, a22 = c(z1) where c means taking the conjugate, a12 = z2, a21 = -c(z2); for octonians replace in the previous coordinates z by quaternions q1, q2. The octonian e7 coordinate can be replaced by a Kaluza-Klein rolled coordinate U(1) loop for EMI exponential ψ-functions exp(iφ).

Observe that the e0,e7 plane is blown up in matrix transformation form to a complex C^2 plane and quaternionic numbers q for coordinates z1= z + ict, z2 = x + iy, x as e1, y as e2, z as e3, t as e4. The q-coordinates extension to octonians does the same and takes two quaternionic coordinates q1, q2 for the octonian matrices, for instance with ej, j = 0, 5,6,7 coordinates. In the dihedral symmetry the n-edge transformations are 2x2-matrices. Using number systems the space extensions arise through the use of coordinates in 2x2-matrices with real (dimension 1), complex (dimension 2), quaternionic (dimension 4) numbers for them. For blow down projections there are different methods beside space projections. Important is also a complex, not real, projective norming where not lines through 0 are mapped to points, but only rays with the origin 0 as endpoint are mapped to a point on a unit sphere which generates dipols +p and -p on the unit sphere.

Repeat: The Hopf fiber bundle projection h: $S^3 \rightarrow S^2$ uses the 3 Pauli matrices of the SU(2) symmetry. The SU(3) symmetry projects with its first 3 GellMann GF matrices of rgb-gravitons the complex SI space with (z1,z2,z3) coordinates down to spacetime coordinates (z1,z2) Setting time t = 0 it projects S^2 of space down to a loop S^1. S^1 is blown down by the matrix with coordinates a11 = 1, a22 = a12 = a21 = 0 to S^0 for 2 poles and the 0-matrix projects this down to a middle point or center which can carry scalar as weight. For the other two SU(3) GF the diagonal matric is replaced by an extended σ3 Pauli matrix and the projections of (z1,z2,z3) have (z1,z3) or (z2,z3) coordinates, z3 is an energy plane for frequency and mass with the line $mc^2 = hf$, z1 in (z1,z3) has radius r and time coordinates for the Minkowski metrical cone lines $r^2 = c^2t^2$. If they are closed at infinity by 2 points, a lemniscate for quarks with 2 poles is obtained. Z2 in (z2,z3) has a loop $x^2 + y^2 = 1$ as quadric for measuring distances. Other projections in use are the stereographic maps, deleting from spheres a point at infinity, central projections, projection operators onto octonian subspaces. The octonians first projection is by norming the
z4 = (e0,e7) plane to 1 for the SI space (z1,z2,z3). It means for the compass in figure 7 that the needle and the circumference are missing. The associated G-matrix or the D3 symmetry sets in this case invariant qualities or numbers for 6 kinds of energies EM and GR potentials (the POL motor), frequency energies, kinetic or rotation (WI or SI motor) and heat (SI motor), magnetic energy (WI motor). 6 masses are set for the fermionic series through a GF measuring triple, 6 number or

charges orbits for EM, for color charges. Sometimes only degenerate orbits occure like the 3 basic spin lengths ½, , 2 for fermions, bosons gravitons. The D3 symmetry of a nucleon triangle is also added. It has two more degenerate numerical orbits as reference points 0,1,∞ for the cross ratios with a fourth complex number z added, generating the D3 matrices; the cubic roots orbit is for the nucleon triangle as vertices. As D3 invariants of the MT symmetry its matrices occur as the complex cross ratios. A combined listing is found for the (z1,z2,z3) space in [1] p.32, inlcuding the corresponding informations for SU(2) with Euclidean (z1,z2) coordinates.

The real cross product for the GF as measuring Gleason operators is quaternionic generated with the SU(2) symmetry of WI. The color charge QCD theory has another interpretation of the octonian vector space for its 8 gluons with the SU(3) symmetry of SI. Gravity and mass GR is presented by the MT symmetry of S^2, the Higgs particles Hj, *rgb*-graviton whirls, central and projective projections (figure 7 right) and the geometry of homogeneous, projective coordinates. They allow changing dimensions by their correlation transformations and geometrical normal forms with quadrics. Above are quoted genus 0 for S^2, genus 1 surfaces for three kinds of tori, genus 2 surfaces for quarks having 2 poles for its EM, GR potetntials, genus 3 brezels for nucleons with 3 quark poles. The genus 4 surface is in figure 4 for EM poles with two EM charges +, -, magnetic S-,N-poles or neutral and spin/momentum orientations. A genus 6 brezel can be used for the hedgehog in figure 8, driven by 3 motors EM+GR POT (x-axis caps). SI (y-axis caps) and WI (z-axis caps). Spheres are used in dimensions 1 (loop), 2 (Riemannian sphere), 3 (genus j = 0,1,2,3,4,6 for Heegard decompositions of S^3). The 5-dimensional sphere of the SI geometry is projectively



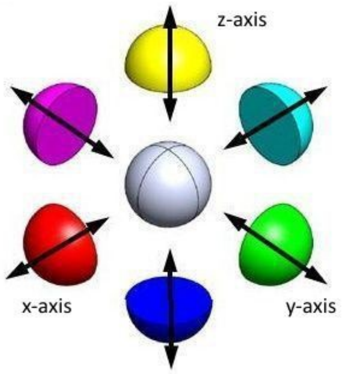


Figure 8 concentric Bohr shells (left) and polar vectorial color charge caps for the energy exchange of deuteron with its environment

normed to the deuteron inner spacetime CP^2 with boundary S^2, observable as a grid in spacetime. Fiber bundle geometries with fiber S^1 are the SU(2) Hopf map for S^3, projecting it down to S^2 and the trivial fiber bundle S^3xS^5 of SU(3).

We have now for SI motor the octahedron as a triangulation of a sphere S^2, for WI motor a tetrahedron, for octonians (figure 17, also the hedgehog caps) a covering with rectangles. The eight or four sides can carry coordinate vectorial units for the octonian or quaternionic numbers. For the proposed POT motor we take a division of S^2 into 2 half spheres and 2 vectors for complex numbers. As a particle for POT was suggested quarks. If no triangulation of S^2 is used for a scalar field which is represented by a vector, carrying a real scalar as weight like length, mass, time, frequency for linear or angular momentum, heat. POT has attached a potential field and its EM or GR vector. For WI a spinor field can be added to the 3 WI bosons as particles with a spin vector. For SI in its SU(3) form, to the 8 gluons a tensor vector for tensor fields and rgb-graviton whirls can be a added which generate no extra gluon coordinate. The handles attached at the vertices of the tetrahedron and octahedron are carrying energies, like charges or whirls and are quality measures (MG, EM charge, mass, frequency, color charges) for scalar weights of vectorial octonian coordinate units. A quark has genus 2 handles for mass and EM. The handles can be presented as rolls of a 2-, 4- or 6-roll mill, driven by the POT, WI, SI motors for a potential flow or plasma dynamics located as a kind of fluid around the rolls.

In higher dimensions the 5-dimensional Schmutzer field and its SU(3) geometry S^5 is interpreted as a Higgs boson H5 for a unified EM + GR field. As particles 5 Higgs bosons are suggested, the self-dual Horn torus H1 with no spin in figure 1 for heavy mass, two are for intertial mass with an up H2 or down spin vector H3 attached to the singular point of H1, which is oriented (as for the neutral leptons helicity) according to outer push or pull force vectors applied to H1. A 4-dimensional projection of S^5 as the deuteron inner spacetime for deuterons dynamics can account for another Higgs boson with an S^5 fiber S^1 normed to 1.

The Hopf fiber bundle for its geometry S^3 with fiber S^1 projects S^3 down to S^2. In figure 9 a possible location and dynamics for the 4 tetrahedron energies is drawn: MG and spin is attached at a north pole of S^2, EM is a scalar point charge rotating mpo or cw oriented on a latitude circle with the PM vector attached and GR sits as a mass scalar or vector attached at the south pole.

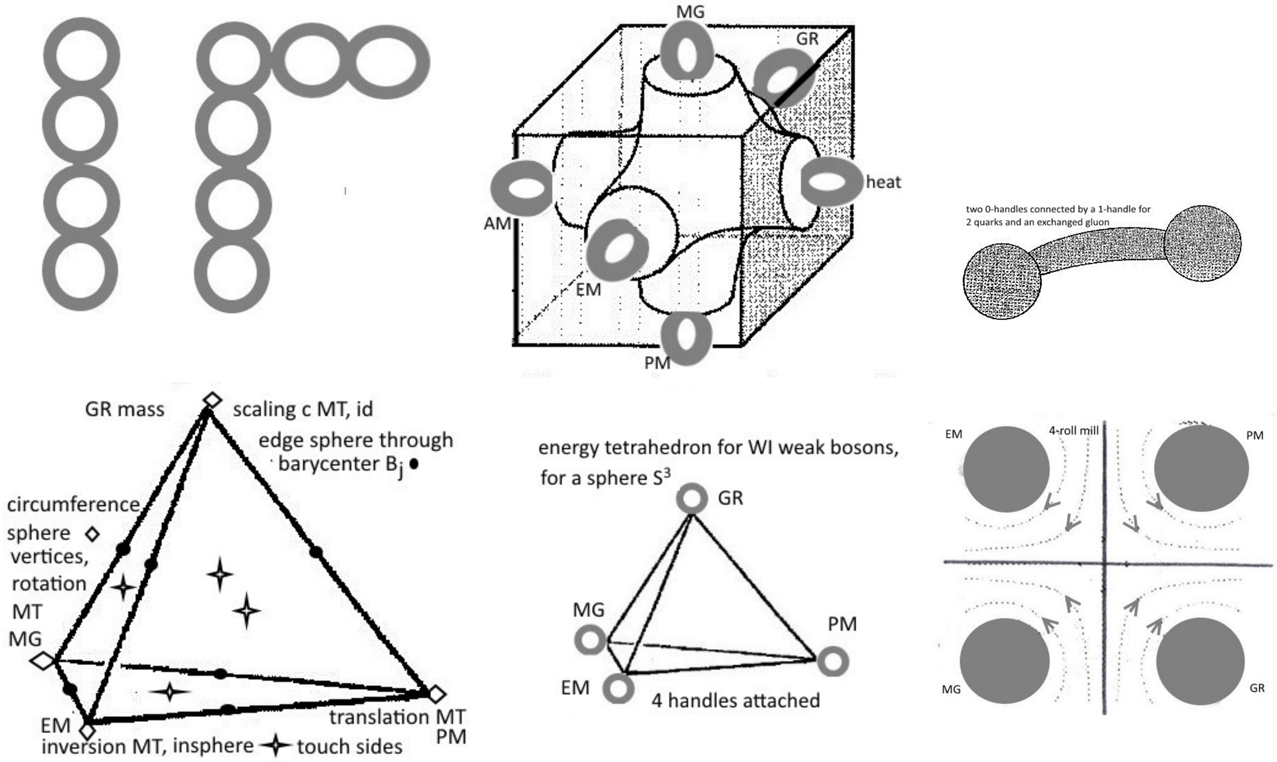


Figure 9 first line: EM genus 4 (also the figures below) and hedgehog genus 6 brezels, quark-gluon handle as a nucleon triangles side

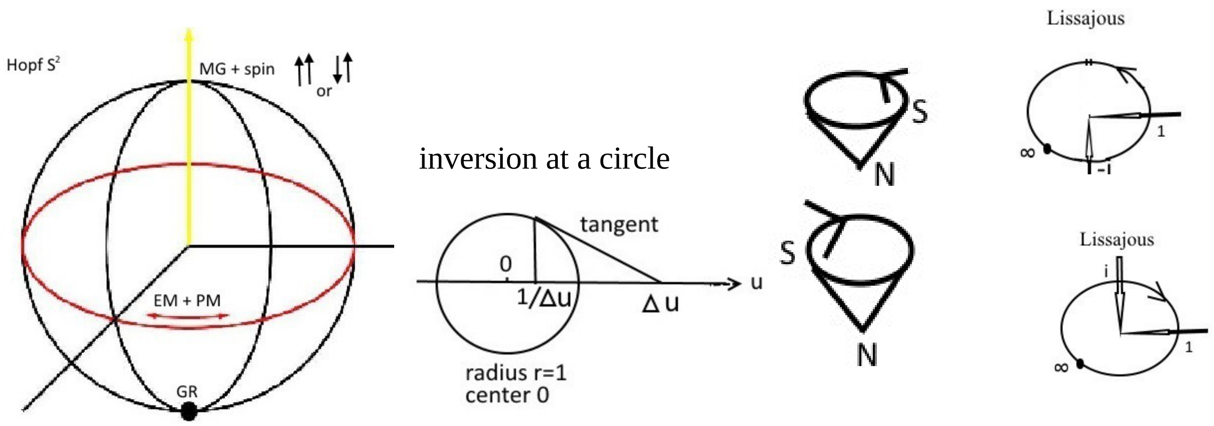


Figure 10 Hopf sphere S^2 for an EM charged lepton, mathematical inversion, MG cones, Lissajous

Magnetic momentum in figure 10 is a vector, at right it is presented as a rotating vector with endpoint S as south pole and initial point N as north pole tracing out a cone surface of a magnetic flow quantum whirl. Its rotation can be generated by two orthogonal hitting frequencies as x⋆sin ωt and y⋆sin(ωt+φ0) which generates a cw or mpo rotating circular Lissajius figure for S with tip N of the cone. For the dihedral representation D4 of SU(2) and the 4 MT in figure 9 (left tetrahedron) we present the scaling MT for GR as exponential function exp(iφ). If an external push or pull to H1 is applied as environmental force or momentum vector p = mv, for inertial mass H2, H3 the exp(iφ), -exp(i(φ + π)) functions rotation matrices a⋆id, -a⋆ id of D4/SU(2) are used. They set at the singular middle point ∞ of H1 a spin 2 up vector with the outer vector p in up or down direction attached at ∞ (helicity orientation and rotation in figure 2, see neutral leptons for dark matter in the universe). The other D4/SU(2) MT are presented for the MG up/down vector on the vertical z-axis as -exp(iφ) for π-rotation, and with the EM mpo rotation i⋆exp(iφ) for (π/2)-rotation, and with the EM cw rotation -i⋆exp(iφ) for (3π/2)-rotation in figure 10 at right is a figure for the MG + spin location of an EM charged lepton in the Hopf S^3 sphere with $h(S^3) = S^2$; for the D4 reflections exp(-iφ) (EM inversion) with a 0 angle to the x-axis, PM translations along the S^2 latitude circle in figure 10 as -exp(-iφ) with a (π/2)-angle to the x-axis, i⋆exp(-iφ) with a (π/4)-angle to the x-axis, -i⋆exp(-iφ) with a (3π/4)-angle to the x-axis.

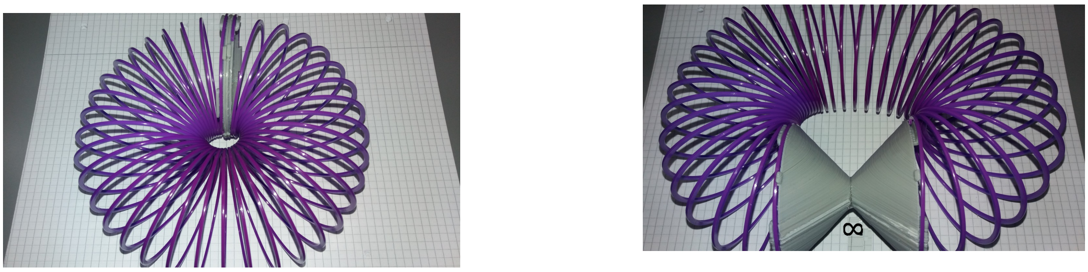


Figure 11 H1 for mass as weight with spin 0, with the singular point (see figure 2) drawn apart for an EM torus it is the inverse image of the Hopf S^2 latitude circle in figure 10 (the circular rotating frequency of EM charged leptons), at right: dark energy – light frequency (wave) cylinder projective closed by a light cone

Other wave presentations, using the exponential function exp set by the octonian e7-coordinate as a Kaluza-Klein coordinate are: for MG harmonic waves ψ = a⋆exp(-i(ωt +φ0), t time, ω = dφ/dt cicular frequency, oscillations/vibrations (use the D4/SU(2) rotation Pauli matrices +σ2, -σ2), ψ = a⋆exp(-i(ωt ± kx +φ0), k = 2π/λ, λ wave length for transversal PM/EMI waves (light, photons), with a left- or right handed helix on a cylinder for its frequency (figure 5 left, use the D4/SU(2) reflection Pauli matrices +σ3, -σ3, for Minkowski rescaling of frequency f ' = f⋆cos θ), EM matter wave (Schrödinger equation) ψ = a⋆exp(-2πi/h(Et-px)) ((use the D4/SU(2) reflection Pauli matrices +σ1, -σ1, Doppler effect for longitudinal wave frequency). The frequency of light is drawn as spring for dark energy in the universe; for the light cylinder of EMI, the cylinder is closed at projective infinity by a point ∞ (in figure 5 at right drawn as a Minkowski light cone LC with the singular point ∞ as presence for time, the left-, right handed helix can be used for the time as past in LC at left and future time in LC at right. For Higgs bosons ψ*ψ, the probability distribution of waves, can be used with mass energy in $mc^2 = E = hf$ presented as graviton wave GR spring (Horn torus and figure 11 left - with a singular point in the middle). The ψ equation for light is taken for H1 in figure 11, but this can also be replaced by a Hopf EM torus rotation for the EM charge, - a circle is rotated about another circle for the S^1xS^1 torus. Instead of an EMI spring ψ equation, a torus equation in space xyz- and spherical coordinates is applied $x(\theta,\varphi) = (R + r\cdot\cos\theta)\cos\varphi$, $y(\theta,\varphi) = (R + r\cdot\cos\theta)\sin\varphi$, $z(\theta,\varphi) = r\cdot\sin\theta$ where φ is a polar angle in the xy-plane and θ the spherical angle towards the z-axis, R is the distance from the center of the vertical circle S^1 of radius r to the center of the torus. The vertical xz-circle S^1 for $\varphi = 0$ is rotated about the horizantal circle z = 0. For a wave descritption set θ = 0, φ = φ0 at time t = 0 and add for the rotation in time $\psi = (x - iy)\cdot\exp(-i\omega t) = (R+r)\cdot\exp(-i(\omega t + \varphi 0))$ with the frequency ω. Hence from the above equations a harmonic wave $\psi = 2R\cdot\exp(i\omega t)$ or $\psi = 2R\cdot\exp(-i\omega t)$ for H1 is obtained. This applies also to leptonic tori and spindle tori. The Schrödinger equation for hydrogens electron gives not this ψ wave. Leptons have another frequency ω inside their toroidal location. In physics publications up to 2019 this Hopf generated frequency ω is not used for the Schr<ödinger ψ-wave. Since also the gyromagnetic relation is not well understand, the Hopf interpretation for ψS is here as follows: we take the physics from [13] chapter 7.1, pp.110-112 and pp. 500-501 since no newer suitable publication is found; in the assumption 2 the discrete winding numbers n = 1,2,... are related to the electrons Bohr radius in the atom by using a wave length λ for the electrons matter wave. In the assumption 5 the emitted energy ΔE from the electron by jumping from one Bohr radius r to another r' with the spin windings $n\lambda = 2\pi r$, $n'\lambda' = 2\pi r'$ is generating the released photons EMI freqeuncy f = ΔE/h. We take for the photons f the $\omega = 2\pi f'$ from the Hopf ψ wave, scaled by the Rydberg constant $R\infty$ (scaled fine strucure constant) to the photons frequency in hydroben with one electron as $f = R\infty(1/n^2 - 1/n'^2) = \Delta E/2\pi h = (En - En')/2\pi h$. The released light with speed c is emitted from the Schwarzschild radius of the electron as its second cosmic speed. According to the assumption 2, the kinetic energy of the electron occurs as $En = R\infty/(2\pi n^2)$, the potential energy Ep = 2En and uses for this equation the related speed $v = \lambda\,\omega/2\pi$ and radius equations $n\lambda = 2\pi r$. For the v, r values, occuring then in $R\infty$, the assumption for the existence of a Schrödinger ψ-wave ψS is is that on an electron orbit its Coulomb force as attraction to the kernel has to be equal to tis centrifugal force. In ψS the substitutions in an ordinary wave equation is for matter waves then taken by using the Hopf frequency $\omega = 2\pi f'$ for E = hf' and replacing λ in the speed $v = \lambda f'$ by by $p = h/\lambda$. The second substitution means that in the Hopf wave description the speed from p = mv (as kinetic energy E(kin)) of the leaning circles rotation is replacing a wave length. The energy preservation E = E(kin) + U, U potential energy, adds in the ψS differential equation on the right side then + U ψS. The U potential is from the electrons gravitational kernel attraction and its circular orbit for the first cosmic speed of the kernel.used in E

The scaled differentiations of ψS are on the left side of the equation by time $\partial/\partial t$, on the right side by the x-direction for the waves momentum expansion in space as $\partial^2/\partial x^2$ (see [13] p. 500).

H1 and neutral leptons are responsible for dark matter in the universe. For H1 its radius is inverted at its Schwarzschild radius $Rs = 2Gm/c^2$, 2R<Rs means for the H1 with wave length $\lambda<2R$ and speed $v0 = \lambda\cdot\omega/2\pi < R\cdot\omega/\pi$ for a circular motion satisfies for frequency $f = \omega/2\pi > c/Rs\sqrt{2}$, using $(Rs\cdot\omega/2\pi c)^2 > v0^2/c^2 = Gm/2Rc^2 > Gm/c^2Rs = ½$.

There are two versions used for the differential calculus, here differential equations, earlier difference equations. Both can have a characteristic polynomial. For D2 the 0-unit sphere +1,-1 from the $z^2 -1 = 0$ roots set dipoles and sets with its symmetry four operators, the id as CPT-invariance, in form of diagonal 4x4-matrices diag[-1,-1,-1] for the conjugation operator C, diag[1,-1-1-1] for space parity and diag[-1,1,1,1] for time reversal. If $z^2 - 1$ or z^2+1 are used as characteristic polynomial for a differential equation then complex solutions in the form $\exp(ax)\cdot\cos(bx)$, $\exp(ax)\cdot\sin(bx)$ or $\exp(i\cdot ax)$ arise; for the cubic polynomial z^3-1 the general exp-solution is of the form $c1\cdot \exp(x) + c2\cdot\exp(ix) + c3\cdot\exp(-ix)$, cj constants. General exp-solutions have also for the 4th and 6th roots of unity polynomials 4 or 6 independent solutions of this kind. For difference equations with (Fibonacci like) sequences s0, s1, s2,...sn,.. , as solutions the real roots of unity xj of the characteristic polynomial have for sn an $cj \cdot xj^n$ solution and the conjugate complex roots replace in the sn summing of all solutions xj, xk by their real solutions $\rho^n \cos(n\varphi)$, $\rho^n \sin(n\varphi)$. The constants can be determined by adding a suitable number of initial conditions for the sequence. An example: the third roots of unity repeat with $p1^2 = p2$, $p2^2 = p1$. In the series only the constants multiplied to 1,p1,p2 give different members of the finite series, presenting a finite nuber of pure nucleon states. If normed to 0,+1,-1 there are 27 possibilities. A warning is necessary: If ∞ is allowed as number, the permutation group of D3 for the triangle with 3 quarks as vertices on 1,p1,p2 in a complex plane allow in addition other states, integrations of the cross ratio energies; there are measures with GF triples which change the states and two driving SI, POT motors for the inner dynamics, also geometrical changes or properties are added. The weak interaction is in the second planned Tool Bag, SI, POT in this first Tool Bag. Geometries from the WI decays of its S^3 are also included here in some figures. Three characters of the six energies allow whirls for the rgb-graviton with pulsation, magnetic momenta, spins, conic vector rotations for field and a generated harmonic wave, particles with mass or without mass. The geometry can include obstacles/poles about which a flow or field lines as in the 6-roll mill move, polar energy exchange of the nucleon energies with its environment arise, the setting of barycentrical coordinates and barycenters are other outputs, blow up or down of dimensons occur. through projections. As a system, a nucleon needs much more for its states than a wave equation or a difference equation can give.

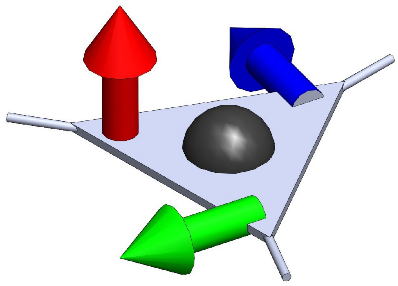
4 Presenting Energy systems

As a universe for describing the energy systems of physics in an octonian vector space is based on different kinds of mathematics. This involves first the use of noncommutative operators and the associated matrix or matrix systems. From the projective geometry **projections, matrix transformations, correlations** are involved, but also other kinds of projections occur like the fiber bundle Hopf map which uses the Pauli 2x2-matrix symmetry of SU(2), WI. In a second description energy systems get through **measurements** of the systems states a measurement aparatus attached which can change after the measurement the state of the system as described in the Copenhagen interpretation. As input of such an exeriment the systems part to be measured ahs to be prepared. The output of a measurement is sometimes through a sclar, in other cases a vector or matric or a functional desription is recorded. In the GellMann form of gluons SI, SU(3) symmetry of 3x3-matrices as Pauli matrix blow ups, octonians seven not only one spin SU(2) GF for **Gleason operator** measurements occur. SU(3) matrices have three more GF. This is newly recommended through the Tool Bag and not used until the year 2019 in physics. These measurements relate on the real cross product where two noncommutative operaotrs generate a third vector having as length the areas spanned by the other two vectors. Beside the new GF **quadrics** with geometrical forms attached are often used for other kinds of measures where an operator is applied to a coordinate vector in a bilinear form. As a newly introduced **symmetry** for gravity the Moebius transformations **MT** with their four possible actions are used for describing the Tool Bag models. Concerning quadris: The U(1) loop S^1 as symmetry of EMI, the electromagntic interaction together with the vector space coordinate extensions can be used for generating local coordinate systems for in describing energies. S^1 is a transversl cross cut through an elliptic cylinder where one cyclindrical coordinate coincides with the energies world line in space. It is a linear axis of the cylinder for EMI whose wave in form of an exponential function has its frequency expanding in time as helix line on the cylinders surface. If closed at infinty by a Minkowski light cone a finite location for dark energy in particle form is sugested as a Horn torus (figure 11 right). Inside the tube the universes matter speed with $v < c$ is inverted (figure 10) at the Minkowski lines $v = c$ (figure 4), c speed of light, to dark energy speeds $v' > c$ with the inverting MT in $v'v = c^2$. At the left in figure 11 a similar figure shown a mathematical torus $S^1 x S^1$. In the Hopf inverse image of S^2 there is a latitude circle of S^2 blown up with the S^1 Hopf bundle fiber to a leaning transversal circle which traces out in rotation the torus surface. For dark mass (figure 11) in the universe can be used a graviton wave helix as in the EMI figures of 1,5 or the transversal circle is moving in time a long the tube (no helix involved). The three kinds of tori are shown in figure 2 where the cross cut circle has a smaller radius r than the distance R from the torus center to the center of the tube which is a geometry for EM charged leptons, for dark mass holds r = R and for neutral leptons r > R. If the r = R case is as particle interpreted as a Higgs H1 boson, the universes dark matter is in part generated by these two kinds of particles, Higgs bosons and neutral leptons. Also the mass of WI bosons can be explained through their Hopf blown up S^3 geometry. In the Heegard handle body decays (figure 9) of S^3 the tori surfaces are of genus 1. The genus of the decaying S^3 sphere in 2 handle bodies allows through the use of the discrete dihaedral symmetries (figure 4) for the polar handle surfaces attached the values 0 for a solid ball with a S^2 boundary and the values 2,3,4,6, (figures 3,9). 2 is for quarks, 3 for nucleons, 4 for leptons EM and WI, 6 for the SI hedgehog (figure 8). Whirls or flows about an obstacle are also in other connections observed, for instance as a heat gnerated eye of a hurricane or magnetic field handles in ths suns erruptions. For generating the quadric S^1 of **EMI**, the Lissajous figure in 10 was used. If the 2 toroidal circles have equal radius the rotating cross cut circle traces out a genus 0 S^2 sphere for **GR** with the MT symmetry; in figure 10 this sphere is also used for charged leptons Hopf S^2 geometry. The Hopf blown up S^3 sphere presents the **WI** bosons geometry. For the SI geometry the quadric S^5 is for a similar trivial fiber bundle with fiber S^3. In this case Minkowski space time in form of the complex Hopf matrix coordinates (z1,z2) is homogenous blown up to complex (z1,z2,z3) octonian coordinates where z3 presents an energy plan for frequency and mass. This **SI** space for SU(3) gluons GellMann transformation matrices has S^5 as unit sphere. The S^3 factor in the SI geometry S^3xS^5 is GR projected down by *rgb*-graviton whirls to the Hopf S^3 or linear space by deleting a stereographic point at infinity. In another field projection S^5 has as stereograohic projection a real 5-dimensional space which is used for the unified Schmutzer potential field for EM + GR. Schmutzer has for his field a projector which maps this potential down to three 4-dimensional fields for EM, for GR and for a scalar field which could be used for a mass-Higgs field, attaching a mass weight to systems. Its sets masses of fermions through GFs, one for neutral leptons masses, one for EM charged masses and two for quarks u,d,s and c,b,t. Use in octonians (e1,...,e5)-coordinates and for its nonzero mass scalar in the universe that for tis Gleason operator T measure has a mass 0 subspace in the [e0,e6,e7] space for vectors, frequency and the light symmetry U(1). T has a support. The (e5,e6) plane is for mass, frequency and the energy line $mc^2 = hf$.

From the harmonic waves, also available for the SI rotor, the rotated momentum vector p blue sits in up/down directions at the vertices of the quark triangle. Since the Hamiltonian for kinetic energy has the form $H = p^2/2m = mv^2/2$ this can be interpreted for the Schrödinger matter waves, setting in ψ as exp values $\omega = 2\pi f$, $v = \lambda f$, $f = E/h$ and $\lambda p = h$. The energy preservation E = E(kin) + E(pot) remains valid and is used for instance together with the preservation of angular momentum for orbiting planets P to integrate a Kepler ellipse orbit. The relativistic scaling with an GR accelerated rosetts orbit of P means that the diameter of the Kepler ellipse is after one rotation shifted by an angle. The general relativistic acceleration of the radius vector of P is due to an unsymmetric distance measure to a central sun Q where Q measures the distance as QP = r and P meassures the distance PQ = r – Rs, Rs the sunsSchwarzschild radius. In homogeneous coordinates the norming can be done and a pari of coordinates (r – Rs, r) can be normed to the Schwarzschild metric scaling factor $(1 – Rs/r)^{½}$ (see the G-compass matrix figure 7) to ((r-Rs)/r,1).

Accociated with the quadrics are coordinate systems, for EMI cylindrical, for GR (with the Schwarzschild metric, tensor matrices) barycentral and SI generated, for WI Euclidean for space (time extended to the Minkowski metric, spinor matrices, figure 6), for SI octonians.

The use of quadrics occurs in this connection for the basic four interactions of physics on a large scale, the coordinates for them and symmetries are different. There are field quantums for all, the photon for EMI as E = hf particle and helix line winding on a cylinder, for GR the neutral color charge of nucleons with rgb-graviton whilrls, for EM the conic magnnetic flow quantum whirls (figure 3), for WI the weak bosons and for SI the gluons. The fermionic particle series arise through the WI decays into two handle bodies The field presentation for EMI, GR, SI, WI can be learned from physics and is not presented in the Tool Bag.



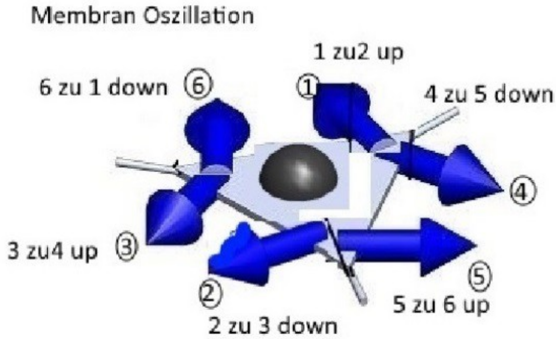


Figure 12 left: SI rotor with three *rgb*-graviton whirls rotated as a representation of the D3 symmetry, right; the rotated momentum blue vector has value 0 for quark vertices at rest.

5 Remarks for the new structures

In a more systematic way with comments added, earlier items are repeated.

Numbers: For the mathematical construction of numbers, starting with natural numbers N consult [4] chapter 6, N arises in phyics as the discrete spin winding numbers, Getting integers, an mpo +1 and cw -1 orientation on a circle is added. For rational numbers the discrete n-the roots (see the dihedral groups) are used in measuring arc length 2πk/n, k = 1,...,(n-1) on the circle. For real numbers limits with ∞ are used for seqeunces of rational numbers which in decimal form allow transcendental numbers like e for the exponential funciotn or π. For vector space coordinates in the dimensions n =1,2,4,6,8 real numbers are added to a set of unit vectors, 1 for reals n =1, complex 1,i for n= 2, quaternioic 1,i,j,k for n =4 and octonian e0, 1,i,j,k,e5,e6,e7 where in this model e7 is closed to a Kaluza-Klein circle U(1) . Number systems are also associated with symmetries which generate numerical orbits. In use are +1, -1 for two polar systems like quarks, EM charges signs, SN south/north poles of magnetic momentum. The degenertate orbit 0,1,∞ of D3 sets the reference points for complex cross ratios with a fourth complex number z added. The invariants are listed as the 6th roots in the G-compass as color charges, they also can be for six EM charges, for 6 fold masses of the fermionic series, 6 energies E(pot) for GR, EM(pot) for EM forces, E(kin) for kinetic and E(rot) for angular/rotation energy, E(heat) for heat with phonons as pseudoparticles without spin, E(magn) for magnetic energy. The degenerate D3 orbit of the 3rd roots of unities is for the nucleon triangle and for 3 charges of the W+, W-, Z0 EM and neutral charges. For the D4 orbit 1,-1,i,-i in figure 9 the tetrahedron vertices are named EM, GR, MG and PM (explained earlier).

Gleason measures as GF triples: The reference is [5] chapter 3. Gleason operators are bounded, self-adjoint, positive of trace class 1. They are generated by GF triples as frame functions which carry numerical weights. The weights can be for 1, 2 or 3 units measured. For length the spin triple, in octonian 123 coordinates measures in meters. For mass in kg the neutrino mass GF is on the octonian 356 triple which is also for the SI rotor. For heat its basic equation suggests the triple volume as a radius measured, pressure measured with N and temperature in Kelvin K or Celsius C measured and the octonian GF triple 246. For rotational energy 347 the triple L = rxp suggests a r radius/ length meaasure, a kinetic and angular momentum energy measure (inverse seconds 1/s) for linear or orbital/rotational energies. For time the measure is in seconds s and a GF triple 145 adds for magnetic energy a vectors length with 1 and a weight for different kinds of conic whirls with 5. In use are magnetic field quantums, *rgb*-graviton whirls (which itself has a GF) , color charge whirls and photons helix windings generating whirls. The barycentrical setting of mass has the 257 GF. The Higgs GF has 3 masses as weights for setting fermionic series masses. The 167 GF is for waves with a wave length on 1 in meters, a frequency in 1/s on 6 and a circle for an exp function description exp(iφ), φ having an angular measure. In the cross product form, other GF can be suggested. The formulas from physics for λp =h (weights for length, kg·m/s, m^2), E = hf (weights for Joule, m^2,1/s), φJ = h (replacing L = rxp, weights for angles rad, kg·m^2, m^2) have for the cross product an area F measured by the Planck constant h as unit and these equations introduce the Heisenberg uncertainies HU. A similar equation for magnetic field quantums ϕo·e0 = h/2 is not a HU. An associated area measure in VS/Am is for EM induction and a magnetic field crossing the area inside an electrical loop current. A magentic measuure is in Am^2, for an electrical charge e0 in As. Another EM cross product is F = Q(vxB), measures as EM force, speed v of an electrical charge in motion towards a magnetic field and induction B. - Gleason measures have a probabiliy μ asssociated through measuring though its von Neumann operator T the subspace N probability values μ(N) = tr(TprN)) where tr is trace and pr(N) the projection of the space which contains N. These projections are noncommuting, for reading the book [6] is recommended. A warning is necessary since this noncommuting means for an underlying orthomodular, not Boolean logic that the deduction theorem is not valid and modus ponens in classical form can only be applied when the projections commute.

Projection operators and associated projective shapes of figures with a normal form. This can be obtaines by a projective correlation which has a quadric listed as normal form. Also central projections like stereographic projections are used. They can be linearlized by a homogeneous projective extended coordinate system. For correlations the projective duality is used where a subspace N with dimension k in a projective space of dimension (n+1) has associated a subspace of dimension n - k. For the inversion (figure 10 middle) in a projective plane a polar point and a tangent line as polar are asociated; the polar point is in this case the point where the tangent touches the inversions circle quadric.
The shapes of normal forms with their quadrics are for spheres, cones, pairs of interseting lines, 2 parallel lines, elliptic cylnders, 2 nonparallel planes, two parallel planes, lines, points, parabolas and hyperbolas with a circles intersection on a line at infinity in 1 or 2 points (figure 13). The parallel lines or planes can be suitably closed up to 2 concentric circles or 2 concentric light cylinders or 2 concentric Bohr spheres for radial expansions. The normal forms ([1] chapter 9) can also be closed at infinity for other shapes: the torus closes a 2-dimensional cylinder by a circle and has itself differnt shapes for a doughnut, Horn and spindle torus. Another Horn torus is obtained by closing a Minkowsk double cone surface at infinity by a circle, For decays like the Heegard decompositions the genus n surfaces are in use. Surgery as in figure 13 guides the decays of the Hopf sphere S^3 which for genus 0 are 2 solid 3-dimensional balls. The physics generation and annihilation operators A and A† are used to lower or rise the numbers of particles by one.
Projective geometry is not mentioned here in more details. There are good books for this available.

Energy bifurcation: in figure 13 the Feigenbaum bifurcation FB is drawn as a Pascal configuration on the ellipse this arises as mentioned above through the 6 invariant complex cross ratio MTs. The first bifurcation from a common input, a Schmutzer potential as a POT motor e0 vector and as part of the 6-roll mill is for the EM 1 and GR 5 potentials, the two next bifurcations are from1 into 2, 4 for heat and magnetic energy, from 5 into kinetic 6 and rotational 3 energy, and as output 7 light and the U(1) symmetry is added. FB itself bifurcates after the 6 energies into 8 gluons, then heat chaos occurs. The bifurcation of energies has as geometry the G-compass in figure 7 where color charged areas as segments between the 6th roots of unity are cc colored. In the form of poles and polar vectors for the 6 energies the former 6 segments are drawn as polar caps about a deuteron sphere where the deuteron exhcanges its energies with the environment, EM force red, heat green, rotational energy magenta, magnetic energy yellow, kinetic momentum blue, mass/GR potential turquoise. The complex or real integrations are applied for integrating forces, accelerations, … to potentials, speeds, …, also area integrations are used when a vector field crosses a charged loop.

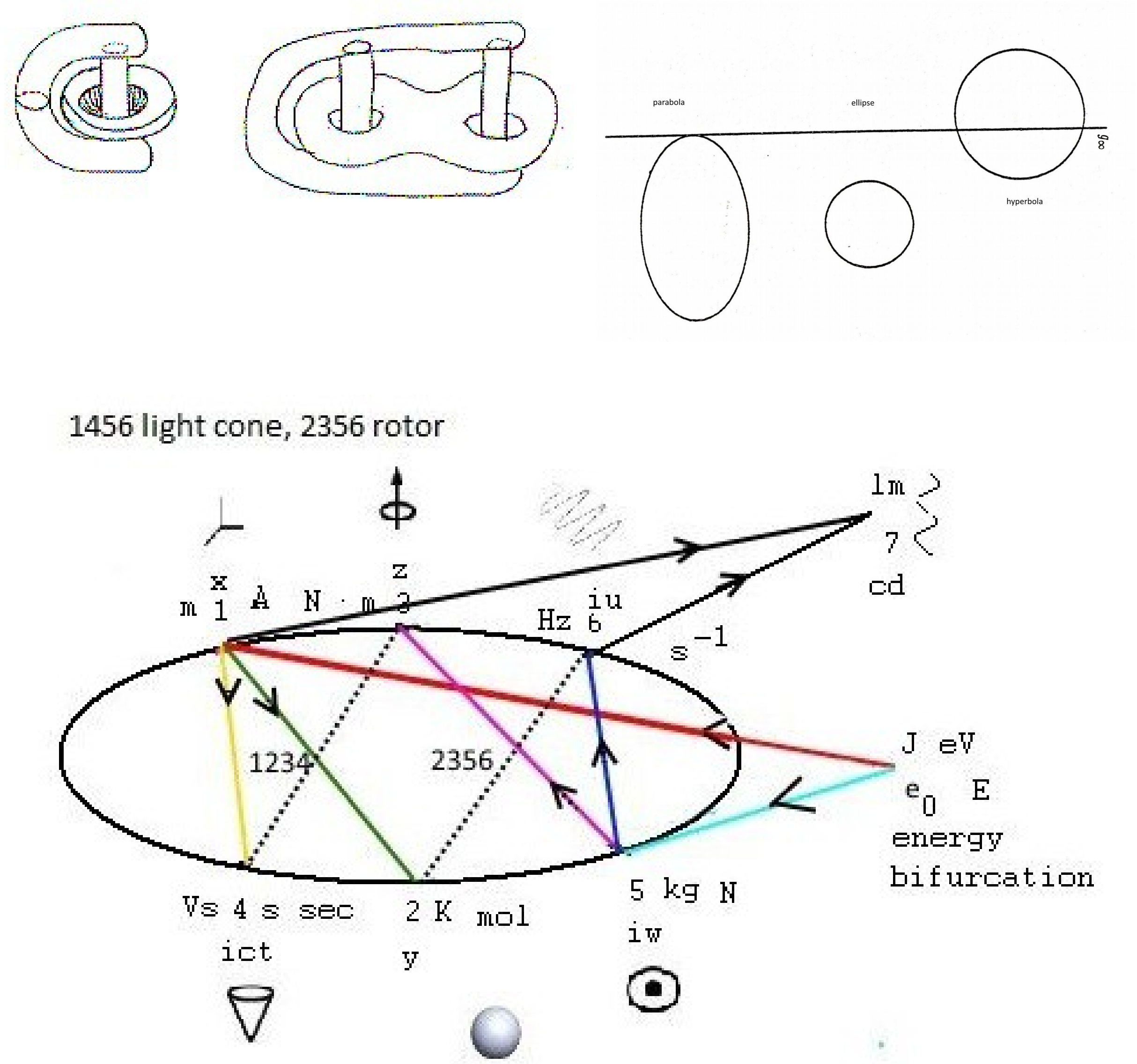


Figure 13 genus 2 shapes for solid lepton tori and for solid quark brezels; a line in the real projective plane at infinity and quadrics; bifurcation of energies with a vectors setting input e0 and a U(1) circle setting for light as output 7 at right

This applies for a magnetic field crossing an electrical currents loop. Potentials are radius dr integrated from scaled potential forces a/r^2 as accelerations (also for the general relativistic Schwarzschild facter for the Schwarzschild metric which adds to the diameter of a Kepler ellipse orbit of a planet rotating about a sun after one revolution a fixed angle as its increased GR speed) and speeds are integrated from time accelerations a/dt^2. Heat spreads out for instance through different shapes of areas and presents entropy in volumes. In discrete local form the gases, fluids or solid states for matter occurs. In figure 5 are shapes or other geometrical objects added to the six energies and light with frequency. For 1 is added the Euclidean spin triple for xyz-coordinates of space and the GF triples. For 2 is added a volume with a ball surface, for 3 a rotation axis for a circluar orbital rotation about the axis, for 4 a cone as a rotatng vector for whirls, for 5 a barycenter in a volume, for 6 a cosine function observed for waves exp functions.Th e0 input vector can be stochastically set somewhere in the octonian space when needed for an energy or field.

Videos available for the model: the V1 SI rotor (figure 12 left), the WI wheel V2 for generating the xyz-axes of space (figure 15). The axes are used in the weak isospin exchange between a deuterons proton and neutron where a WI decay is applied and a u-quark on an axis decays, emitting a W+ boson whose energy is absorbed by the partner d-quark on this axis. Other videos are for the membran oscillation V3 of the nucleons triangle sides where a hals-cone area is traces out by the sides conic rotation. The barcentrical coordinates as axes are generated. and three cones surfaces as whirls by the six SI rotors rotations. For the wheel the rotations are flat about the center of the Eiuclidan metrics coordinate axis. Two videos pulsation V4, V5 show the GR length scalings which are in SI coordinates central drawn for the nucleons triangle in 3 sides sizes for the basic spin length ½, 1, 2. For the WI coordinates an additional spiral rotation is assumed where three dogs in the quark vertices run with constant speed head to tail on a spiralic contraction and reverse their orbit on that spiral again from large to middle to small length and back. The SI and WI coordinates are

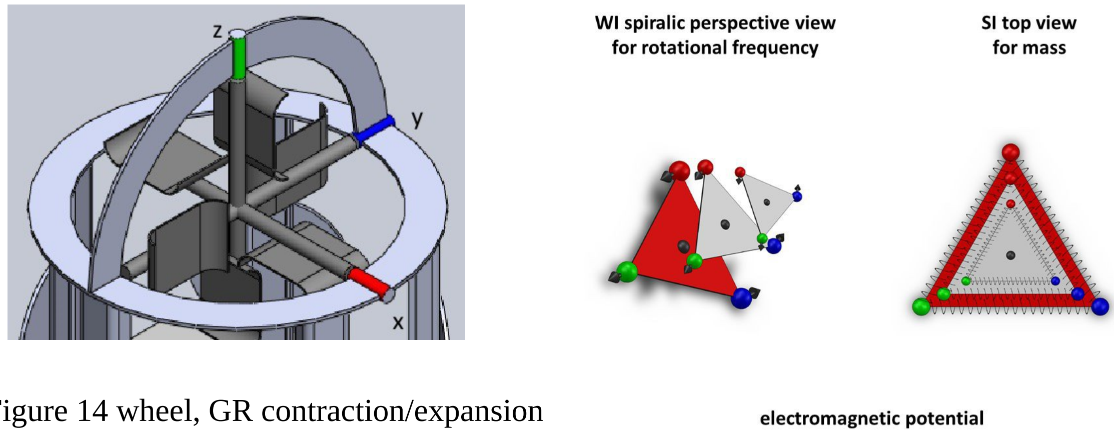


Figure 14 wheel, GR contraction/expansion

of a nucleon triangle as pulsation in time

assumed to be in special relativistic motion their SI and wheel rotors run with different speeds and for a nucleons parts common speed v the rescaling of coordinate units is necessary. This occurs also in atoms with electrons mass and in atomic kernels with neutrons mass, but in nucleons with the u-quark mass as mass defect where the mass is rescaled by the relativistic factor using v as speed. Another video 12 roll mill V6 shows a cycle in time for the 6 gluon exchanges between a nucleons paired quarks which generate the SI rotor rotations. In between is shown that a vector field drawn as color charge is crossing the nucleon triangle and the associated color charge energy vectors or energies are integrated as in a list above. The color charges on triangles replace the polar caps in the hedgehog figure. Possible a new video V7 shows the 8 modles for those who want to buy the Tool Bag.

Dihedral groups

In the bifurcation a method to draw Feynman graphs for weak decays is used. This is not like the Lissajous figures for hitting frequencies where their integer proportion generates different shapes in form of a circle (figure 10) or several touching or intersecting circular rings.

In figure 15 the weak boson is a point or spring which absorbes the local energies, the hitting particles are drawn by their momentum vectors and world lines. In the bifurcation of energies one input vector is set as octonian e0 coordinate unit somewhere in the 8-dimensional vector space. As output two generated systems are drawn as momentum vectors.

For the dihedral number constructions the graph has been a circle on which n = 1,2,3,... rays for the nth roots of unity are drawn and the vector is a rotating needle of a compass (figure 7). These are discrete vectorial rotations, not of the kind in figure 15.

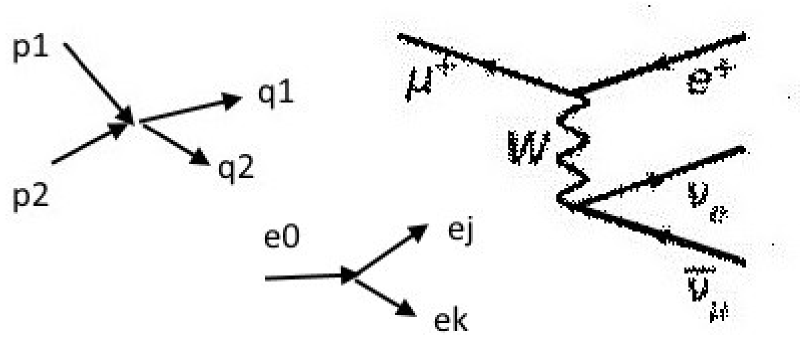


Figure 15 Feynman diagrams and bifurcation

Dihdral groups Dn have associated a characteristic equation $z^n -1 =0$ for a difference equation, replacing a differential equation. The n solutions are presented as scalar charges, as oriented vectors, rotations of charges or vectors, eigenvectors of matrices etc.

D0 is a taken for a point rotation as the needles initial point, tracing out with its endpoint the circle. Without a vector attached D0 sets scalars at a point, for instance mass at a barycenter. Scalar vector fields set in addition vectors. Instead of a central flat needle rotation are conic whirls (figure 3) used where the initial vector is at the tip of the cone and the endpoint traces out an exponential functions circle carrying an angular frequency energy, for instance of a rotating charge. It can be used as a 1-roll mill (see [1] chapter 3 and the compass) as obstacle, grid or pole in a circular (potential) flow (figure 8) around this (rotating) obstacle or as polar caps for complex integrations and winding numbers about the center. The circle occurs also as the light/EMI symmetr U(1), as fiber S^1 of the inverse Hopf map blown up from a point ot the S^5 SI fiber blown up from a point in the complex inner space time grid of deuteron.

D1 has a stereographic ∞ point (figure 14) marked on a blown up D0 circle C. It can project the circle onto a (real coordinate) line. GR uses such central projections, also for length as two proportions b,c fraction b/c. ∞ can be used for renorming of scalar measures which run towards ∞ . To ∞ a vectorial momentum can be added with which the cooroll C moves along a world line in the universe. C is at rest and can get an externals forces push or pull. Phonons for heats pseudo-particles without spin are of this kind of energy carrier. Neutral leptons are blown up 3-dimensional particles (figure 6) where a spin vector is set on a orthogonal located point 0 to ∞ on the circle. Rotating ∞ on C is often used, the D1 symmetry of order 2 is for a cw or mpo orinetation.

D2 has 2 poles or coorolls and is used for quarks, as a flat 1-dimensional retract (figure 16 left) lemniscate for the 3-dimensional brezel (figure 3). (It is mentioned that mathematical lines, circles and lemniscaes are the only complex or plane algebraic curves.) Quarks carry a POT motor for EM +GR, presenting a common potential (see the Schmutzer field). In the bifurcation (figure 13) there are the 3 motors: POT, SI, WI which drive each two rolls as listed in the 6-roll mill. POT is for EM +GR, SI with gluons is for heat + rotational energy, WI with weak bosons is for magnetic and kinetic energy. The D2 symmetry as Klein group of order 4 is for the discrete physics C, P, T operators and an id as CPT invariance.

D3 has 3 poles or coorolls for nucleons with 3 quarks. (figutr 16 right). It has *rgb*-graviton whirls for the neutral color charge of nulceons and with force/energy vectors added at the quarks points.The rotations of these vectors are a presentation of the D3 symmetry in form of the SI rotor Barycentrical coordinates are generated (figure 12). The WI motor as weak isospin exchange beween paired quarks of deuteron has in its wheel presentation (for generating the spaces Euclidan coordinate axes) three unit circles as rolls in the xy, xz or yz plane with 3 force vectors in opposite

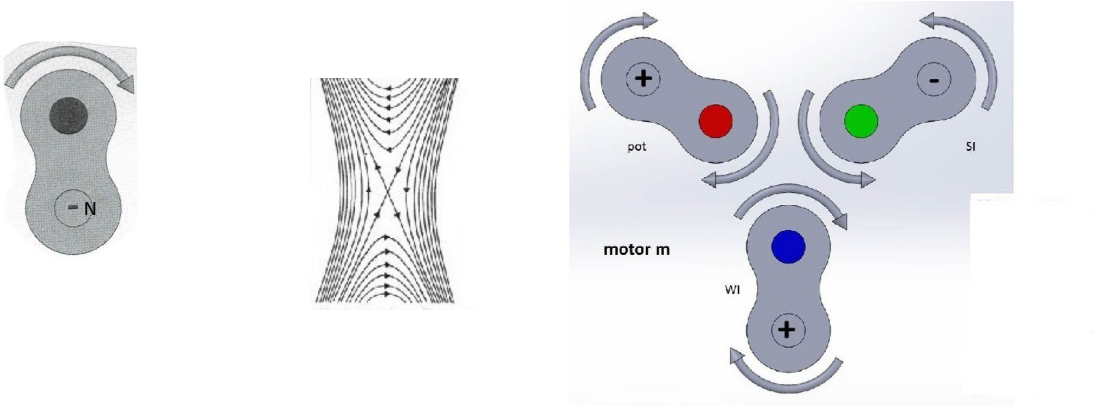


Figure 16 a quark with 2 poles or coorolls and flow about the poles, a nucleon with 3 quarks

direction added at the circle. They generate the whirl rotation and the rotation axis. The flow around the 3 rolls runs in potential circle whirls with the xy, xz or yz circle about the axis z,y,or x. Both SI and WI rotors use 3 quarks as with energy/color charge whirl vectors attached or as rolls.
In this geometry another use of the n-edge Dn can be explained. Take as homogeneous normal form 2 intersecting lines $x^2 - y^2 = 0$, close the lines at projective infinity by 2 points. The lemniscate of D2 is obtained. Its singular point can bifurcate to two disjoint circles for Cooper pairs. If the 2 points of D2 are identified, 3 circles arise with a common singular point where they touch. It can split and the 3 WI xy, xz, yz circles pairwise orthogonal positions. The weak isospin exchange occurs on the 3 axes intervals between paired u-, d-quarks of deuteron. The 3 circles occur also by adding to the 3 radii lines of D3 to its 3 points on the circle points at infinity.
As geometrical figures for quarks an interval (handle) for a gluon on its side and with the 2 quarks as endpoints can be drawn (figure 9). For nucleons the quark triangle is drawn. 3 circles as retracts of 3-dimensional Hopf handles (figure 3, a genus 3 brezel) can be attached at the vertices. For EM charged leptons, the Hopf sphere S^2 the 3 points, vectors or circles are located (figure 10) as equator on which a point charge (EM for instance) rotates in cw or mpo direction. A magnetic momentum μ is attached at the S^2 north pole having with the spin vector this pole as initial point. The Hopf fiber S^1, blowing up points on S^2 to the S^3 leaning circles is described by the Hopf map which uses the 3 Pauli matrices. The central circle of the Hopf S^3 tori is blown up form the S^2 south pole (figure 6 right, left: the S^3 version applies for observing). On S^2 the direction of μ changes (see above he gyromagnetic relation) with mpo for an opposite direction and for cw with equal direction. At the stereographic south pole on S^2 sits a mass charge set by an higgs field. Another S^2 is drawn for the helicity of neutral leptons where μ is replaced by the momentum vector (figure 6).
From n = 3 on the Dn groups are noncommutative. Mostly the reflections P and rotations Q are composed with $PQ \neq QP$. The symmetry axes are for n = 2k diametrical, for n = 2k +1 from a vertex to the midpoint of the opposite side (barycentrical).
D4 has 4 poles or coorolls. The former handle geometry suggests that a tetrahedron is drawn as triangulation of a S^2 (figure 9). At the 4 vertices 4 handles can be attached for a Heegard/Hopf genus 4 brezel. As 4 cooroll mill its rolls for magnetism MG, electrical charge EM, momentum PM and GR drive a flow (figure 17}. MG is a whirl (figure 3), EM a roll, PM is a vector for the motion (speed, world line) of the system in space and GR is a mass barycenter with a Schwarzschild radius ball in the center of the system (figure 8). There are 3 Bohr radii for the system, for this ball, another concentric sphere bounding its inner energies as grid in spacetime and an environmental energy sphere as for D6 described now. For D4 two SI polar caps are missing. MG + GR rolls can be for a combined magnetic-gravitational field similar as the EM + GR field of Schmutzer. EM + PM is for electrical currents. The D4 symmetry G8 of order 8 can be magnetic interpreted (figure 4): For EM charged systems, spin s and magnetic momentum μ have as common initial point, the

north pole of μ. The aligned orientation is up-up for s,μ and a + EM charge, up-down for a – EM charge (6 members of G8 for quarks, EM charged leptons, W+,W- bosons). As magnetic whirl quantums with a mpo or cw rotation on the cones circle ther are 2 members of G8 for left- or right handed screws and neutral charges, as known for neutral leptons with helicity (figure 6), but also for the weak Z0 boson.
D6 has 6 poles and coorolls as in the 6-roll mill (driving a potential flow (see [11]) with 3 motors POT, SI, WI, there rolls, also poles, are like obstacles about which a flow/fluid runs) and the hedgehog with 6 polar caps. To the rolls of the D4 cooroll mill are added the SI rolls heat and rotation, as octonian rolled ccordinates e2, e3. The D4 rolls are on e1, e4, e5, e6. The 6 coordinates are for a complex 3-dimensional SI space ([1] p. 32), coupled by the Heisenberg uncertainties (figure 8). The POT, SI motors run with the same speed, the WI motor has another speed. In order to get a common group speed for deuterons parts it is necessary that the mass of u-quarks is special relativistic speed v rescale, known as mass defect. Euclidean WI and barycentrical SI + GR coordinates are in special relativistic motion (figure 15, 4). The Minkowki watch (figure 17) shows an angle θ between 2 rays with sin θ = v/c for rescaling coordinate units by an orthogonal projection between the lines. In the tangent spaces for two systems in GR interaction a similar rescaling (figure 0, p.1) of radius and time differentials dr, dt occurs for the Schwarzschild nonlinear scaling of the Minkowski metric, using an angle β between the 2 rays with sin β = Rs/r. The metric is for a central sun or barycenter about which the second system is rotating with a speed between the first and second cosmic speed belonging to the mass of the barycenter. The D6 symmetry D3xZ2 of order 12 is for the fermionic series and the generator of Z2 can be the conjugate operator of physics.
The hedgehog drawn as octahedron (figure 9), the 8 sides can carry unit vectors of the octonians opposite to the cubes vertices in figure 9, noted by left L, right R, F front, H hidden, B bottom, top T, for instance in figure 17 as: RHB for e6/world lines/EMI 167, RFT for e0 GR/vectors 356, LFT for e3 E(rot)/orbits about an axis 347, RHT for e4 MG/whirls 145, LHB for e7 light/EMI/U(1)/waves/cylindrical coordinates (no Fano line, a GF for wave length λ, helix frequency ω, speed v = λω/2π), LFB for e1 EM/Euclidean coordinates 123, LHT for e5 GR/mass/particles/barycentrical coordinates 257, RFB for e2 heat/spherical coordinates 246. Octonian Fano lines are added (figure 18). Notice that SU(3) GellMann matrices have 3 GF triples like the 123 Pauli spin matrices, one for *rgb*-graviton whirls. Another one may be for light, and one for Higgs bosons with a Horn torus geometry.

Conclusions

The Tool Bag has a rich supply how geometries, symmetries, subspaces and local geometric forms and geometries of the octonians occur. The GF triples show other measurements to be used and many symmetries are added to the classical ones. With these additions the model for a first set of deuteron states is presenting a unification of the four basic interactions of physics. It was mentiones that WI properties are mostl missing here. This needs a second Tool Bag to be produced. Already the figures for the S^3 decays in form of its Heegard decompositions provide new geometrical models, also the wheel dynamics and geometry for generating Euclidean coordinates, the WI isospin exchange where a proton and neutron couple their quarks, the use of two speeds for the WI motor and the common speed of the POT, SI motors which give rise to a special relativistic coordinate rescaling. Single u-quarks are heavier in measurement than the mass of the nuleons allow. The inclusion of this Minkowski metric general relaticitic rescaling in a tangent space with differentials as coordinates is not fully discussed. This part of model is not in the Tool Bag.

The index of the handbook is adding to a word marked in blue additional informations which can be obtained through internet articles appearing on the screen of a computer. Click on the word. The index is also on the DVD included in the Tool Bag which contains also the videos. We hope for a good response, suggestions what can be made or explained better, what has to be corrected and what can be added to this new model. To have a good view to the nuclear range may help to handle better our macroscopic applications and our energy consumations.

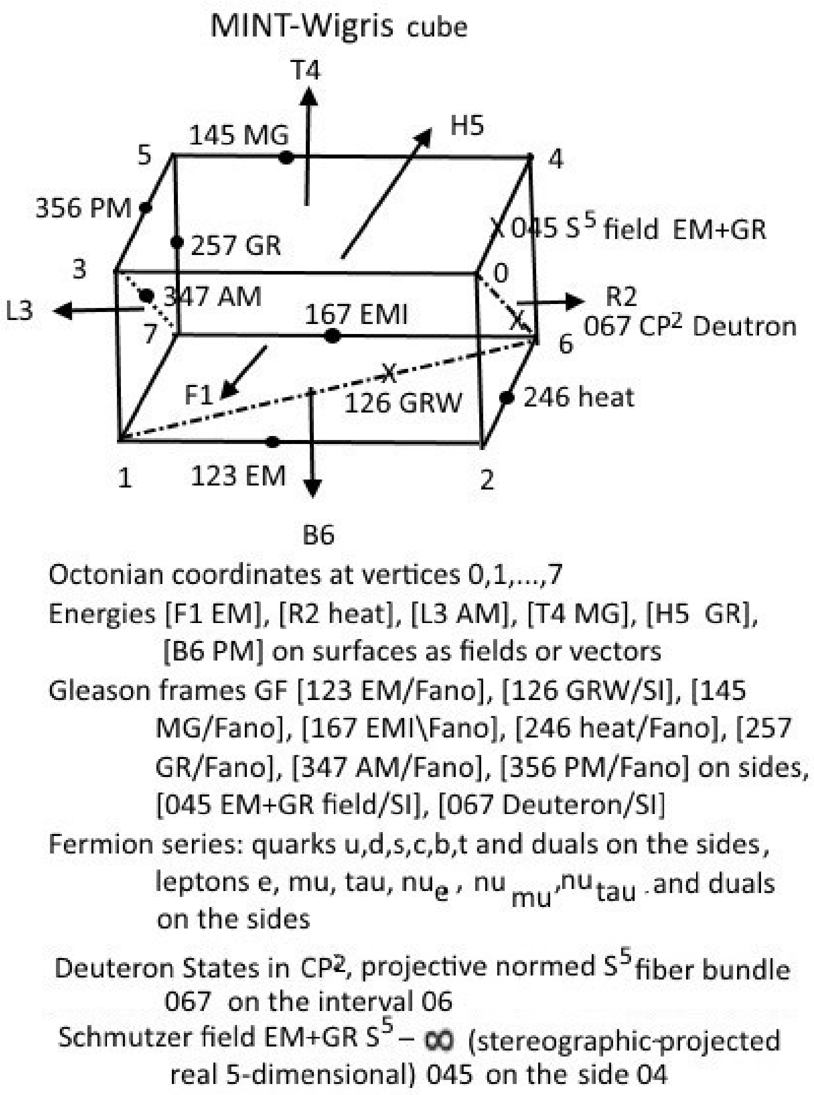


Figure 17 Memo for coordinates, energies, GF, fermions, deuteron, EM+GR field

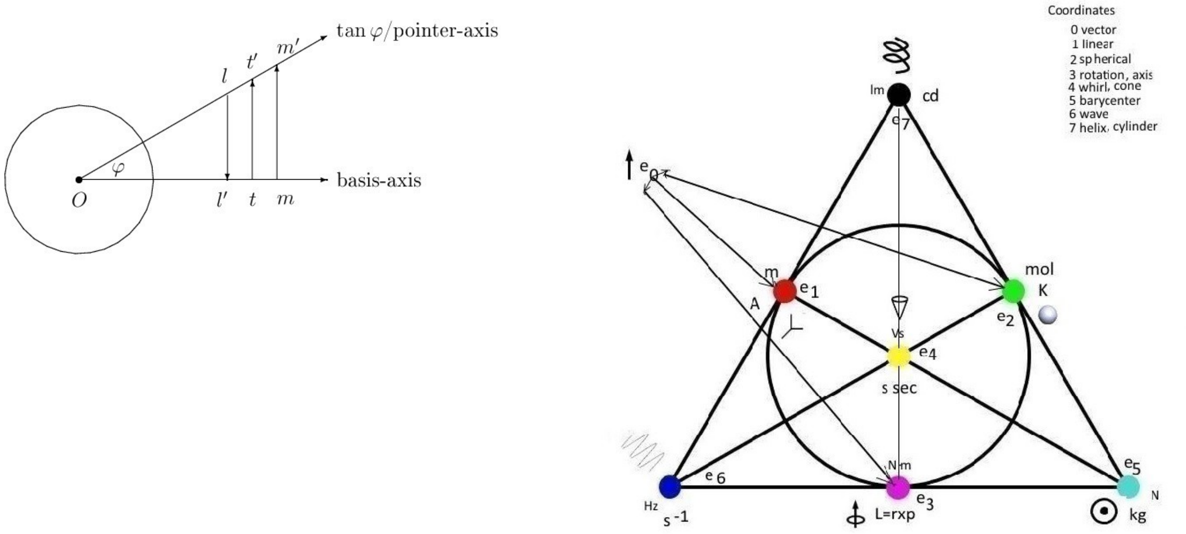


Figure 18 Minkowski watch as orthogonal projection for coordinate measures (length, time, mass), an e0 vectorial energy input at left and seven Fano lines measures as octonian GF triples

References

[1] G. Kalmbach H.E., MINT-WIGRIS, MINT Verlag, Bad Woerishofen, 2017, available under: https://www.scholars-press.com

[2] Many related MINT articles and parts of books in the 104 items long list of the authors (mostly scientific) publications. Often they are stored in the internet and can be downloaded or are available in the authors MINT (Mathematik, Informatik, Naturwissenschaften, Technik) book editions, vol. 1-38, 1997-2018, - the Deutsche Nationalbibliothek Frankfurt/M.

[3] G. Kalmbach H.E., Deuteron States, Nessa Journal of Physics, 2017, vol. 1, issue 2, pp. 1-17

[4] G. Kalmbach and R. Schweizer, Diskrete Mathematik, Vieweg/Springer, Wiesbaden, 1988 and Mathematik – bunt gemischt, Becker Velten, 1996

[5] G. Kalmbach, Quantum Measures and Spaces, Kluwer, Dordrecht, 1998

[6] G. Kalmbach, Orthomodular Lattices, Academic Press, London, 1983

[7] G. Kalmbach with U. Eberspaecher, MINT-Wigris Tool Bag, with a handbook, Bad Woerishofen, 2019

[8] Angular momentum operator - WikiVisually.html 2019

[9] Hering/Martin/Stohrer, Physikalisch-technisches Handbuch, VDI, Duesseldorf, 1995

[10] Internet video under YouTube: Moebius Transformations Revealed 2014; internet articles obtained by clicking on a blue word in the index of the tool bag for general informations

[11] T. Poston and I. Stewart, Catastrophe theory and its applications, Pitman, London, 1978

[12] E. Schmutzer, Projektive einheitliche Feldtheorie, Harry Deutsch, Frankfurt, 2004, (the unified electrical and gravity fields is here 5-dimensional with three 4-dimensional projections similar to the above octonians coordintes in [1] into spacetime (e1,e2,e3,e4), the deuteron space (e2,e3,e5,e6) and a scalar field (e1,e4,e5,e6).)

[13] K. Stierstadt, Physik der Materie, VCH, Weinheim, 1989

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