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Research Paper

Modeling and Analysis of the Interaction of Neutral Populations and Drug and Gang Populations: A Competing Species Model

A. Kazmierczak

T. H. E. Institute

1111 E. Brooks St Norman, OK 73071

Corresponding author: * A. Kazmierczak
Tel.: ----- Email: akazmierczak1949@gmail.com

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ABSTARCT

The Previous work has modeled interaction between neutral citizens and one other population, the drug population, the gang population or the violent police population. Though interesting, these models do not reflect reality, neutral citizens must interact with all three other populations. This paper addresses the interaction between neutral populations and drug, and gang populations. The formulation is based on models of interactions between competing species type dynamics. An exploration of the long-term dynamics and stability of homogeneous equilibrium solutions and their stability is given.

KEYWORDS

Drugs, gangs, competing species model, equilibrium solutions, stability at equilibrium solutions.

Mathematica subject classification: 62J12, 62G99

Computing Classification: I.4

1. INTRODUCTION

Drugs and gangs are not new phenomenon. However, there is a marked and exponential increase in the growth of drug and gang populations. These populations wreak havoc to native citizens. These groups affect political and social policies and create serious issues. Consequently, countries are faced with extremely difficult, complex, and contentious political and social decisions on the issues of drugs and gangs.

The acceptance of drug and gangs provides a tolerance for more drugs and gangs. Hence, countries face the possibility of ever increasing drugs and gangs. Despite these impending threats, there is not much literature that takes a dynamical systems approach to understanding the spread of these populations. Our primary objective is to bridge the gap.

In our framework, we let C represent the level of police violence. This police violence denoted by C: C can be viewed as the amount of police violence in a community. The drug population is denoted by D and the gang population by G. This paper is a first step in providing a mathematical modeling framework to study the evolution and interaction between these neutral populations and growing, drugs and gangs. The neutral population is modeled by standard population growth models and is represented by N.

The paper is organized as follows. In section 2, we develop and analyze the time-dependent autonomous police violence ordinary differential equation (ODE) model. We examine the equilibrium solutions, the stability of the equilibrium solutions and investigate the dynamics numerically. In section 2, we consider the situation of the current level of police violence in the system. We examine the equilibrium solutions, the stability of the equilibrium solutions and investigate the dynamics numerically for this situation also. In section 3, we consider the situation where the one population grows. Section 4 addresses the scenario when one population declines. Section 5 addresses the scenario when both populations increase. Section 6 addresses the scenario when both drug and gang populations decrease. In part 7 we present our conclusions based on the analysis in the previous sections.

2. Neutral Drug Gang (NDG) ODE Model

Consider the mathematical model;

$N = a_1N/(1+d_1G) - a_{NR}GN/(1+d_2N) + a_1N/(1+d_1D) - a_{NR}DN/(1+d_2N) - b_1N^2 = 0$

$G = a_2G/(1+d_3N) - a_{nr}GN/(1+d_2N) + a_2G/(1+d_4D) - a_{nr}GD/(1+d_4D) - b_2G^2 = 0$

$D = a_2D/(1+d_3N) - a_{nr}DN/(1+d_2N) + a_2D/(1+d_4G) - a_{nr}GD/(1+d_4G) - b_2D^2 = 0$

The populations N(t) represents the neutral population, D(t) represents the drug population and G(t) represents the gang population.

The parameters are all assumed to be positive and their descriptions are given in Table 1a.

Symbols	**Meaning**
a_1	Growth rate of the D, G, population
a_2	Growth rate of the Neutral population
b_1	Population loss in N due to intra-species competition and natural mortality
b_2	Population loss in D, G due to intra-species competition and natural mortality
a_{NR}	Maximum per capita loss in N due to D, G
d_1	Measures the effectiveness of D, G, in disrupting the growth rate of N
d_2	Measures the resilience of N, to strategies by G,D,
d_3	Measures the effectiveness of N in disrupting D, G
d_4	Measures the resilience of G, D, to strategies by N and other D, G

Table 1a: List of parameters used in the differential equation model

In the case of $d_i = b_i = 0$, the mathematical model becomes similar to the competing species model. The parameters d_i influence the carrying capacity of the individual populations. Or instance, if $d_1 >> 1$ then the growth rate of C is reduced. This is interpreted as violent police population, which can greatly hinder the growth rate of N. Notice, that if $d_2 >> 1$ then the recruitment by C is small, Also, if $d_3 >> 1$, new members are introduced into the police population more slowly The values chosen for the variables in this model are listed in Table 1b.

Table1b: Values of parameters

a_1	a_2	a_{nr}	b_1	b_2	d_1	d_2	d_3	d_4
2	2	2	0.5	0.5	2	2	2	2

2.1 Neutral Drug, Gang, (NDG) ODE Model

Consider the mathematical model

$$N = a_1N/(1+d_1G) - a_{NR}GN/(1+d_2N) + a_1N/(1+d_1D) - a_{NR}DN/(1+d_2N) - b_1N^2 = 0$$

$$G = a_2G/(1+d_3N) - a_{nr}GN/(1+d_2N) + a_2G/(1+d_4D) - a_{nr}GD/(1+d_4D) - b_2G^2 = 0$$

$$D = a_2D/(1+d_3N) - a_{nr}DN/(1+d_2N) + a_2D/(1+d_4G) - a_{nr}GD/(1+d_4G) - b_2D^2 = 0$$

Since this system is nonlinear, the first step is linearization using the Jacobian.

The Jacobian for this system is defined as

$$J = \begin{vmatrix} \partial N/\partial N & \partial N/\partial G & \partial N/\partial D \\ \partial G/\partial N & \partial G/\partial G & \partial G/\partial D \\ \partial D/\partial N & \partial D/\partial G & \partial D/\partial D \end{vmatrix}$$

The partial derivatives are:

$$\partial N/\partial N = a_1/(1+d_1G) - (a_{nr}G(1+d_2N) - a_{nr}d_2GN)/(1+d_2N)^2 + a_1/(1+d_1D) - (a_{nr}D(1+d_2N) - a_{nr}d_2DN)/(1+d_2N)^2 - 2b_1N$$

$$\partial N/\partial G = -a_1d_1N/(1+d_1G)^2 - a_{nr}N/(1+d_2N)$$

$$\partial N/\partial D = -a_1d_1N/(1+d_1D)^2 - a_{nr}N/(1+d_2N)$$

$$\partial G/\partial N = -a_2d_3G/(1+d_2N)^2 - (a_{nr}G(1+d_2N) - a_{nr}d_2GN)/(1+d_2N)^2$$

$$\partial G/\partial G = a_2/(1+d_3N) - a_{nr}N/(1+d_2N) + a_2/(1+d_4D) - a_{nr}D/(1+d_4D) - 2b_2G$$

$$\partial G/\partial D = -a_2d_4G/(1+d_4D)^2 - (a_{nr}G(1+d_4D) - a_{nr}d_4GD)/(1+d_4D)^2$$

$$\partial D/\partial N = -a_2d_4D/(1+d_1N)^2 - (a_{nr}D(1+d_2N) - a_{nr}d_2DN)/(1+d_2N)^2$$

$$\partial D/\partial G = -a_2d_4/(1+d_4G) - (a_{nr}D(1+d_4G) - a_{nr}d_4DG)/(1+d_4G)^2$$

$\partial D/\partial D = a_2/(1+d_3N) - a_{nr}N/(1+d_2N) - a_{nr}GN/(1+d_4G) - 2b_2D$

Using these partial derivatives, the Jacobian becomes:

J =

| $a_1/(1+d_1G) - (a_{nr}G(1+d_2N) - a_{nr}d_2GN)/(1+d_2N)^2 + a_1/(1+d_1D) - (a_{nr}D(1+d_2N) - a_{nr}d_2D)/(1+d_2N)^2 - 2b_1N$ |

| $-a_1d_1N/(1+d_1G)^2 - a_{nr}N/(1+d_2N)$ |

| $-a_2d_4G/(1+d_4D)^2 - (a_{nr}N/(1+d_2N)$ |

| $-a_2d_3G/(1+d_2N)^2 - (a_{nr}G(1+d_2N) - a_{nr}d_2GN)/(1+d_2N)^2$ |

| $a_2/(1+d_3N) - a_{nr}N/(1+d_2N) + a_2/(1+d_4D) - a_{nr}D/(1+d_4D) - 2b_2G$ |

| $-a_2d_4G/(1+d_4D)^2 - (a_{nr}G(1+d_4D) - a_{nr}d_4GD)/(1+d_4D)^2$ |

| $-a_2d_4D/(1+d_3N)^2 - (a_{nr}D(1+d_2N) - a_{nr}d_2DN)/(1+d_2N)^2$ |

| $-a_2d_4/(1+d_4G) - (a_{nr}D(1+d_4G) - a_{nr}D/(1+d_4G) - 2b_2G$ |

| $a_2/(1+d_3N) - a_{nr}N/(1+d_2N) - a_{nr}GN/(1+d_4G) - 2b_2D$ |

2.2 Equilibrium Points

Using the Maple CAS the following real valued equilibrium points were obtained:

{D=0.,G=2.886000936,N=2.886000936},
{D=0.,G=-1.386000936,N=-1.386000936},
{D=1.500000000,G=3.500000000,N=0.},
{D=0.,G=3.179449472,N=2.043559577},
{D=0.,G=-1.179449472,N=-1.443559577},

D=0.6663492972,G=0.8750730641,N=0.8026799460},
{D=0.2653920796,G=0.5682408219,N=4.124768937},
{D=-7.158932779,G=-3.780423019,N=3.976946247},
{D=-0.5973136278,G=-0.7089388654,N=-0.4120730198},
{D=-7.916697282,G=-4.982715615,N=-5.688310577},

{D=0.,G=0.,N=0.},
{D=-0.8000000000,G=-8.,N=0.},
{D=0.,G=0.,N=8.},
{D=0.,G=8.,N=0.},
{D=12.,G=0.,N=0.}

2.3 Analyzing equilibrium points for stability

In this section we use the equilibrium points to generate the eigen values for the system and establish whether the equilibrium point is stable or unstable.

Table 2 summarizes the results for the current population levels.

Table 2 – Results for Current Population Levels

Equilibrium Points	Eigenvalues	Node Type	Stability
{D=0., G=2.886000936, N=2.886000936},	-.981729408472435, 1.34996041158318, -6.64609108611074	Saddle	Unstable
{D=0., G=-1.386000936, N=-1.386000936},	3.37228605902377+2.02684604974168*I, 3.37228605902377-2.02684604974168*I, -17.7097675990475	Saddle	Unstable
{D=1.50, G=3.50, N=0.},	-1.94060148566472, -6.57050962533528, -9.250	Attracting	Stable
{D=0., G=3.179449472, N=2.043559577},	-.480739598545902, 1.55203019982847, -6.91802482728257	Saddle	Unstable
{D=0., G=-1.179449472, N=-1.443559577},	3.08296676859391+1.30701987585797*I, 3.08296676859391-1.30701987585797*I, -29.7025326141878	Saddle	Unstable
D=0.6663492972, G=0.8750730641, N=0.8026799460},	0.581177228598734, -3.74240958142869, -.751071867170040	Saddle	Unstable
{D=0.2653920796, G=0.5682408219, N=4.124768937},	-2.07979473553723, -3.13019263056733, 0.402667332504560	Saddle	Unstable
{D=-7.158932779, G=-3.780423019, N=3.976946247},	-3.95645432641909, 2.04685291960921, -1.17483211519011	Saddle	Unstable
{D=-0.5973136278, G=-0.7089388654, N=-0.4120730198},	55.2162666499610+46.4182336071202*I, 55.2162666499610-46.4182336071202*I, -12.2473811979218	Saddle	Unstable
{D=-7.916697282, G=-4.982715615, N=-5.688310577},	5.62895884086411, 2.77966693558065, 0.285396884555236	Repelling	Unstable
{D=0., G=0., N=0.},	4., 2., 0	Repelling	Unstable
{D=-0.80, G=-8., N=0.},	11.1142857145000+22.2747042383162*I, 11.1142857145000-22.2747042383162*I, 14.133333340	Repelling	Unstable

{D=0., G=0., N=8.},	-4., -.82352941180, . 941176470600000	Saddle	Unstable
{D=0., G=8., N=0.},	1.99999999940984, -11.5555555554098, -13.882352940	Saddle	Unstable
{D=12., G=0., N=0.}	-10., -21.920, -3.6923076920	Attracting	Stable

In short, of the fifteen equilibrium points, only three are attracting nodes, nine are saddle points and three are repelling point. The overall system is more unstable than stable.

3.0 Growth of a Single Population

In this section, we consider the situation where there is a 25% increase in one of the populations, arbitrarily the drug population. The gang and violent police populations remain as they are. The mathematical model now becomes.

$$N = a_1N/(1+d_1G) - a_{NR}GN/(1+d_2N) + a_1N/(1+d_1D(1.25)) - a_{NR}D(1.25)N/(1+d_2N) - b_1N^2 = 0$$

$$G = a_2G/(1+d_3N) - a_{nr}GN/(1+d_2N) + a_2G/(1+d_4D(1.25)) - a_{nr}GD(1.25)/(1+d_4D) - b_2G^2 = 0$$

$$D = a_2D(1.25)/(1+d_3N) - a_{nr}D(1.25)N/(1+d_2N) + a_2D(1.25)/(1+d_4G) - a_{nr}GD(1.25)/(1+d_4G) - b_{2(}(1.25D)^2 = 0$$

3.1 Equilibrium Points

Using the Maple CAS) we obtained the following real valued equilibrium points:

{D=0.,G=0.,N=0.},
{D=-0.8000000000,G=-8.,N=0.},
{D=1.500000000,G=3.500000000,N=0.},
{D=0.,G=8.,N=0.},
{D=0.,G=3.179449472,N=2.043559577},

{D=0.,G=-1.179449472,N=-1.443559577},
{D=0.6663492972,G=0.8750730641,N=0.8026799460},
{D=0.2653920796,G=0.5682408219,N=4.124768937},
{D=-7.158932779,G=-3.780423019,N=3.976946247},
{D=-0.5973136278,G=-0.7089388654,N=-0.4120730198},

{D=-7.916697282,G=-4.982715615,N=-5.688310577},
{D=0.,G=2.886000936,N=2.886000936},

{D=0.,G=-1.386000936,N=-1.386000936},
{D=12.,G=0.,N=0.},
{D=0.,G=0.,N=8.}

3.2 Analyzing equilibrium points for stability

In this section we use the equilibrium points to generate the eigenvalues for the system and establish whether the equilibrium point is stable or unstable. The real valued equilibrium points are:

Table 3 summarizes the results for an increased drug population.

Table 3 – Results for Increased Drug population

Equilibrium Points	Eigenvalues	Node Type	Stability
{D=0., G=0., N=0.},	4., 2., 0.	Repelling	Unstable
{D=-0.80, G=-8., N=0.},	.100000000350+12.0842559535239*I, .100000000350-12.0842559535239*I, 15.866666670	Repelling	Unstable
{D=1.50, G=3.50, N=0.},	-1.31464835938447, -6.24785164061553, -10.078947370	Attracting	Stable
{D=0., G=8., N=0.},	5.43983947615177, -13.3221924173518, -13.882352940	Saddle	Unstable
{D=0., G=3.179449472, N=2.043559577},	-.169877692829532, 3.59421823866899, -7.95635883283946	Saddle	Unstable
6. {D=0., G=-1.179449472, N=-1.443559577},	3.13073927808989+2.05283423504000*I, 3.13073927808989-2.05283423504000*I, -6.97946026817977	Saddle	Unstable
{D=0.6663492972, G=0.8750730641, N=0.8026799460},	.411562866999724, -3.42002370806810, -.889904263931623	Saddle	Unstable
{D=0.2653920796, G=0.5682408219, N=4.124768937},	-2.21241416611010, -3.14050695678477, 0.470989905994870	Saddle	Unstable
{D=-7.158932779, G=-3.780423019, N=3.976946247},	-3.94420363081456, 5.96617904173929, 1.04900517407528	Saddle	Unstable
{D=-0.5973136278, G=-0.7089388654, N=-0.4120730198},	57.2484117865965+55.0509625075048*I, 57.2484117865965-55.0509625075048*I, -42.6939544431929	Saddle	Unstable
{D=-7.916697282, G=-4.982715615, N=-5.688310577},	5.46891153152769, 6.56009854801399, 2.59982159645832	Saddle	Unstable

{D=0., G=2.886000936, N=2.886000936},	-.820977358652597, 3.42247226876414, -7.61335717811155	Saddle	Unstable
{D=0., G=-1.386000936, N=-1.386000936},	3.35180536646949+2.45142427441773*I, 3.35180536646949-2.45142427441773*I, -6.43480402193897	Saddle	Unstable
{D=12., G=0., N=0.},	-13., -27.935483870, -3.120	Attracting	Stable
{D=0., G=0., N=8.}	-4., -.82352941180, 0.94117647060	Saddle	Unstable

In short, there are fifteen equilibrium points. Of these, points two are attracting, nine are saddle points and two repelling node. An increase of one population does have some negative effect on the overall stability of the system.

4.0 Decrease in the Drug Population

In this section, we consider the situation where the drug population is reduced by 25%. The mathematical model now becomes:

$$N = a_1N/(1+d_1G) - a_{NR}GN/(1+d_2N) + a_1N/(1+d_1D(0.75)) - a_{NR}D(0.75)N/(1+d_2N) - b_1N^2 = 0$$

$$G = a_2G/(1+d_3N) - a_{nr}GN/(1+d_2N) + a_2G/(1+d_4D(0.75)) - a_{nr}GD(0.75)/(1+d_4D) - b_2G^2 = 0$$

$$D = a_2D(0.75)/(1+d_3N) - a_{nr}D(0.75)N/(1+d_2N) + a_2D(0.75)/(1+d_4G) - a_{nr}GD(0.75)/(1+d_4G) - b_{2(}(0.75D)^2 = 0$$

4.1 Equilibrium Points

Using the Maple CAS obtained the following real valued equilibrium points::

{D=0.,G=2.886000936,N=2.886000936},

{D=0.,G=-1.386000936,N=-1.386000936},

{D=0.9681454523,G=0.8001862195,N=0.7747395738},

{D=0.3573969791,G=0.5501984868,N=4.157002013},

{D=-7.028627381,G=-3.884660968,N=3.531846117},

{D=-0.7975871716,G=-0.7094899737,N=-0.4113564284},

{D=-7.901889872,G=-5.169923083,N=-5.361668229},

{D=12.,G=0.,N=0.},

{D=0.,G=0.,N=8.},

{D=0.,G=8.,N=0.},

{D=-1.200000000,G=-5.500000000,N=0.},
{D=0.,G=0.,N=0.},
{D=0.,G=3.179449472,N=2.043559577},
{D=0.,G=-1.179449472,N=-1.443559577},
{D=1.333333333,G=4.,N=0.}

4.2 Analyzing Equilibrium Points for Stability

In this section we use the equilibrium points to generate the eigenvalues for the system and establish whether the equilibrium point is stable or unstable.

Table 4 summarizes the results for a decreased drug population.

Table 4 – Results for Decreased Drug Population

Equilibrium Points	Eigenvalues	Node Type	Stability
{D=0., G=2.886000936, N=2.886000936},	-.825405575926250, 3.43324546378208, -7.61970215585583	Saddle	Unstable
{D=0., G=-1.386000936, N=-1.386000936},	5.26260928836890, 1.50827854273281, -6.50208112010172	Saddle	Unstable
{D=0.9681454523, G=0.8001862195, N=0.7747395738},	.657624268165402, -3.34884162076320, -.754599161202198	Saddle	Unstable
{D=0.3573969791, G=0.5501984868, N=4.157002013},	-1.97219696463732, -3.18970763501768, 0.442617993255002	Saddle	Unstable
{D=-7.028627381, G=-3.884660968, N=3.531846117},	-3.48238470337450, 2.43594781393668, 1.43928349743782	Saddle	Unstable
{D=-0.7975871716, G=-0.7094899737, N=-0.4113564284},	290.248459874468, -173.878031421148, -49.2617418323199	Saddle	Unstable
{D=-7.901889872, G=-5.169923083, N=-5.361668229},	5.35997912664614, 1.96913784528716, 3.60961713406670	Repelling	Unstable
{D=12., G=0., N=0.},	-7., -15.894736840, -2.640	Attracting	Stable
{D=0., G=0., N=8.},	-4., -.82352941180, 0.94117647060	Saddle	Unstable
{D=0., G=8., N=0.},	5.43983947615177, -13.3221924173518, -13.882352940	Saddle	Unstable

{D=-1.20, G=-5.50, N=0.},	.742857142500000+2.26834624848752*I, .742857142500000-2.26834624848752*I, 10.10	Repelling	Unstable
{D=0., G=0., N=0.},	4., 2., 0.	Repelling	Unstable
{D=0., G=3.179449472, N=2.043559577},	-.176951238268382, 3.60813329531591, -7.96320034404753	Saddle	Unstable
{D=0., G=-1.179449472, N=-1.443559577},	6.45333202030004, -0.0308354923670802, -7.14047823993296	Saddle	Unstable
{D=1.333333333, G=4., N=0.}	-.312571131860554, -6.46520664573945, -9.1111111110	Attracting	Stable

In short, of the fifteen equilibrium points, two are attracting points, eleven are saddle points and three are repelling point. A decline in one population has little effect on the stability of the entire system.

3 Growth of Two Populations

In this section, we consider the situation where there is a 25% increase in two of the populations, arbitrarily the drug and gang populations. The mathematical model now becomes

$$N = a_1N/(1+d_1G(1.25)) - a_{NR}G(1.25)N/(1+d_2N) + a_1N/(1+d_1D(1.25)) - a_{NR}D(1.25)N/(1+d_2N) - b_1N^2 = 0$$

$$G = a_2G(1.25)/(1+d_3N) - a_{nr}G(1.25)N/(1+d_2N) + a_2G(1.25)/(1+d_4D(1.25)) - a_{nr}G(1.25)D(1.25)/(1+d_4D) - b_2(1.25G)^2 = 0$$

$$D = a_2D(1.25)/(1+d_3N) - a_{nr}D(1.25)N/(1+d_2N) + a_2D(1.25)/(1+d_4G(1.25)) - a_{nr}G(1.25)D(1.25)/(1+d_4G(1.25)) - b_{2(}(1.25D)^2 = 0$$

5.1 Equilibrium Points

Using the Maple CAS) we obtained the following real valued equilibrium points:

{D=-6.825508444,G=-3.778104588,N=4.538572709},
{D=0.5514238389,G=0.6965354493,N=0.8799445614},
{D=0.2322891399,G=0.4624443307,N=3.999690490},
{D=-0.4770701941,G=-0.5652193108,N=-0.4132326659},
{D=-7.608072302,G=-4.866322713,N=-6.124690515},

{D=0.,G=8.,N=0.},
{D=0.,G=-1.325514418,N=-1.402116071},
{D=0.,G=-0.8549725776,N=-1.550798184},

{D=0.,G=0.,N=0.},
{D=1.243398113,G=3.460388679,N=0.},
{D=-0.6433981132,G=-7.860388679,N=0.},
{D=0.,G=0.,N=8.},
{D=12.,G=0.,N=0.}

5.2 Analyzing Equilibrium Points for Stability

In this section we use the equilibrium points to generate the eigenvalues for the system and establish whether the equilibrium point is stable or unstable. The real valued equilibrium points are:

Table 5 summarizes the results for an increased drug and gang populations

Table 5 – Results for Increased Drug and Gang Population

Equilibrium Points	Eigenvalues	Node Type	Stability
{D=-6.825508444, G=-3.778104588, N=4.538572709},	-4.52805533750428, 5.59706756242742, 2.00140884107686	Saddle	Unstable
{D=0.5514238389, G=0.6965354493, N=0.8799445614},	.516293167327847, -3.46250764877014, -.583100327957707	Saddle	
{D=0.2322891399, G=0.4624443307, N=3.999690490},	-1.94335618418178, -3.28193666699896, .647482654780733	Saddle	
{D=-0.4770701941, G=-0.5652193108, N=-0.4132326659},	-11.6146830136172+45.8488230351971*I, -11.6146830136172-45.8488230351971*I, 168.552590267235	Attracting	Stable
{D=-7.608072302, G=-4.866322713, N=-6.124690515},	5.72668363788299, 6.42725692719325, 3.68455453992376	Repelling	Unstable
{D=0., G=8., N=0.},	5.68235015264247, -15.5871120576425, -17.90476190	Saddle	
{D=0., G=-1.325514418, N=-1.402116071},	3.49298783266368+2.28046422070449*I, 3.49298783266368-2.28046422070449*I, -5.74625878232737	Saddle	
{D=0., G=-0.8549725776, N=-1.550798184},	2.17247903196303, 3.74977479942698, -7.28715062539000	Saddle	
{D=0., G=0. ,N=0.},	4., 2., 0	Repelling	Unstable
{D=1.243398113, G=3.460388679, N=0.},	-.890921838447044, -7.09949089655296, -11.065437730	Attracting	Stable
{D=-0.6433981132,	-2.02979363970000+29.0489518771471*I,	Saddle	

G=-7.860388679, N=0.},	-2.02979363970000-29.0489518771471*I, 17.865437730		
{D=0., G=0., N=8.},	-4., -.82352941180, .94117647060	Saddle	
{D=12., G=0., N=0.}	-13., -27.935483870, -3.120	Attracting	Stable

In short, of the thirteen equilibrium points, three are attracting points, eight are saddle points, and two are repelling. An increase of two populations again does not affect the overall stability of the system.

6.0 Decline in Two populations

In this section, we consider the situation where the drug and gang populations are reduced by 25%. The mathematical model now becomes:

$N = a_1N/(1+d_1G(0.75)) – a_{NR}G(0.75)N/(1+d_2N) + a_1N/(1+d_1D(0.75))– a_{NR}D(0.75)N/(1+d_2N)- b_1N^2 = 0$

$G = a_2G(0.75)/(1+d_3N) - a_{nr}G(0.75)N/(1+d_2N) + a_2G(0.75)/(1+d_4D(0.75)) – a_{nr}G(0.75)D(0.755)/(1+d_4D(0.75)) – b_2(0.75G)^2 = 0$

$D = a_2D(0.75)/(1+d_3N) - a_{nr}D(0.75)N/(1+d_2N) + a_2D(0.75)/(1+d_4G(0.75)) – a_{nr}G(0.75)D(0.75)/(1+d_4G(0.75)) – b_{2(}(0.75D)^2 = 0$

6.1 Equilibrium Points

Using the Maple CAS obtained the following real valued equilibrium points:

{D=0.,G=0.,N=0.},
{D=12.,G=0.,N=0.},
{D=0.,G=3.805141256,N=1.161919803},
{D=0.,G=2.685795468,N=3.874482104},
{D=0.8281937217,G=1.186053465,N=0.6951151836},

{D=0.3046397579,G=0.7414476255,N=4.313224807},
{D=-0.7986726153,G=-0.9508638111,N=-0.4100471378},
{D=-7.787004684,G=-3.775286081,N=3.314440188},
{D=-8.418036609,G=-5.149481362,N=-5.236017926},
{D=0.,G=0.,N=8.},

{D=0.,G=8.,N=0.},
{D=1.890983834,G=3.563935338,N=0.},
{D=-1.057650501,G=-8.230602004,N=0.}

6.2 Analyzing equilibrium points for stability

In this section we use the equilibrium points to generate the eigenvalues for the system and establish whether the equilibrium point is stable or unstable.

Table 6 summarizes the results for a decreased drug and gang populations.

Table 6 – Results for Decreased Drug and Gang Populations

Equilibrium Points	Eigenvalues	Node Type	Stability
{D=0., G=0., N=0.},	4., 2., 0	Repelling	Unstable
{D=12., G=0., N=0.},	-7., -15.894736840, -2.640	Attracting	Stable
{D=0., G=3.805141256, N=1.161919803},	.331996319418428, 3.80032971953449, -7.46669223795292	Saddle	Unstable
{D=0., G=2.685795468, N=3.874482104},	-1.62597244850920, 3.16211591697277, -6.45357052746357		
{D=0.8281937217, G=1.186053465, N=0.6951151836},	.671740800376438, -3.41301737917158, -.540114068804861		
{D=0.3046397579, G=0.7414476255, N=4.313224807},	-2.22408690336067, -3.10650884597160, .648980136232268		
{D=-0.7986726153, G=-0.9508638111, N=-0.4100471378},	51.1105877207415+36.4904858901933*I, 51.1105877207415-36.4904858901933*I, -37.1374895994829		
{D=-7.787004684, G=-3.775286081, N=3.314440188},	-3.33894851835633, 2.75727326627312, .515333939083215		
{D=-8.418036609, G=-5.149481362, N=-5.236017926},	5.28637924601138, 2.84454843356426, 1.60588211142437	Repelling	Unstable
{D=0., G=0., N=8.},	-4., - .82352941180, .94117647060		
{D=0., G=8., N=0.},	5.13000143299376, -10.9761552789938, -9.846153850		
{D=1.890983834, G=3.563935338, N=0.},	-.849616800923489, -5.10133152607651, -7.3459030070	Attracting	Stable

{D=-1.057650501, G=-8.230602004, N=0.}	.787106108400000+3.09564839890968*I, .787106108400000-3.09564839890968*I, 10.345903020	Repelling	Unstable

In short, of the thirteen equilibrium points, two are attracting, eight are saddles and three are repelling. The overall system remains unstable despite a reduction in two populations. Even a reduction in two populations has little if any effect on the overall stability.

7 Conclusions

As the tables show, the overall system is more unstable in the starting scenario. A reduction in one or both populations has little effect on the overall system stability. An increase in one or two populations also has little effect on the overall system stability. It would seem that the more populations in the model would yield results that a change in a small number of populations would have very limited impact on overall system stability.

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